**Remark 1** Sometimes a radical expression is a multiple of another radical expression, but we can’t tell because the radical is not in a simplified form.

\[
\sqrt{50} = \sqrt{25 \cdot 2} = \sqrt{25} \cdot \sqrt{2} = 5 \cdot \sqrt{2}
\]

So, \(\sqrt{50} = 5\sqrt{2}\). This means that the value of \(\sqrt{50}\) is actually \(5\) times \(\sqrt{2}\)!

In this topic we will discuss how to breakdown radicals into their simplest form, which will help us recognize when two radicals are equivalent.

**Formula** Suppose \(a\) and \(b\) are nonnegative real numbers and \(n\) is a positive whole number then:

**Product Rule**  
(for radicals)

\[
\sqrt[n]{a \cdot b} = \sqrt[n]{a} \cdot \sqrt[n]{b}
\]

**Remark 2** Remember, radicals are like “reverse” exponents. So, many of the rules that apply to exponents also apply to radicals. For example, the formula above is very similar to one of the power rules for exponents.

Remember? \((a \cdot b)^m = a^m \cdot b^m\)

**Remark 3** Let’s see if we can apply our new rule for radicals.

\[
\sqrt{20} = \sqrt{2 \cdot 2 \cdot 5} = \sqrt{2 \cdot 2} \cdot \sqrt{5} = 2 \cdot \sqrt{5}
\]

Using a factor tree we see that \(20 = 2 \cdot 2 \cdot 5\). This means \(\sqrt{20} = \sqrt{2 \cdot 2 \cdot 5}\). According to the product rule we can break this square root apart.

\[
\sqrt{2 \cdot 2 \cdot 5} = \sqrt{2 \cdot 2} \cdot \sqrt{5}. \text{ But we know that a square root turns a pair of two identical factors into only one of those factors: } \sqrt{2 \cdot 2} = 2. \text{ All together we have:}
\]

\[
\sqrt{20} = \sqrt{2 \cdot 2 \cdot 5} = \sqrt{2 \cdot 2} \cdot \sqrt{5} = 2\sqrt{5}
\]

Let’s try to sum this process up in our main idea.
Main Idea

To simplify $n$th roots:

1. Use a factor tree to completely factor the radicand (the number “inside” the radical sign).
2. Eliminate each group of $n$ identical factors by pulling a single factor outside the radical sign.
   - The factor pulled out will be connected by multiplication.

Example 1

Completely simplify each radical.

- $a \sqrt{72}$
- $b \sqrt{432}$
- $c \sqrt{625}$
- $d \sqrt{30}$

Example 2

Completely simplify each radical.

- $a \sqrt[3]{54}$
- $b \sqrt[4]{48}$
- $c \sqrt[5]{-160}$