Math 095--Solving Quadratic Equations–page 1

Name_____ **Date**_____

To solve quadratic equations, make sure the equation is in standard form of $ax^2 + bx + c = 0$. Standard form implies descending order, first term positive, and equals zero. Then factor, set each factor (that contains a variable) equal to zero, and isolate the variable.

a. $x^2 + 7x - 18 = 0$
$x^2 - 2x + 9x - 18 = 0$
$x(x - 2) + 9(x - 2) = 0$
$(x - 2)(x + 9) = 0$
$x - 2 = 0$ or $x + 9 = 0$
$x = 2$ or $x = -9$
$\{2, -9\}$

b. $2x^2 = 5x + 12$
$2x^2 - 5x - 12 = 0$
$2x^2 - 8x + 3x - 12 = 0$
$2x(x - 4) + 3(x - 4) = 0$
$(x - 4)(2x + 3) = 0$
$x - 4 = 0$ or $2x + 3 = 0$
$x = 4$ or $2x = -3$
$x = 4$ or $x = -\frac{3}{2}$
$\left\{4, -\frac{3}{2}\right\}$

c. $m^2 = 16$
$m^2 - 16 = 0$
$(m + 4)(m - 4) = 0$
$m + 4 = 0$ or $m - 4 = 0$
$m = -4$ or $m = 4$
$\{-4, 4\}$

d. $9y^2 - 25 = 0$
$(3y + 5)(3y - 5) = 0$
$3y + 5 = 0$ or $3y - 5 = 0$
$3y = -5$ or $3y = 5$
$y = -\frac{5}{3}$ or $y = \frac{5}{3}$
$\left\{-\frac{5}{3}, \frac{5}{3}\right\}$

Sometimes you need to factor out a GCF when you are solving equations.

e. $2x^2 + 18x - 44 = 0$
$2(x^2 + 9x - 22) = 0$
$2(x + 11)(x - 2) = 0$
$x + 11 = 0$ or $x - 2 = 0$
$x = -11$ or $x = 2$
$\{-11, 2\}$

Notice on (e) that the GCF 2 did NOT contain a variable; therefore, I didn't set the $2 = 0$. Even if you do write $2 = 0$, it isn't true so doesn't affect the solution set.

f. $6x^3 - 21x^2 - 27x = 0$
$3x(2x^2 - 7x - 9) = 0$
$3x(2x^2 + 2x - 9x - 9) = 0$
$3x[\, 2x(x + 1) - 9(x + 1)\,] = 0$
$3x(2x - 9)(x + 1) = 0$
$3x = 0$ or $2x - 9 = 0$ or $x + 1 = 0$
$x = \frac{0}{3}$ or $2x = 9$ or $x = -1$
$x = 0$ or $x = \frac{9}{2}$ or $x = -1$
$\left\{0, \frac{9}{2}, -1\right\}$

Notice on (f) that the GCF 3x DID contain a variable so set $3x = 0$ and isolate the variable.

g. $-x^3 + 81x = 0$

$-x(x^2 - 81) = 0$

$-x(x + 9)(x - 9) = 0$

$-x = 0$ or $x + 9 = 0$ or $x - 9 = 0$

$x = \frac{0}{-1}$ or $x = -9$ or $x = 9$

$x = 0$ or $x = -9$ or $x = 9$

$\{0, -9, 9\}$

Sometimes, you have to simplify the equation before you begin factoring. You might need to multiply to remove parentheses, combine like terms, rearrange to get the equation into standard form, etc.

h. $z(z + 8) - 3(z + 2) = 8$

$z^2 + 8z - 3z - 6 = 8$

$z^2 + 5z - 6 = 8$

$z^2 + 5z - 6 - \mathbf{8} = 0$

$z^2 + 5z - 14 = 0$

$(z + 7)(z - 2) = 0$

$z + 7 = 0$ or $z - 2 = 0$

$z = -7$ or $z = 2$

$\{-7, 2\}$

i. $6a^2 + 4(2a + 7) = 5a + 8 + 3a + 2a^2 + 45$

$6a^2 + 8a + 28 = 2a^2 + 8a + 53$

$6a^2 + 8a + 28 - \mathbf{2a^2} - \mathbf{8a} - \mathbf{53} = 0$

$4a^2 - 25 = 0$

$(2a + 5)(2a - 5) = 0$

$2a + 5 = 0$ or $2a - 5 = 0$

$2a = -5$ or $2a = 5$

$a = -\frac{5}{2}$ or $a = \frac{5}{2}$

$\left\{-\frac{5}{2}, \frac{5}{2}\right\}$

Solve the following equations.

1. $x^2 + 10x + 9 = 0$

2. $3a^2 - a - 2 = 0$

3. $m^2 = 49$

4. $k^2 - 13k - 48 = 0$

5. $81y^2 = 25$

6. $2x^2 - 64x = 66$

7. $8z^2 + 18z = -9$

8. $c^2 - 13c + 30 = 0$

9. $-x^2 = x - 30$

10. $3m^2 + 78m + 144 = 0$

11. $20q^3 - 90q^2 = 50q$

12. $e^2 + 3e - 40 = 0$

13. $z^2 = -8z - 7$

14. $10y^2 - 3 = 6y^2 + 6$

15. $a^2 - 5a = 0$

16. $8x^2 + 21x - 9 = 0$

Math 095--Solving Quadratic Equations–page 3

17. $m^2 - 16m = -28$

18. $10k^2 = k + 3$

19. $10x^3 + 5x^2 - 75x = 0$

20. $a^2 = -12a$

21. $27x^3 = 90x^2 + 72x$

22. $z^2 + 8 = 12$

23. $m(m - 5) - 3(m - 6) = -2m + 45$

24. $9y^2 - 7y + 2 = 4 + y + 5y^2 + 7 + 8y$

25. $2a(a - 3) = 4a(3a + 1) + 2a + 19 - 9a - 23$

Answer Key.

1. $\{-9, -1\}$
2. $\left\{-\frac{2}{3}, 1\right\}$
3. $\{-7, 7\}$
4. $\{16, -3\}$
5. $\left\{-\frac{5}{9}, \frac{5}{9}\right\}$
6. $\{33, -1\}$
7. $\left\{-\frac{3}{4}, -\frac{3}{2}\right\}$
8. $\{10, 3\}$
9. $\{-6, 5\}$
10. $\{-24, -2\}$
11. $\left\{0, -\frac{1}{2}, 5\right\}$
12. $\{-8, 5\}$
13. $\{-7, -1\}$
14. $\left\{-\frac{3}{2}, \frac{3}{2}\right\}$
15. $\{0, 5\}$
16. $\left\{-3, \frac{3}{8}\right\}$
17. $\{14, 2\}$
18. $\left\{-\frac{1}{2}, \frac{3}{5}\right\}$
19. $\left\{0, \frac{5}{2}, -3\right\}$
20. $\{0, -12\}$
21. $\left\{0, -\frac{2}{3}, 4\right\}$
22. $\{-2, 2\}$
23. $\{-3, 9\}$
24. $\left\{-\frac{1}{2}, \frac{9}{2}\right\}$
25. $\left\{-\frac{4}{5}, \frac{1}{2}\right\}$

Suppose I wanted to check answers that were fractions. I'll illustrate with problem #18. First, you should always perform the check in the ORIGINAL equation. Also, you need to be careful to use parentheses for the solution, to be careful with signs, and to remember all fraction rules (such as multiplying fractions or adding/subtracting fractions which requires a common denominator).

18. Original problem: $10k^2 = k + 3$ Solution: $\left\{-\frac{1}{2}, \frac{3}{5}\right\}$

check x = $-\frac{1}{2}$

$$10\left(-\frac{1}{2}\right)^2 = \left(-\frac{1}{2}\right) + 3$$

$$10\left(\frac{1}{4}\right) = -\frac{1}{2} + \frac{3}{1}$$

$$\frac{10}{4} = -\frac{1}{2} + \frac{6}{2}$$

$$\frac{5}{2} = \frac{5}{2}$$

check x = $\frac{3}{5}$

$$10\left(\frac{3}{5}\right)^2 = \left(\frac{3}{5}\right) + 3$$

$$10\left(\frac{9}{25}\right) = \frac{3}{5} + \frac{3}{1}$$

$$\frac{90}{25} = \frac{3}{5} + \frac{15}{5}$$

$$\frac{18}{5} = \frac{18}{5}$$

These both check so now I know that the solution set is $\left\{-\frac{1}{2}, \frac{3}{5}\right\}$.