I. Write an equation of a circle in standard form given the following conditions.

1. Center: (-2, 4) and Radius: 4
   \[(x - h)^2 + (y - k)^2 = r^2\]
   \[(-2)^2 + (4 - 4)^2 = 4\]

2. Center: (3, -1) and Diameter: 18
   \[r = \frac{18}{2} = 9\]
   \[(x - h)^2 + (y - k)^2 = r^2\]
   \[(x - 3)^2 + (y + 1)^2 = 81\]

II. Given the following functions. Perform the indicated operations.

\[f(x) = -3x\]
\[g(x) = |x - 2|\]
\[h(x) = \frac{1}{x + 1}\]

1. Find \((f - h)(x)\) and state its domain.
   \[f(x) - h(x) = -3x - \frac{1}{x + 1} \implies \text{Domain: } (-\infty, -1) \cup (-1, \infty)\]

2. Find \((g \cdot h)(x)\) and state its domain.
   \[g(x) \cdot h(x) = \frac{|x - 2|}{x + 1} \implies \text{Domain: } (-\infty, -1) \cup (-1, \infty)\]

3. Find \((g \circ f)(x)\) and state its domain.
   \[g(f(x)) = g(-3x) = |-3x - 2| \implies \text{Domain: } (-\infty, \infty)\]

4. Evaluate \((f \circ g)(5)\).
   \[f(g(5)) = f(3) = -9\]

5. Evaluate \((h \circ g)(4)\).
   \[h(g(4)) = h(2) = \frac{1}{3}\]
III. Determine the domain and write the answer in interval notation.

1. \( f(x) = \frac{x-2}{x-5} \)  
   \[ D: (-\infty, 5) \cup (5, \infty) \]

2. \( f(x) = \sqrt{2-x} \)  
   \[ \begin{align*} 62-x & \geq 0 \\ -x & \geq -2 \\ x & \leq 2 \end{align*} \]  
   \[ D: (-\infty, 2] \]

3. \( f(x) = 4x^2 - 5x + 1 \)  
   \[ D: (-\infty, \infty) \]

4. \( f(x) = \frac{4}{\sqrt{x+4}} \)  
   \[ \begin{align*} 6x+4 & \geq 0 \\ x & > -4 \end{align*} \]  
   \[ D: (-4, \infty) \]

IV. Use the given conditions to write an equation of a line in slope intercept form.

1. \( m = -\frac{2}{3} \) and passes through \((3, -9)\)  
   \[ \begin{align*} y - y_1 &= m(x - x_1) \\ y + 9 &= -\frac{2}{3}(x - 3) \end{align*} \]

2. Passes through \((4, -2)\) and \((-12, -4)\)  
   \[ \begin{align*} m &= \frac{y_2 - y_1}{x_2 - x_1} = \frac{-2 - (-4)}{-12 - 4} = \frac{1}{8} \\ y + 2 &= \frac{1}{8} x - \frac{1}{2} \end{align*} \]  
   \[ y = \frac{1}{8} x - \frac{5}{2} \]
3. Passes through \((2, -6)\) and the line is parallel to the line defined by \(2x - y = 4\).

\[
\begin{align*}
2x - y &= 4 \\
-6 &= 2(2) + b \\
b &= -10
\end{align*}
\]

\[
\begin{align*}
\frac{y - y_1}{x - x_1} &= m \\
y - (-6) &= 2(x - 2) \\
y &= 2x - 10
\end{align*}
\]

V. Solve

1. A car has a 15-gal tank for gasoline and gets 30 mpg on a highway while driving 60 mph. Suppose that the driver starts a trip with a full tank of gas and travels 450 mi on the highway at an average speed of 60 mph.

a. Write a linear model representing the amount of gas \(G(t)\) left in the tank \(t\) hours into a trip.

\[
G(t) = 15 - 2t
\]

\[
m = \frac{60}{30} = 2
\]

b. Evaluate \(G(4.5)\) and interpret the meaning in the context of this problem.

\[
G(4.5) = 15 - 2(4.5) = 6
\]

After 4.5 hours of driving, the tank has 6 gal of gas left.

2. Given \(f(x) = -x^3 + 4\), determine the average rate of change of the function on the interval \([2, 4]\).

Average rate of change:

\[
\frac{f(x_2) - f(x_1)}{x_2 - x_1} = \frac{f(4) - f(2)}{4 - 2} = \frac{-64 - (-8)}{2} = -28
\]

VI. Evaluate the piecewise function.

1. \(f(x) = \begin{cases} 
-4x - 3 & \text{for } x < 0 \\
x^2 & \text{for } x \geq 0
\end{cases}\)

a. \(f(1) = (1)^2 = 1\)

b. \(f(0) = 0^2 = 0\)

c. \(f(-4) = -4(-4) - 3 = 13\)
2. \( f(x) = \begin{cases} 
-4x+2 & ; x < -1 \\
x^2 & ; -1 \leq x \leq 2 \\
5 & ; x > 2 
\end{cases} \)

a. \( f(-4) = -4(-4) + 2 = 18 \)

b. \( f(-1) = (-1)^2 = 1 \)

c. \( f(3) = 5 \)

d. \( f(20) = 5 \)

VII. Determine over what interval(s) each function is increasing, decreasing, or constant.

1. Increasing: \((-\infty, -3), (-3, 0)\),
Decreasing: \((-3, -2), (0, 3)\)
Constant: \((3, \infty)\)
2.

Increasing: \((2, \infty)\)
Decreasing: \((-\infty, 2)\)
Constant: None

VIII. Graph each of the following functions. (Blue represents the transformed)

1. \(f(x) = -2(x-1)^3 + 4\)

2. \(g(x) = |x-2| - 1\)
1. \( f(x) = x^4 - 4x^2 + 1 \)

2. \( f(x) = x^3 - 5x^3 + 4x - 1 \)

IX. Use a graphing calculator to find the relative maximum(s) and/or minimum(s) for each function.

3. \( f(x) = -\sqrt{x+1} + 3 \)

4. \( g(x) = (x+2)^2 + 2 \)
X. Find the following:
   a. vertex
   b. x-intercept
   c. y-intercept
   d. determine if the vertex is a max/min
   e. state the domain and range
   f. graph
   for each problem.

1. \( f(x) = x^2 - 8x + 15 \)
   a) \( \text{Vertex: } (4, -1) \)
      \[ h = -\frac{b}{2a} = -\frac{-8}{2(1)} = 4 \]
      \[ k = f(4) = 16 - 32 + 15 = -1 \]
   b) \( \delta = x^2 - 8x + 15 \)
      \[ \delta = (x - 5)(x - 3) \]
      \[ x = 5, \ x = 3 \]
      \( (5, 0), (3, 0) \)
   c) \( (0, 15) \)
   d) \( \text{min.} \)

2. \( f(x) = -2(x+1)^2 + 8 \)
   a) \( V(-1, 8) \)
   b) \( \delta = -2(x+1)^2 + 8 \)
      \[ -8 = -2(x+1)^2 \]
      \[ \sqrt{-4} = \sqrt{(x+1)^2} \]
      \[ \pm 2 = x + 1 \]
      \[ x = -1 \pm 2 \]
      \[ x = -1 + 2 = 1 \]
      \[ x = -1 - 2 = -3 \]
      \( (1, 0), (-3, 0) \)
   c) \( (0, 6) \)
   d) \( \text{max} \)
   e) \( D: (-\infty, \infty) \)
      \[ R: (-\infty, 8] \]
XI. Find the following:
   a. end behavior
   b. zero(s) and multiplicity
   c. determine if the graph crosses or touches at each zero

for each function.

1. \( f(x) = 2x^2(x-1)^3(x+1)^4 \)

   a. Degree: 9

   Leading coefficient is positive

   Behavior is down to left, up to right

   \( x \to -\infty, f(x) \to -\infty \)
   \( x \to \infty, f(x) \to \infty \)

   b/c) \( x^2_2 = 2(x-1)^3(x+1)^4 \)

   \( x_2 = 0 \) (multi. 2)
   \( x^2 = 0 \) (multi. 3)
   \( x = 1 \) (multi. 4)
   \( x = -1 \)

2. \( f(x) = -3(x-2)^3(x+1)^2 \)

   a) Degree: 5

   Leading coefficient is negative

   Behavior is up to the left, down to the right

   \( x \to -\infty, f(x) \to \infty \)
   \( x \to \infty, f(x) \to -\infty \)

   b/c) \( 0 = -3(x-2)^3(x+1)^2 \)

   \( x_2 = 0 \) (multi. 3)
   \( x = -1 \) (multi. 2)

XII. Solve each inequality and write the solution in interval notation.

1. \( x(x-3) \geq 18 \)

   \( k^2 - 3k - 18 \geq 0 \)
   \( (k - 6)(k + 3) \geq 0 \)
   \( (k - 6)(k + 3) = 0 \)

   \( k = 6 \quad k = -3 \)

   \(-\infty, -3\] \( U \] [6, \infty)
2. \( \frac{6-2x}{x^2} \geq 0 \)

\[
\begin{array}{c|c|c|c}
& T & F & T \\
\hline
-1 & 0 & 1 & 2 \\
\end{array}
\]

\((-\infty, 0) \cup (0, 3]\)

3. \( \frac{8}{3x-4} < 1 \)

\[
\frac{8}{3x-4} - 1 < 0
\]

\[
\frac{8}{3x-4} - \frac{3x-4}{3x-4} < 0
\]

\[
\frac{-3x+12}{3x-4} < 0
\]

\[
\begin{array}{c|c|c|c}
& T & F & T \\
\hline
0 & \frac{4}{3} & 2 & 4 \\
\end{array}
\]

\((-\infty, \frac{4}{3}) \cup (4, \infty)\)