

Math 95--Quadratic Word Problems--page 1

To solve word problems, you need to have a procedure:

- label the unknown(s)
- write the equation in words or write the formula being used
- replace the words with algebra or replace the letters in the formula with algebra
- solve
- state the solution(s)

This worksheet will present word problems that are quadratic (2nd degree). Usually you don't know the problem is quadratic until you've written a formula and removed any parentheses. Then if the equation is a 2nd degree equation, you need to get the equation equal to zero so you can factor to solve.

Part A--Pythagorean Formula. The first type problem is the Pythagorean formula, which deals with the lengths of the sides of a right triangle. The formula says $a^2 + b^2 = c^2$, where a and b are legs and c is the hypotenuse. So an alternate formula says $\text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2$. You may find that drawing and labeling a right triangle helps so you can identify the legs and hypotenuse. Look at the examples to follow the thinking.

- a. A TV has a base of 32 inches and a height of 24 inches. What is the length of the diagonal?

base 32 inches = leg
height 24 inches = leg
diagonal = x = hypotenuse

Write the formula: $\text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2$
Substitute: $(32)^2 + (24)^2 = (x)^2$
Simplify: $1024 + 576 = x^2$
 $1600 = x^2$
Rearrange: $0 = x^2 - 1600$
Factor: $0 = (x + 40)(x - 40)$
Set to zero: $x + 40 = 0$ or $x - 40 = 0$
Solve: $x = -40$ or $x = 40$ Only x = 40 is logical so...
Answer: diagonal = x = hypotenuse = 40 inches

The diagonal of the TV is 40 inches.

- b. A 39 foot ladder is resting against a building. The top of the ladder reaches a height of 36 feet. Find how far the ladder is from the bottom of the building.

width of bottom of ladder from building = x = a leg
36 ft = height of top of ladder from the ground = a leg
39 ft = ladder = the hypotenuse

Write the formula: $\text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2$
Substitute: $(x)^2 + (36)^2 = (39)^2$
Simplify and solve: $x^2 + 1296 = 1521$
 $x^2 + 1296 - 1521 = 0$
 $x^2 - 225 = 0$
 $(x + 15)(x - 15) = 0$
 $x + 15 = 0$ or $x - 15 = 0$
 $x = -15$ or $x = 15$ Only x = 15 is logical so...
Answer: width of bottom of ladder from building = x = 15 feet

The bottom of the ladder is 15 feet from the bottom of the building.

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Homework for Part A--Set up and solve.

1. In a right triangle, the legs measure 8 inches and 6 inches. Find the hypotenuse.
2. In a right triangle, the legs measure 5 inches and 12 inches. Find the hypotenuse.
3. In a right triangle, one leg is 24 inches and the hypotenuse is 25 inches. Find the other leg.
4. In a right triangle, one leg is 9 inches and the hypotenuse is 41 inches. Find the other leg.

Hint: on 5, 6, 7, you need to decide if you're finding the leg or the hypotenuse.

5. A rectangular garden has a diagonal of 20 feet; the length is 16 feet. Find the width.
6. A ladder leans against a building and reaches a height of 48 feet on the building. The bottom of the ladder is 14 feet from the bottom of the building. How long is the ladder?
7. A television screen has a diagonal of 30 inches. The bottom of the screen measures 24 inches. What is the height of the screen?

Part B--Other Quadratic Word Problems. This part of the worksheet will present other quadratic word problems. Go through the examples carefully.

- c. In a rectangle, the width is 7 inches less than the length. The area is 120 sq inches. Find the length and width.

$$\begin{aligned}\text{length} &= x \\ \text{width} &= x - 7\end{aligned}$$

$$\begin{aligned}\text{length}(\text{width}) &= \text{area} \\ (x)(x - 7) &= 120 \\ x^2 - 7x - 120 &= 0 \\ x^2 + 8x - 15x - 120 &= 0 \\ x(x + 8) - 15(x + 8) &= 0 \\ (x + 8)(x - 15) &= 0 \\ x + 8 = 0 &\quad \text{or} \quad x - 15 = 0 \\ x = -8 &\quad \text{or} \quad x = 15 \text{ (which is the only logical result for} \\ &\quad \text{length)}\end{aligned}$$

$$\begin{aligned}\text{length} &= x = 15 \text{ inches} \\ \text{width} &= x - 7 = 15 - 7 = 8 \text{ inches}\end{aligned}$$

- d. The product of two positive integers is 33. One number is 8 more than another number. Find the numbers.

$$\begin{aligned}\text{another number} &= x \\ \text{one number} &= x + 8\end{aligned}$$

$$\begin{aligned}\text{one number}(\text{another number}) &= 33 \\ (x + 8)(x) &= 33 \\ x^2 + 8x &= 33 \\ x^2 + 8x - 33 &= 0 \\ x^2 - 3x + 11x - 33 &= 0 \\ x(x - 3) + 11(x - 3) &= 0 \\ (x - 3)(x + 11) &= 0 \\ x - 3 = 0 &\quad \text{or} \quad x + 11 = 0 \\ x = 3 &\quad \text{or} \quad x = -11 \\ x = 3 &\text{ is the only result that fits the parameters of the problem}\end{aligned}$$

$$\begin{aligned}\text{another number} &= x = 3 \\ \text{one number} &= x + 8 = 3 + 8 = 11\end{aligned}$$

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- e. The product of two consecutive positive odd integers is 99. Find the integers.

$$1^{\text{st}} \text{ odd integer} = x$$

$$2^{\text{nd}} \text{ odd integer} = x + 2$$

$$1^{\text{st}} \text{ odd} (2^{\text{nd}} \text{ odd}) = 99$$

$$(x)(x + 2) = 99$$

$$x^2 + 2x = 99$$

$$x^2 + 2x - 99 = 0$$

factor and solve

$$x = 9 \text{ or } x = -11$$

$x = 9$ is the only result that fits the parameters of the problem

$$1^{\text{st}} \text{ odd integer} = x = 9$$

$$2^{\text{nd}} \text{ odd integer} = x + 2 = 11$$

Homework for Part B--Set up and solve.

8. In a rectangle, the width is 10 inches less than the length. The area is 39 sq inches. Find the length and width.
9. In a rectangle, the length is 4 inches more than the width. The area is 96 sq inches. Find the length and width.
10. The product of two positive integers is 45. One number is 4 more than the other. Find the numbers.
11. The product of two positive integers is 84. One number is 5 less than the other. Find the numbers.
12. The product of two consecutive positive odd integers is 63. Find the numbers.
13. The product of two consecutive positive even integers is 48. Find the numbers.

Part C--More Challenging Quadratic Word Problems. Sometimes, you have a word problem that is more challenging. Observe.

- f. In a right triangle, the long leg is 3 inches longer than the short leg. The hypotenuse is 6 inches more than the short leg. Find the length of all three sides.

$$\text{short leg} = x$$

$$\text{long leg} = x + 3$$

$$\text{hypotenuse} = x + 6$$

$$\text{leg}^2 + \text{leg}^2 = \text{hypotenuse}^2$$

$$(x)^2 + (x + 3)^2 = (x + 6)^2$$

$$x^2 + (x + 3)(x + 3) = (x + 6)(x + 6)$$

$$x^2 + x^2 + 3x + 3x + 9 = x^2 + 6x + 6x + 36$$

$$2x^2 + 6x + 9 = x^2 + 12x + 36$$

$$2x^2 + 6x + 9 - x^2 - 12x - 36 = 0$$

$$x^2 - 6x - 27 = 0 \quad \text{now factor and solve}$$

$$x = -3 \quad \text{or} \quad x = 9$$

Only $x = 9$ is logical so...

$$\text{short leg} = x = 9 \text{ inches}$$

$$\text{long leg} = x + 3 = 12 \text{ inches}$$

$$\text{hypotenuse} = x + 6 = 15 \text{ inches}$$

14. Set up and solve: In a right triangle, the long leg is 5 inches longer than the short leg. The hypotenuse is 10 inches more than the short leg. Find the length of all three sides.

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Answer Key. On this answer key, I'll show the label(s), the initial equation, the equation equal to zero, and the solution. All solving is left up to you! On #14, I'll show more details.

1. hypotenuse = $x = 10$ inches
 $(8)^2 + (6)^2 = (x)^2$
 $0 = x^2 - 100$
 $x = 10$
2. hypotenuse = $x = 13$ inches
 $(5)^2 + (12)^2 = (x)^2$
 $0 = x^2 - 169$
 $x = 13$
3. leg = $x = 7$ inches
 $(x)^2 + (24)^2 = (25)^2$
 $x^2 - 49 = 0$
 $x = 7$
4. leg = $x = 40$ inches
 $(x)^2 + (9)^2 = (41)^2$
 $x^2 - 1600 = 0$
 $x = 40$
5. width = $x = \text{leg} = 12$ feet
 $(16)^2 + (x)^2 = (20)^2$
 $x^2 - 144 = 0$
 $x = 12$
6. length of ladder = $x = \text{hypotenuse} = 50$ feet
 $(48)^2 + (14)^2 = (x)^2$
 $0 = x^2 - 2500$
 $x = 50$
7. height = $x = \text{leg} = 18$ inches
 $(24)^2 + (x)^2 = (30)^2$
 $x^2 - 324 = 0$
 $x = 18$
8. length = $x = 13$ inches
width = $x - 10 = 3$ inches
 $(x)(x - 10) = 39$
 $x^2 - 10x - 39 = 0$
 $x = 13$
9. width = $x = 8$ inches
length = $x + 4 = 12$ inches
 $(x + 4)(x) = 96$
 $x^2 + 4x - 96 = 0$
 $x = 8$
10. other number = $x = 5$
one number = $x + 4 = 9$
 $(x + 4)(x) = 45$
 $x^2 + 4x - 45 = 0$
 $x = 5$
11. other number = $x = 12$
one number = $x - 5 = 7$
 $(x - 5)(x) = 84$
 $x^2 - 5x - 84 = 0$
 $x = 12$
12. 1st odd = $x = 7$
2nd odd = $x + 2 = 9$
 $(x)(x + 2) = 63$
 $x^2 + 2x - 63 = 0$
 $x = 7$
13. 1st even = $x = 6$
2nd even = $x + 2 = 8$
 $(x)(x + 2) = 48$
 $x^2 + 2x - 48 = 0$
 $x = 6$
14. short leg = $x = 15$ inches
long leg = $x + 5 = 20$ inches
hypotenuse = $x + 10 = 25$ inches
 $(x)^2 + (x + 5)^2 = (x + 10)^2$
 $x^2 + (x + 5)(x + 5) = (x + 10)(x + 10)$
 $x^2 + x^2 + 5x + 5x + 25 = x^2 + 10x + 10x + 100$
 $2x^2 + 10x + 25 = x^2 + 20x + 100$
 $2x^2 + 10x + 25 - x^2 - 20x - 100 = 0$
 $x^2 - 10x - 75 = 0$
 $x^2 + 5x - 15x - 75 = 0$
 $x(x + 5) - 15(x + 5) = 0$
 $(x + 5)(x - 15) = 0$
 $x = 15$ (the only logical answer)