Example 1:

Find all of the Zeros of \( p(x) = 15x^3 + 14x^2 - 3x - 2 \) and then use the Zeros to write the polynomial function as a product of linear factors.

STEP 1:

Find all potential rational Zeros:

the factors of the constant term \(-2\) are: \(\pm 1, \pm 2\)

the factors of the leading coefficient \(15\) are: \(\pm 1, \pm 3, \pm 5, \pm 15\)

Potential rational Zeros are, therefore, of the form

\(\pm \frac{1}{1}, \pm \frac{2}{1}, \pm \frac{1}{3}, \pm \frac{1}{5}, \pm \frac{2}{3}, \pm \frac{2}{5}, \pm \frac{1}{15}, \pm \frac{2}{15}\)

That is, we divided each factor of the constant term, in turn, by every factor of the leading coefficient. There are sixteen potential rational Zeros.

Please note that just because we can find potential rational Zeros, this does not mean that this polynomial function has actual rational Zeros!

STEP 2:

Next, we have to replace the variable in the polynomial with each of the sixteen possibilities. If one replacement results in \( P(x) = 0 \), we will have found an actual rational Zero of the polynomial function.

If your number replacements DO NOT result in \( P(x) = 0 \), you can discard this number replacement. It will NOT be a Zero of the polynomial function.

YES, we do have to plug ALL of the potential Zeros into the polynomial function until we find one that result in \( P(x) = 0 \). It is absolutely BEST to let your calculator do this job! To avoid confusion, it is suggested that you first try the rational possibilities from the largest to the smallest number.

Let’s use the calculator to see whether or not \( x = 2 \) is an actual rational Zero.

\[
15 \, (2) \, ^3 + 14 \, (2) \, ^2 - 3 \, (2) - 2 \, \text{ Enter}
\]

... the answer is 168 !!!

Since the variable replacement \( 2 \) DID NOT result in \( P(x) = 0 \), it is NOT an actual rational Zero and can be discarded.
.... after trying more potential rational Zeros from the largest to the smallest in the lengthy list in Step 1 above .... sigh .... we finally arrive at \( x = -1 \).

That is, we will find the answer to \( p(-1) = 15(-1)^3 + 14(-1)^2 - 3(-1) - 2 \)

\[
15 \ (-1)^3 + 14 \ (-1)^2 - 3 \ (-1) - 2 \ 
\]

... the answer is 0 !!!

Since the result is 0, the number -1 is an actual rational Zero of the polynomial function.

STEP 3:

Since -1 is an actual rational Zero, then \( x - (-1) = x + 1 \) is a factor of the polynomial function.

Next, we divide \( x + 1 \) into the polynomial to find another factor. We accomplish this by using polynomial long division.

\[
\begin{array}{c|ccccc}
 & 15x^2 & -x & -2 \\
\hline
x+1 & 15x^3 & +14x^2 & -3x & -2 \\
  & -(15x^2 + 15x^2) & & & \\
  & -15x^2 & -3x & & \\
  & -( -15x^2 - x) & & & \\
  & -2x & -2 & \\
  & -(-2x - 2) & & & \\
  & 0 & & & \\
\end{array}
\]

We call \( 15x^2 - x - 2 \) the quotient, and 0 the remainder. We already know that \( x + 1 \) is a factor of the function \( P \). Now we know another factor of the function, namely, the quotient \( 15x^2 - x - 2 \).

Incidentally, since both are factors of \( P \), we can write the polynomial as follows

\[ p(x) = 15x^3 + 14x^2 - 3x - 2 = (x + 1)(15x^2 - x - 2) \]

STEP 4:

We form a new equation by setting the quotient equal to 0:

\( 15x^2 - x - 2 = 0 \)

We will use factoring to find the remaining two Zeros

\( (5x - 2)(3x + 1) = 0 \)
Then \(3x + 1 = 0\) or \(5x - 2 = 0\)

and \(x = -\frac{1}{3}\) or \(x = \frac{2}{5}\)

The Zeros of the given polynomial function are \(-1, -\frac{1}{3},\) and \(\frac{2}{5}\). They are also the x-intercepts of the function.

Now we will use the Zeros to write the polynomial function as a product of linear factors. Note: "Linear factors" are of the form \(ax + b\), where \(a\) and \(b\) are constants.

Remember, if a number \(r\) is a Zero of the polynomial function, then \((x - r)\) is a linear factor of the function. Therefore, the polynomial can now be written as a product of linear factors as follows:

\[
p(x) = 15[x - (-1)][x - (-\frac{1}{3})][x - \frac{2}{5}]
\]

Please be aware that the polynomial in unfactored form (see original polynomial) has a leading coefficient of 15. It has to show up in the factored form also.

and finally, \(p(x) = 15(x + 1)(x + \frac{1}{3})(x - \frac{2}{5})\)

In this case, we might want to do the following to "eliminate" the fractions. Since \(15 = 3(5)\), we can multiply the 3 with the second factor and the 5 with the third factor as follows:

\[
p(x) = 3 \cdot 5(x + 1)(x + \frac{1}{3})(x - \frac{2}{5})
\]

Eliminating fractions is customarily attempted, whenever possible!

Please note that constant factors are NOT multiplied into each linear factor. Instead, YOU chose with which factor you want to multiply them. In this case, eliminating fractions is customarily attempted, whenever possible, therefore, we ALWAYS chose the factors with the fractions for multiplication purposes!