Part A--Factoring Using a Greatest Common Factor (GCF). We are going to be using several different techniques for factoring. The first technique is factoring using a greatest common factor (GCF). Factoring reverses the distributive property; factoring is related to division ideas. Once you have factored, you can check by multiplying to see if you get the original polynomial.

a. Greatest common factors can be numbers, variables, both numbers and variables, and phrases. The GCF divides into all the terms of the polynomial, and it is written in front of a set of parentheses. The result of dividing by the GCF is written inside the parentheses. So your result is written in this form: GCF(the other factor).

b. To factor a polynomial using a GCF number, find the largest number that divides into all coefficients all the way across the polynomial.

\[
egin{align*}
18x + 24 &= 2(9x + 12) \\
28a^2 + 35a - 49 &= 7(4a^2 + 5a - 7) \\
22m^2 - 33m + 11 &= 11(2m^2 - 3m + 1)
\end{align*}
\]

Homework.

1. \( 24y + 32 \) 
2. \( 21m - 7 \)
3. \( 4x^2 + 12x + 20 \) 
4. \( 18z^3 + 27z - 36 \)

Homework.

5. \( z^4 + 9z^3 + 7z^2 \) 
6. \( 3m^5 - 8m^3 \)
7. \( 5x^2 + 2x \) 
8. \( 4x^7 + 3x^5 - 11x^3 \)
d. To factor a polynomial using a GCF that has both a number and a variable, you look for the largest number that divides into all coefficients and you look for the smallest exponent of the variable that appears in all the terms. The numbers will divide; the exponents will subtract.

\[
\begin{align*}
20a^5 + 35a^4 & - 15a^3 \\
5a^4(4a^2 + 7a - 3) & \quad 8m(2m^2 - m - 4) \quad 18y^8 + 36y^7 + 9y^6 \\
15a^2 & \quad 16m^3 - 8m^2 - 32m \\
9y^5(2y^2 + 4y + 1) & 
\end{align*}
\]

**Homework.**

9. \(7x^4 + 14x^3 + 35x^2\)

10. \(8a^5 - 10a^4 + 12a^3\)

11. \(36x^3 - 24x\)

12. \(32m^7 + 72m^6 - 16m^5\)

e. To factor a polynomial using a GCF that is a phrase, look for an identical phrase that’s written in parentheses that is present in all terms of the polynomial. The GCF phrase is written in parentheses on the left; the result is written in parentheses on the right. (This order can be reversed and be correct.)

\[
\begin{align*}
x(x + 8) + 9(x + 8) & \quad x^2(x - 7) + 9x(x - 7) + (x - 7) \\
(x + 8)(x + 9) & \quad x^2(x - 7) + 9x(x - 7) + 1(x - 7) \quad \text{remember the understood 1!} \\
& \quad (x - 7)(x^2 + 9x + 1)
\end{align*}
\]

**Homework.**

13. \(x(x + 3) - 4(x + 3)\)

14. \(x(x + 9) + 2(x + 9)\)

15. \(x(x - 5) - 3(x - 5)\)

16. \(x^2(x + 1) + 2x(x + 1) - (x + 1)\)

**Part B--Factoring by Grouping.** Factoring by grouping is usually used when your polynomial has four or more terms. We’ll start by looking at polynomials with four terms. You begin by looking at the first two terms and finding the GCF of these first two only. You then bring down the next sign. Then you look at the last two terms and find the GCF of these two only. There should now be a common phrase that you factor out. This will make more sense after a couple of examples.

f. \(ax + bx + ay + by\) The original four terms

\[
\begin{align*}
& \quad \text{Factor first two; keep sign; factor last two} \\
& \quad \text{Factor out the common phrase}
\end{align*}
\]

\[
\begin{align*}
x(a + b) & + y(a + b) \\
(a + b)(x + y) & 
\end{align*}
\]

g. \(x^2 + 7x + ax + 7a\) The original four terms

\[
\begin{align*}
x(x + 7) & + a(x + 7) \\
(x + 7)(x + a) & 
\end{align*}
\]

h. \(9x^2 + 6x + 21xy + 14y\) The original four terms

\[
\begin{align*}
3x(3x + 2) & + 7y(3x + 2) \\
(3x + 2)(3x + 7y) & 
\end{align*}
\]

i. \(4x^2 + 2x + 10x + 5\) The original four terms

\[
\begin{align*}
2x(2x + 1) & + 5(2x + 1) \\
(2x + 1)(2x + 5) & 
\end{align*}
\]

Factor first two; keep sign; factor last two

Factor out the common phrase
j. \[10y^2 + 6y + 5y + 3\] The original four terms
\[2y(5y + 3) + 1(5y + 3)\] Factor first two; keep sign; factor last two
\[(5y + 3)(2y + 1)\] Factor out the common phrase

k. \[ax + ay - bx - by\] The original four terms
\[a(x + y) - b(x + y)\] Factor first two; keep sign; factor last two
\[(x + y)(a - b)\] Factor out the common phrase

l. \[3x + 3y - ax - ay\] The original four terms
\[3(x + y) - a(x + y)\] Factor first two; keep sign; factor last two
\[(x + y)(3 - a)\] Factor out the common phrase

m. \[7x^2 - 7x - 3x + 3\] The original four terms
\[7x(x - 1) - 3(x - 1)\] Factor first two; keep sign; factor last two
\[(x - 1)(7x - 3)\] Factor out the common phrase

n. \[16x^2 - 20x - 28x + 35\] The original four terms
\[4x(4x - 5) - 7(4x - 5)\] Factor first two; keep sign; factor last two
\[(4x - 5)(4x - 7)\] Factor out the common phrase

Homework.

17. \[mx + my + 7x + 7y\]
18. \[x^2 + 9x + ax + 9a\]
19. \[12x^2 + 8x + 27xy + 18y\]
20. \[18x^2 + 6x + 21x + 7\]
21. \[14x^2 + 35x + 2x + 5\]
22. \[mx + my - px - py\]
23. \[9x + 9y - kx - ky\]
24. \[12x^2 - 12x - 7x + 7\]
25. \[30x^2 - 18x - 55x + 33\]
Answer Key.

1. 8(3y + 4)  2. 7(3m - 1)
3. 4(x^2 + 3x + 5)  4. 9(2z^2 + 3z - 4)
5. z^2(z^2 + 9z + 7)  6. m^3(3m^2 - 8)
7. x(5x + 2)  8. x^3(4x^4 + 3x^2 - 11)
9. 7x^2(x^2 + 3x + 5)  10. 2a^3(4a^2 - 5a + 6)
11. 12x(3x - 2)  12. 8m^6(4m^2 + 9m - 2)
13. (x + 3)(x - 4)  14. (x + 9)(x + 2)
15. (x - 5)(x - 3)  16. (x + 1)(x^2 + 2x - 1)
17. (x + y)(m + 7)  18. (x + 9)(x + a)
19. (3x + 2)(4x + 9y)  20. (3x + 1)(6x + 7)
21. (2x + 5)(7x + 1)  22. (x + y)(m - p)
23. (x + y)(9 - k)  24. (x - 1)(12x - 7)
25. (5x - 3)(6x - 11)