Math 95–Factoring Hints--page 1

First, how do you organize your strategies for factoring? Here is a list of helpful hints.

1. Look for a GCF (greatest common factor) first. A GCF is something that divides into each term. It could be a coefficient, variable, phrase, or combination of these. Coefficients divide; exponents of same base subtract.

   a. $18x^2 + 24x + 30$
   $6(3x^2 + 4x + 5)$ (Coefficient)

   b. $2x^7 + 3x^6 - 4x^5$
   $x^5(2x^2 + 3x - 4)$ (Variable)

   c. $8x(2x - 5) + 9(2x - 5)$
   $(2x - 5)(8x + 9)$ (Phrase)

   d. $24x^8 - 16x^7 + 48x^6$
   $8x^6(3x^2 - 2x + 6)$ (Coefficient and variable)

2. See if you need to rewrite a phrase, using a -1 as a factor. Keep factoring as necessary.

   a. $8 - x$
   $-x + 8$ rewrite with $-x$ in front
   $-1(x - 8)$ divide out neg 1

   b. $16 - x^2$
   $-x^2 + 16$ rewrite with $-x^2$ in front
   $-1(x^2 - 16)$ divide out neg 1
   $-1(x + 4)(x - 4)$ factor into conjugates

   c. $16 - x^2$ an alternative; used sometimes
   $(4 + x)(4 - x)$

3. How many terms are there?

   a. Two terms: Difference of squares? $x^2 - 64$
   $(x + 8)(x - 8)$ factor into conjugates

   Sum of squares? $x^2 + 25$
   prime doesn't factor

   Difference of cubes? $x^3 - 8$
   $(x - 2)(x^2 + 2x + 4)$

   Sum of cubes? $64x^3 + 27$
   $(4x + 3)(16x^2 - 12x + 9)$

   b. Three terms: Use $a \cdot c$ and grouping (first two, last two) so re-write with 4 terms

$x^2 - 12x - 28$	$a \cdot c = 28$	Factors of 28	Subtract to -12
$x^2 - 14x + 2x - 28$		$1 \cdot 28$	$1 - 28 = -27$
$x(x - 14) + 2(x - 14) = -27$		$2 \cdot 14$	$2 - 14 = -12$ Works!
$(x - 14)(x + 2)$			

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Three terms: $4x^2 - 8x - 21$ $4(-21) = -84$; use 6 and -14 since
$4x^2 + 6x - 14x - 21$ $6 \cdot -14 = -84$ and $6 - 14 = -8$
$2x(2x + 3) - 7(2x + 3)$
$(2x + 3)(2x - 7)$

Remember that the signs in the original polynomial determine the signs in the factors. If the original problem ends in a PLUS sign, you add to the middle number (so both pos or both neg, depending on middle sign). If the original problem ends in a SUBTRACT sign, you subtract to the middle number (so one pos and one neg). Here's an organized list.

Polynomial		Factors
+ +	=	(+)(+)
- +	=	(-)(-)
+ -	=	(+)(-) or reverse order
- -	=	(+)(-) or reverse order

c. Four or more terms: Use grouping.

1) $2x + 2y + ax + ay$
$2(x + y) + a(x + y)$
$(x + y)(2 + a)$

2) $m^3 + m^2n - 2m - 2n$
$m^2(m + n) - 2(m + n)$
$(m + n)(m^2 - 2)$

4. At the end of the problem, check for additional GCFs. Also check to see if anything else factors again.
$x^4 - 16$
$(x^2 + 4)(x^2 - 4)$
$(x^2 + 4)(x + 2)(x - 2)$ Can you see why?

5. Sometimes you use several different processes in the same problem.

a. $3x^2(2x + 4) - 13x(2x + 4) - 10(2x + 4)$
$(2x + 4)(3x^2 - 13x - 10)$ (GCF phrase)
$(2x + 4)\ (3x^2 - 15x + 2x - 10)$ (rewrite with 4 terms)
$(2x + 4)\ [3x(x - 5) + 2(x - 5)]$ (factor 1st two terms; factor last two terms)
$(2x + 4)\ (x - 5)(3x + 2)$ (factor by grouping)
$2(x + 2)(x - 5)(3x + 2)$ (GCF coefficient on first phrase)

b. $x^3 + 7x^2 - 9x - 63$ (4 terms so factor by grouping)
$x^2(x + 7) - 9(x + 7)$ (find GCF of 1st two terms; GCF of last two)
$(x + 7)(x^2 - 9)$ (factor GCF phrase to front)
$(x + 7)(x + 3)(x - 3)$ (difference of squares factors into conjugate)

6. SUMMARY:
Factor out GCF first
Factor anything else that needs factoring; keep factoring until you can't factor anything else

7. Prime polynomials: polynomials that don't factor. How can you tell? Look at the example.

a. $x^2 + 3x + 14$ $14 = 1(14)$ or $2(7)$. Which of these adds to 3?
prime Nothing! You can't get the middle 3! Prime.

b. $4x^2 + 2x - 5$ $4(5) = 20 = 1(20)$ or $2(10)$ or $4(5)$. What subtracts to
prime 2? Nothing! You can't get the middle 2! Prime.

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Now, how do you think when you have to factor? Observe the following.

	The problem:	**Think:**
8.	$x^2 - 25$	no GCF; difference of squares; factor into conjugates
	$(x + 5)(x - 5)$	
9.	$4x^2 - 49$	no GCF; difference of squares; factor into conjugates
	$(2x + 7)(2x - 7)$	
10.	$2x^2 - 18$	GCF
	$2(x^2 - 9)$	now inside ( ), it's difference of squares; factor into conjugates
	$2(x + 3)(x - 3)$	
11.	$5x^2 + 13x + 6$	no GCF; trinomial so use a · c and grouping. Think: 5(6) = 30; what are the factors of 30 that will add to the middle number 13? 10 and 3 work.
	$5x^2 + 10x + 3x + 6$	re-write with 4 terms
	$\frac{5x^2}{5x} + \frac{10x}{5x} + \frac{3x}{3} + \frac{6}{3}$	find GCF of 1st two terms; GCF of last two terms
	$5x(x + 2) + 3(x + 2)$	factor out the GCF phrase
	$(x + 2)(5x + 3)$	
12.	$24x^3 - 4x^2 - 48x$	GCF
	$4x(6x^2 - 1x - 12)$	now inside ( ), trinomial so use a · c and grouping
		carry down 4x while you factor the trinomial
	$4x\ [6x^2 + 8x - 9x - 12]$	6(12) = 72; find factors of 72 that subtract to –1 so 8 and –9
	$4x\left[\frac{6x^2}{2x} + \frac{8x}{2x} - \frac{9x}{-3} - \frac{12}{-3}\right]$	
	$4x\ [\ 2x(3x + 4) - 3(3x + 4)\ ]$	
	$4x\ [\ (3x + 4)(2x - 3)\ ]$	
	$4x(3x + 4)(2x - 3)$	
13.	$x^2 + 2x - 15$	no GCF; trinomial so use a · c and grouping
	$x^2 - 3x + 5x - 15$	
	$x(x - 3) + 5(x - 3)$	
	$(x - 3)(x + 5)$	
14.	$3x^3 - 21x^2 + 36x$	GCF
	$3x(x^2 - 7x + 12)$	now inside ( ), trinomial so use a · c and grouping
	$3x\ [x^2 - 3x - 4x + 12]$	
	$3x\ [\ x(x - 3) - 4(x - 3)\ ]$	
	$3x\ [\ (x - 3)(x - 4)\ ]$	
	$3x(x - 3)(x - 4)$	
15.	$125x^3 + 64$	Sum of cubes; just factor
	$(5x + 4)(25x^2 - 20x + 16)$	