We’re going to solve mixture word problems with two equations (so we need two variables). One way to organize your information is to use a chart. Observe carefully. By the way, all the “math” will be left up to you so you have the opportunity to “do the math”. The answers for examples a-g appear before the homework.

**Part A: Chemistry Mixture Problems**

**a.** A chemist mixes some 70% solution with some 40% solution to obtain 120 gallons of 50% solution. Find the number of gallons of each solution.

Think: 

- x = gallons of 70% solution
- y = gallons of 40% solution

<table>
<thead>
<tr>
<th>Strength (in %)</th>
<th>How Many Gallons</th>
<th>Amount of Pure (% times how many gallons)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st solution</td>
<td>0.70</td>
<td>x</td>
</tr>
<tr>
<td>2nd solution</td>
<td>0.40</td>
<td>y</td>
</tr>
<tr>
<td>Final sol.</td>
<td>0.50</td>
<td>120</td>
</tr>
</tbody>
</table>

The “how many gallons” column gives one equation; the “amount of pure” column gives the second equation.

\[
\begin{align*}
x + y &= 120 & \text{because } & \text{gal + gal = total gal} \\
0.70x + 0.40y &= 0.50(120) & \text{because } & \text{% (gal) + % (gal) = % (total gal)}
\end{align*}
\]

**b.** A pharmacist mixes two different strengths of peroxide. If she mixes a 1% with a 9% solution to obtain 60 ml of a 4% solution, how many ml of each type solution does she use?

Think: 

- x = ml of 1%
- y = ml of 9%

<table>
<thead>
<tr>
<th>Strength (in %)</th>
<th>How Many ml</th>
<th>Amount of Pure (% times how many ml)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st solution</td>
<td>0.01</td>
<td>x</td>
</tr>
<tr>
<td>2nd solution</td>
<td>0.09</td>
<td>y</td>
</tr>
<tr>
<td>Final sol.</td>
<td>0.04</td>
<td>60</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
x + y &= 60 & \text{because } & \text{ml + ml = total ml} \\
0.01x + 0.09y &= 0.04(60) & \text{because } & \text{% (ml) + % (ml) = % (total ml)}
\end{align*}
\]
c. A chemist has to make 80 liters of 15.5% alcohol solution from mixing a 32% solution and a 10% solution. How many liters of each type are mixed?

Think:  
\[ x = \text{liters of 32\% solution} \]
\[ y = \text{liters of 10\% solution} \]

<table>
<thead>
<tr>
<th>strength (in %)</th>
<th>how many liters</th>
<th>amount of pure (% times how many liters)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1\text{st} solution</td>
<td>0.32 (x)</td>
<td>(0.32(x))</td>
</tr>
<tr>
<td>2\text{nd} solution</td>
<td>0.10 (y)</td>
<td>(0.10(y))</td>
</tr>
<tr>
<td>Final sol.</td>
<td>0.155</td>
<td>80</td>
</tr>
</tbody>
</table>

\[ x + y = 80 \quad \text{because} \quad \text{liters} + \text{liters} = \text{total liters} \]
\[ .32x + .10y = .155(80) \quad \text{because} \quad \%(\text{liters}) + \%(\text{liters}) = \%(\text{total liters}) \]

Part B: Investment Mixture Problems

d. A person had $10,000 to invest; some was invested at 6% and some at 8%. If the total annual interest was $730, find the amount invested at each interest rate.

Think:  
\[ x = \$\text{ invested at 6\%} \]
\[ y = \$\text{ invested at 8\%} \]

<table>
<thead>
<tr>
<th>%</th>
<th>how many dollars invested</th>
<th>interest (% times $ invested)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1\text{st} investment</td>
<td>0.06 (x)</td>
<td>(0.06(x))</td>
</tr>
<tr>
<td>2\text{nd} investment</td>
<td>0.08 (y)</td>
<td>(0.08(y))</td>
</tr>
<tr>
<td>overall</td>
<td>10,000</td>
<td>$730</td>
</tr>
</tbody>
</table>

\[ x + y = 10,000 \quad \text{because} \quad \$\text{ invested} + \$\text{ invested} = \text{total \$ invested} \]
\[ .06x + .08y = 730 \quad \text{because} \quad \%(\$\text{ invested}) + \%(\$\text{ invested}) = \text{total interest} \]

e. Jenn invested some money at 5% and $6000 more than this at 7%. If the total annual interest is $1740, find the amount invested at each rate.

Think:  
\[ x = \# \text{ of \$ invested at 5\%} \]
\[ y = \# \text{ of \$ invested at 7\%} \]

<table>
<thead>
<tr>
<th>%</th>
<th>how many dollars invested</th>
<th>interest (% times $ invested)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1\text{st} investment</td>
<td>0.05 (x)</td>
<td>(0.05(x))</td>
</tr>
<tr>
<td>2\text{nd} investment</td>
<td>0.07 (y)</td>
<td>(0.07(y))</td>
</tr>
<tr>
<td>overall</td>
<td>(y = x + 6000)</td>
<td>$1740</td>
</tr>
</tbody>
</table>

\[ y = x + 6000 \quad \text{because} \quad \text{the 7\% is \$6000 more than the 5\% and x represents the 5\%} \]
\[ .05x + .07y = 1740 \quad \text{because} \quad \%(\$\text{ invested}) + \%(\$\text{ invested}) = \text{total interest} \]
Part C: General Mixture Problems

f. A company manufactures some spigots at $25 each and others at $32 each. If the company sold 50 spigots and made $1334, find the number of each type spigot sold.

Think: 
\[ x = \text{spigots at$25 each} \]
\[ y = \text{spigots at$32 each} \]

\[
\begin{array}{ccc}
\text{for} & \text{how many} & \text{total value ($ for one times how many spigots)} \\
\text{one spigot} & \text{spigots} & \\
\hline
1^{st} & 25 & x & 25(x) \\
2^{nd} & 32 & y & 32(y) \\
\text{overall} & 50 & \$1334 & \\
\end{array}
\]

\[
x + y = 50 \quad \text{because} \quad \text{spigots + spigots} = \text{total spigots}
\]
\[
25x + 32y = 1334 \quad \text{because} \quad \$ (spigots) + \$ (spigots) = \text{total}$
\]

g. A grocer mixed $1.80 a pound peanuts with $3.20 a pound chocolate to obtain 50 pounds of a mixture worth $2.92 a pound. Find the number of pounds of each item used.

Think: 
\[ x = \text{pounds of peanuts} \]
\[ y = \text{pounds of chocolate} \]

\[
\begin{array}{ccc}
\text{for} & \text{how many} & \text{total value ($ for one times how many pounds)} \\
\text{one pound} & \text{pounds} & \\
\hline
1^{st} & 1.80 & x & 1.80(x) \\
2^{nd} & 3.20 & y & 3.20(y) \\
\text{overall} & 2.92 & 50 & 2.92(50) \\
\end{array}
\]

\[
x + y = 50 \quad \text{because} \quad \text{pounds + pounds} = \text{total pounds}
\]
\[
1.80x + 3.20y = 2.92(50) \quad \text{because} \quad \$ (pounds) + \$ (pounds) = \$ \text{ending(pounds}_{\text{total}})
\]

You should work through g to make sure you can do the math :) and then check your results (see below). Then you should try the worksheet.

Answers:

a. \[ x = 40 \text{ gallons of 70%;} \quad y = 80 \text{ gallons of 40%} \]
b. \[ x = 37.5 \text{ ml of 1%;} \quad y = 22.5 \text{ ml of 9%} \]
c. \[ x = 20 \text{ liters at 32%;} \quad y = 60 \text{ liters at 10%} \]
d. \[ x = 3500 \text{ at 6%;} \quad y = 6500 \text{ at 8%} \]
e. \[ x = 11,000 \text{ at 5%;} \quad y = 17,000 \text{ at 7%} \]
f. \[ x = 38 \text{ spigots at$25 each;} \quad y = 12 \text{ spigots at$32 each} \]
g. \[ x = 10 \text{ lbs peanuts at$1.80 a pound;} \quad y = 40 \text{ lbs chocolate at$3.20 a pound} \]

Now try these.

1. A 25% acid solution must be added to a 40% solution to get 240 liters of 30% acid solution. How many of each solution must be used?

2. A 50% antifreeze must be mixed with a 20% antifreeze to get 250 gallons of 32% antifreeze solution. How many of each must be used?
3. A certain metal is 40% tin. A 40% tin metal must be mixed with a 70% tin metal to get 150 kilograms of metal that is 52% tin. How much of each must be used?

4. A 1% solution should be mixed with a 4% solution to get 75 ml of 2% solution. How much of each must be used?

5. A 60% acid solution must be mixed with a 75% acid solution to get 20 liters of a 72% solution. How much of each must be used?

6. Gloria invested some money at 18%, and $3000 less than this at 20%. The total annual interest is $3200. How much is invested at each rate?

7. Marty inherited a sum of money from a relative. He deposits some of the money at 16%, and $4000 more than this at 12%. He earns $3840 in interest per year. How much is invested at each rate?

8. Evelyn invested some money at 10%, and $5000 more than this at 14%. Her total annual interest was $3100. How much was invested at each rate?

9. Joe invested some money at 8%, and $3000 more than twice as much at 10%. The total annual interest was $2540. How much was invested at each rate?

10. Larry has $20,000 to invest, some at 5% and some at 7%. If the annual interest is $1240, how much is invested at each rate?

11. Mary has $35,000 to invest, some at 6% and some at 9%. If the annual interest is $2760, how much is invested at each rate?

12. $50,000 is invested, some at 8% and some at 15%. If the average yield is 11%, find the amount invested at each rate. (Round to the nearest hundredth.)

13. Ink worth $100 per barrel will be mixed with ink worth $60 per barrel to get 48 barrels of ink worth $75 per barrel. How many barrels of each type of ink should be used?

14. Helen has 2 more dimes than nickels. Altogether she has $1.70. How many coins of each type does she have?

15. A bank teller has some five-dollar bills and some twenty-dollar bills. The teller has 5 more twenties than fives. The total value of the money is $725. How many of each type of bill does she have?

16. A merchant wishes to mix candy worth $5 per pound with candy worth $2 per pound to get 60 pounds of a mixture that can be sold for $3 per pound. How many pounds of each type of candy should be used?

17. A merchant wishes to mix some candy worth $1.50 a pound with some candy worth $5.50 a pound to get 240 pounds of candy worth $3.50 per pound. How many pounds of the $5.50 candy should she use?

Answer Key.

1. \[ x + y = 240 \]
   \[ .25(x) + .40(y) = .30(240) \]
   \[ x = 160 \text{ liters of 25% solution} \]
   \[ y = 80 \text{ liters of 40% solution} \]

2. \[ x + y = 250 \]
   \[ .50(x) + .20(y) = .32(250) \]
   \[ x = 100 \text{ gallons of 50% antifreeze} \]
   \[ y = 150 \text{ gallons of 20% antifreeze} \]

3. \[ x + y = 150 \]
   \[ .40(x) + .70(y) = .52(150) \]
   \[ x = 90 \text{ kilograms of 40% tin metal} \]
   \[ y = 60 \text{ kilograms of 70% tin metal} \]

4. \[ x + y = 75 \]
   \[ .01(x) + .04(y) = .02(75) \]
   \[ x = 50 \text{ ml of 1% solution} \]
   \[ y = 25 \text{ ml of 4% solution} \]

5. \[ x + y = 20 \]
   \[ .60(x) + .75(y) = .72(20) \]
   \[ x = 4 \text{ liters of 60% solution} \]
   \[ y = 16 \text{ liters of 75% solution} \]

6. \[ y = x - 3000 \]
   \[ .18(x) + .20(y) = 3200 \]
   \[ x = $10,000 \text{ at 18\%} \]
   \[ y = $7000 \text{ at 20\%} \]
7. \(y = x + 4000\)
   \[0.16(x) + 0.12(y) = 3840\]
   \(x = $12,000\) at 16%
   \(y = $16,000\) at 12%

8. \(y = x + 5000\)
   \[0.10(x) + 0.14(y) = 3100\]
   \(x = $10,000\) at 10%
   \(y = $15,000\) at 14%

9. \(y = 2x + 3000\)
   \[0.08(x) + 0.10(y) = 2540\]
   \(x = $8,000\) at 8%
   \(y = $19,000\) at 10%

10. \(x + y = 20,000\)
    \[0.05(x) + 0.07(y) = 1240\]
    \(x = $8,000\) at 5%
    \(y = $12,000\) at 7%

11. \(x + y = 35,000\)
    \[0.06(x) + 0.09(y) = 2760\]
    \(x = $13,000\) at 6%
    \(y = $22,000\) at 9%

12. \(x + y = 50,000\)
    \[0.08(x) + 0.15(y) = 0.11(50,000)\]
    \(x = $28,571.43\) at 8%
    \(y = $21,428.57\) at 15%

13. \(x + y = 48\)
    \[100(x) + 60(y) = 75(48)\]
    \(x = 18\) barrels of $100 ink
    \(y = 30\) barrels of $60 ink

14. \(y = x + 2\)
    \[0.05(x) + 0.10(y) = 1.70\]
    \(x = 10\) nickels
    \(y = 12\) dimes

15. \(y = x + 5\)
    \[5(x) + 20(y) = 725\]
    \(x = 25\) five-dollar bills
    \(y = 30\) twenty-dollar bills

16. \(x + y = 60\)
    \[5(x) + 2(y) = 3(60)\]
    \(x = 20\) pounds of $5 candy
    \(y = 40\) pounds of $2 candy

17. \(x + y = 240\)
    \[1.50(x) + 5.50(y) = 3.50(240)\]
    \(x = 120\) pounds of $1.50 candy
    \(y = 120\) pounds of $5.50 candy