Answer each of the following questions. You must show your work in order to receive partial credit.

Find the exact value.

1) Find the exact value of $\tan(\theta + \phi)$ given that $\sin \theta = \frac{4}{5}$ and $\sin \phi = \frac{5}{13}$ and that $\theta$ and $\phi$ are between $0$ and $\frac{\pi}{2}$.

\[
\tan(\theta + \phi) = \frac{\tan \theta + \tan \phi}{1 - \tan \theta \tan \phi} = \frac{\frac{4}{5} + \frac{5}{13}}{1 - \left(\frac{4}{5}\right)\left(\frac{5}{13}\right)} = -\frac{163}{110}
\]

2) Given that $\sin \theta = -\frac{4}{5}$ with $\theta$ in quadrant IV, find $\cos 2\theta$.

\[
\cos 2\theta = 1 - 2\sin^2 \theta = 1 - 2\left(-\frac{4}{5}\right)^2 = 1 - \frac{32}{25} = -\frac{7}{25}
\]

3) Given that $\cos \theta = -\frac{5}{13}$ with $\theta$ in quadrant II, find $\tan 2\theta$.

\[
\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta} = \frac{2 \left(-\frac{12}{5}\right)}{1 - \left(-\frac{12}{5}\right)^2} = -\frac{24}{5} = \frac{120}{119}
\]

4) $\sin \theta = \frac{1}{4}$. $\theta$ lies in quadrant I. Find $\sin \frac{\theta}{2}$.

\[
\sin \frac{\theta}{2} = \sqrt{\frac{1 - \cos \theta}{2}} = \sqrt{\frac{1 - \frac{15}{16}}{2} \cdot \frac{4}{4}} = \sqrt{\frac{8}{115}}
\]

Simplify the expression.

5) $\frac{9 \sin^2 \alpha - 16}{4 \sin \alpha - 1} \cdot \frac{\sin^2 \alpha - 1}{21 \sin \alpha - 28}$

\[
\frac{(3 \sin \alpha - 4)(3 \sin \alpha + 4)}{4(\sin \alpha - 1)} \cdot \frac{(\sin \alpha - 1)(2 \sin \alpha + 1)}{7(3 \sin \alpha - 4)} = \frac{28}{28}
\]