Math 93–Handout on Fraction Rules--page 1

A. **Reduce to lowest terms.** First, determine the sign. Second, on coefficients (numbers), divide numerator and denominator by the same number/factor OR factor using primes so you can reduce identical factors. Finally, on variables, factor and reduce the variables that are identical in numerator and denominator.

1. $\frac{18}{27} \Rightarrow \frac{18\div 9}{27\div 9} \Rightarrow \frac{2}{3}$

2. $\frac{12a^3}{-18a^5} \Rightarrow -\frac{2\cdot 2\cdot 3\cdot a\cdot a\cdot a}{2\cdot 3\cdot 3\cdot a\cdot a\cdot a\cdot a\cdot a} \Rightarrow -\frac{2}{3a^2}$

or: divide 12 and 18 by 6

B. **Multiply.** You can multiply straight across and then reduce. Or you can reduce (any numerator with any denominator--as long as you have a multiplication symbol) first and then multiply straight across.

3. $\frac{2}{3}\left(-\frac{9}{20}\right) \Rightarrow \frac{1}{1}\left(-\frac{3}{10}\right) \Rightarrow -\frac{3}{10}$

4. $-\frac{14}{25}\left(-\frac{10}{21}\right) \Rightarrow -\frac{2}{5}\left(-\frac{2}{3}\right) \Rightarrow \frac{4}{15}$

on 2 and 20, you divide by 2
on 3 and 9, you divide by 3

on 14 and 21, you divide by 7
on 25 and 10, you divide by 5

Or use prime factors and cancel identical factors in numerator/denominator:

$\frac{2}{3}\cdot -\frac{3\cdot 3}{2\cdot 2\cdot 5} \Rightarrow -\frac{3}{2\cdot 5} \Rightarrow -\frac{3}{10}$

$-\frac{2\cdot 7}{5\cdot 5}\cdot -\frac{2\cdot 5}{3\cdot 7} \Rightarrow \frac{2\cdot 2}{5\cdot 3} \Rightarrow \frac{4}{15}$

C. **Divide.** You invert the divisor and multiply. After you invert, try to reduce and then multiply straight across. You still need to be careful with signs!

5. $\frac{-16}{x^2}\div\frac{24}{x^5} \Rightarrow \frac{-16}{x^2}\cdot\frac{x^5}{24} \Rightarrow \frac{-2}{x\cdot x}\cdot\frac{x\cdot x\cdot x\cdot x\cdot x}{3} \Rightarrow -\frac{2x^3}{3}$

on 16 and 24, you divide by 8 or you use prime factors (16 = 2·2·2·2 and 24 = 2·2·2·3 and reduce accordingly)

D. **Add/Subtract.** Do NOT invert!! Do NOT reduce!! To add or subtract fractions, you MUST have a common denominator. Convert each fraction to one with a common denominator by multiplying the original fraction by a form of 1 to change the looks, not the value. Try to reduce the final result. To find the common denominator, you can list the multiples of the denominators or you can use prime factors.

6. $-\frac{7}{8}+\frac{5}{12} \Rightarrow -\frac{7}{8}\left(\frac{3}{3}\right)+\frac{5}{12}\left(\frac{2}{2}\right) \Rightarrow -\frac{21}{24}+\frac{10}{24} \Rightarrow -\frac{11}{24}$

Multiples of 8 = 8, 16, 24, 32, 40, 48, …
Multiples of 12 = 12, 24, 36, … Common denominator is 24 (1st common # on both lists)

or: $-\frac{7}{2\cdot 2\cdot 2}+\frac{5}{2\cdot 2\cdot 3} \Rightarrow -\frac{7}{2\cdot 2\cdot 2}\left(\frac{3}{3}\right)+\frac{5}{2\cdot 2\cdot 3}\left(\frac{2}{2}\right) \Rightarrow -\frac{21}{24}+\frac{10}{24} \Rightarrow -\frac{11}{24}$

7. $-\frac{4m}{9}-\frac{5m}{12} \Rightarrow -\frac{4m}{9}\left(\frac{4}{4}\right)-\frac{5m}{12}\left(\frac{3}{3}\right) \Rightarrow -\frac{16m}{36}-\frac{15m}{36} \Rightarrow -\frac{31m}{36}$

E. **Mixed Numbers.** Convert mixed numbers to improper fractions; then proceed as above.

8. $-3\frac{3}{4}+7\frac{1}{6} \Rightarrow -\frac{15}{4}+\frac{43}{6} \Rightarrow -\frac{15}{4}\left(\frac{3}{3}\right)+\frac{43}{6}\left(\frac{2}{2}\right) \Rightarrow -\frac{45}{12}+\frac{86}{12} \Rightarrow -\frac{41}{12}$ *or* $-3\frac{5}{12}$

Math 93–Handout on Fraction Rules--page 2

F. **Solving Equations Involving Fractions.** There are many ways to approach equations that contain fractions. One approach is to clear the fractions. Multiply ALL terms by the same common denominator. All denominators should reduce out of the equation. Proceed to solve as usual. Results should be written as reduced fractions.

9. $\frac{5}{6}x - \frac{3}{4} = \frac{1}{2}x - \frac{7}{9}$

$\left(\frac{36}{1}\right)\frac{5}{6}x - \left(\frac{36}{1}\right)\frac{3}{4} = \left(\frac{36}{1}\right)\frac{1}{2}x - \left(\frac{36}{1}\right)\frac{7}{9}$

$6 \cdot 5x - 9 \cdot 3 = 18 \cdot 1x - 4 \cdot 7$
$30x - 27 = 18x - 28$

$30x - 27 = 18x - 28$
$-18x \quad -18x$
$12x - 27 = -28$
$+27 \quad +27$
$12x = -1$

$\frac{12x}{12} = \frac{-1}{12}$

$x = -\frac{1}{12}$

10. $\frac{2}{3}x + 4 = \frac{3}{5}$

$\left(\frac{15}{1}\right)\frac{2}{3}x + (15)4 = \left(\frac{15}{1}\right)\frac{3}{5}$

$5 \cdot 2x + 15 \cdot 4 = 3 \cdot 3$
$10x + 60 = 9$

$10x + 60 = 9$
$-60 \quad -60$
$10x = -51$

$\frac{10x}{10} = \frac{-51}{10}$

$x = -\frac{51}{10}$

11. Sometimes the equations have no fractions, but the result is a fraction. Just solve!

$6x + 11 = 16$
$-11 \quad -11$
$6x = 5$

$\frac{6x}{6} = \frac{5}{6}$

$x = \frac{5}{6}$

12. $3(7k + 8) = 18$
$21k + 24 = 18$
$-24 \quad -24$
$21k = -6$

$\frac{21k}{21} = \frac{-6}{21}$

$k = -\frac{6}{21}$

remember to reduce to lowest terms (think: both numbers divide by 3)

or use primes to reduce (6 = 2·3 and 21 = 3·7; reduce accordingly)

$k = -\frac{2}{7}$