1.3 Models and Applications

In this section we will look at solving word problems. There is a five step strategy for solving word problems:

**Step 1**: Read the problem carefully. Attempt to state the problem in your own words and state that the problem is looking for. Let x (or any variable) represent one of the unknown quantities of the problem.

**Step 2**: If necessary, write expressions for any other unknown quantities in terms of whatever variable you used in step 1.

**Step 3**: Write an equation in x (or whatever variable you chose) that models the verbal conditions of the problem.

**Step 4**: Solve the equation and answer the problem’s question.

**Step 5**: Check the solution in the original wording of the problem, not in the equation obtained from the words.

Use the five-step strategy for solving word problems to find the number or numbers described in the following example problems.

**EXAMPLE**: When two times a number is decreased by 3, the result is 11. What is the number?

**Step 1**: We will let x represent the number.

**Step 2**: There are not any other unknown quantities in this problem.

**Step 3**: The words “decreased by” mean subtraction, and two times a number involves multiplication. The words “the result is” means you will put an equals sign. Here is the resulting equation: \(2x - 3 = 11\).

**Step 4**: Now we solve this. First we add 3 to both sides: \(2x = 14\). Now divide both sides by 2. You will get the answer \(x = 7\).

**Step 5**: So if 2 times 7 is decreased by 3, this should equal 11. \(2(7) - 3 = 11\). This is true, so 7 is our answer.

**EXAMPLE**: When 80% of a number is added to the number, the result is 252. What is the number?

**Step 1**: We will let x represent the number.

**Step 2**: The expression for 80% of a number is 0.8x.

**Step 3**: The words “added to” means addition. The words “the result is” means you will put an equals sign. Here is the resulting equation: \(0.8x + x = 252\).

**Step 4**: Now we solve this. First we add the two x terms. There is a 0.8x and a 1x. This makes 1.8x. So our equation becomes \(1.8x = 252\). Divide both sides by 1.8 to get the answer: \(x = 140\).

**Step 5**: So 80% of 140 plus 140 should equal 252. So \(0.8(140) + 140 = 252\). You get \(112 + 140 = 252\). This is a true statement, so 140 will be our answer.
EXAMPLE: One number exceeds another by 24. The sum of the numbers is 58. What are the numbers?

Step 1: We will let x represent the first number.

Step 2: We need to write an expression of the second number in terms of x. The word “exceed” means you need to use addition. The expression we will use is \( x + 24 \).

Step 3: The words “sum of” means addition. The words “the result is” means you will put an equals sign. So we know: first number + second number = 58. So: \( x + x + 24 = 58 \).

Step 4: Now we solve this. First we add the two x terms. So our equation becomes \( 2x + 24 = 58 \). To solve this, subtract 24 from both sides. You will get \( 2x = 34 \). Divide both sides by 2 to get the answer: \( x = 17 \). This is the first number. The expression for the second number is \( x + 24 \). Put in a 17 for x to get the second number; \( 17 + 24 = 41 \). Therefore the first number is 17 and the second number is 41.

Step 5: To check this, first we see that the number 41 exceeds 17 by 24. Then we know that \( 41 + 17 = 58 \). Therefore our answers are correct.

EXAMPLE: Given: \( 10x + 6 = 12x - 7 \) and \( y_1 \) exceeds \( y_2 \) by 3, find \( x \).

Step 1: We already have an expression for \( y_1 \).

Step 2: We already have an expression for \( y_2 \).

Step 3: We need to come up with an expression involving \( y_1 \) and \( y_2 \). We know that \( y_1 \) is larger than \( y_2 \). We know that if we add 3 to \( y_2 \) then this will equal \( y_1 \). Therefore, \( y_1 = y_2 + 3 \).

Step 4: Now we solve this. In the equation, \( y_1 = y_2 + 3 \), we need to put in the substitutions for \( y_1 \) and \( y_2 \). Therefore, \( 10x + 6 = 12x - 7 + 3 \). Now we need to solve this. First we add the like terms on the right. The resulting equation is: \( 10x + 6 = 12x - 4 \). Now subtract 12x from both sides. You will get: \( -2x + 6 = -4 \). Now subtract 6 from both sides. You will get: \( -2x = -10 \). Now divide both sides by \(-2\). The answer is \( x = 5 \).

Step 5: To check this, we can plug in the \( x \) to find \( y_1 \) and \( y_2 \). So \( y_1 = 10(5) + 6 = 56 \). Also, \( y_2 = 12(5) - 7 = 53 \). In the expression, \( y_1 = y_2 + 3 \), we have \( 56 = 53 + 3 \). This is a true statement, so our answer is \( x = 5 \).

EXAMPLE: Find \( x \) given: \( y_1 = 2.5 \), \( y_2 = 2x + 1 \), \( y_3 = x \) and the difference between 2 times \( y_1 \) and 3 times \( y_2 \) is 8 less than 4 times \( y_3 \).

Step 1: We already have an expression for \( y_1 \).

Step 2: We already have an expression for \( y_2 \) and \( y_3 \).
Step 3: We need to come up with an expression involving \( y_1, y_2 \) and \( y_3 \). The word “difference” means subtraction. The phrase “8 less than 4 times \( y_3 \)” will equate to \( 4y_3 - 8 \). Therefore, the whole equation will be: 
\[ 2y_1 - 3y_2 = 4y_3 - 8. \]

Step 4: Now we solve this. In the equation, \( 2y_1 - 3y_2 = 4y_3 - 8 \), we need to put in the substitutions for \( y_1, y_2, \) and \( y_3 \). Therefore, \( 2(2.5) - 3(2x + 1) = 4(x) - 8 \). Now we need to solve this. First we multiply and distribute on the left side. The resulting equation is: \( 5 - 6x - 3 = 4x - 8 \). Now add like term. The equation now becomes: \( 2 - 6x = 4x - 8 \). Now subtract \( 4x \) from both sides. You will get: \( 2 - 10x = -8 \). Now subtract 2 from both sides. You will get: \( -10x = -10 \). Now divide both sides by \(-10\). The answer is \( x = 1 \).

Step 5: To check this, we can plug in the \( x \) to find \( y_1, y_2 \) and \( y_3 \). So \( y_1 = 2.5 \). Also, \( y_2 = 2(1) + 1 = 3 \) and \( y_3 = 1 \). In the expression, \( 2(2.5) - 3(3) = 4(1) - 8 \), we have \( 5 - 9 = 4 - 8 \). Therefore, \( -4 = -4 \). This is a true statement, so our answer is \( x = 1 \).

EXAMPLE: The average salary for computer programmers is $7740 less than twice the average salary for carpenters. Combined, their average salaries are $99,000. Determine the average salary for each of these jobs.

Step 1: We will let \( x \) represent the average salary for carpenters.

Step 2: We need to write an expression for the average salary of computer programmers in terms of \( x \). The expression we will use is \( 2x - 7740 \). The word “twice” means two times. The word “less than” means subtraction, and you usually reverse the order of how it was worded in the question. That is why the 7740 comes after the variable term here.

Step 3: The word “combined” means addition. The words “the result is” means you will put an equals sign. So we know: computer programmer salary + carpenter salary = 99000. The equation is: \( x + 2x - 7740 = 99000 \).

Step 4: Now we solve this. First we add the two \( x \) terms. So our equation becomes \( 3x - 7740 = 99000 \). To solve this, add 7740 to both sides. You will get \( 3x = 106740 \). Divide both sides by 3 to get the answer: \( x = 35580 \). This is the salary for carpenters. The expression for the salary for computer programmers is \( 2x - 7740 \). Put in 35580 for \( x \). So the salary for computer programmers is \( 2(35580) - 7740 = 63420 \). Another way to get this answer is to subtract 35580 from 99000.

Step 5: To check this, first we see that adding our two answers, 35580 + 63420 = 99000. Also, 7740 less than twice 35580 is: \( 2(35580) - 7740 = 63420 \).

EXAMPLE: You are choosing between two long-distance telephone plans. Plan 1 has a monthly fee of $20 with a charge of $0.05 per minute for all long distance calls. Plan 2 has a monthly fee of $5 with a charge of $0.10 per minute for all long distance calls. For how many minutes of long-distance calls will the costs for the two plans be the same?

Step 1: We will let \( x \) represent the number of minutes for long distance calls.

Step 2: We need to determine the cost equations for each plan. Each plan will have fixed costs plus variable costs. For Plan 1, the equation is \( C = 20 + 0.05x \). The cost for Plan 2 is \( C = 5 + 0.1x \).
Step 3: The question is asking for how many minutes will the costs for both plans be the same. This means we need to set the two costs equal to each other. Therefore, $20 + 0.05x = 5 + 0.1x$ is the correct setup.

Step 4: Now we solve this. First I will subtract $0.1x$ from both sides. The result is: $20 - 0.05x = 5$. Now I will subtract $20$ from both sides. This results in $-0.05x = -15$. Now divide both sides by $-0.05$ to get $x = 300$ min.

Step 5: To check this, we will find the cost for each plan. For Plan 1, $C = 20 + 0.05(300) = \$35$. For Plan 2, $C = 5 + 0.1(300) = \$35$. The costs of both plans are the same, so we know our answer is correct.

EXAMPLE: Including the 12% room tax for Las Vegas Strip hotels, a Las Vegas Strip hotel charges $168 per night. Find the hotel’s nightly cost before the tax is added.

Step 1: We will let $x$ represent the nightly cost before the tax is added.

Step 2: There is no other variable used in this problem.

Step 3: So we start with the original cost of the room. Then we will add 12% of the original cost. In equation form, it will look like this: $x + 0.12x = 168$.

Step 4: Now we solve this. First I will add the like terms on the left side. The result is: $1.12x = 168$. Now I will divide both sides by $1.12$. You will get $x = \$150$.

Step 5: To check this, we will start with $150$ and add 12% of this. So $150 + 0.12(150) = 150 + 18 = 168$. Therefore we know our answer is correct.

EXAMPLE: You invested $11,000 in two accounts paying 5% and 8% annual interest. If the total interest earned for the year was $730, how much was invested at each rate?

Step 1: We will let $x$ represent the amount invested at 5%.

Step 2: We know that the total invested in both the 5% and 8% is $11,000$. Therefore, $x + 8\% = 11,000$. If we solve for the 8% we get $11000 - x$. This means the amount invested at 8% should be equal to $11000 - x$.

Step 3: For this problem, we need to use the formula: Interest = Rate * Time (I = PRT). Since we have the word ‘annual’ then our time is equal to one year. We need to calculate the interest earned for each rate. Let’s start with the interest earned at 5%. In the formula $I = PRT$, the principle is the amount we are investing. From Step 1, this is $x$. Therefore $I = x*0.05*1$. For the interest earned at 8%, the principle was the expression from Step 2, which is $11000 - x$. Therefore $I = (11000 - x)*0.08*1$. The sum of these two interests is $730$. So our equation will be $0.05x + 0.08(11000 - x) = 730$.

Step 4: Now we solve this. First I will distribute the 0.08. The result is: $0.05x + 880 - 0.08x = 730$. Now I will add the like terms: $-0.03x + 880 = 730$. Now subtract 880 from both sides to get: $-0.03x = -150$. Divide both sides by $-0.03$ to get $x = \$5000$. This is the amount invested at 5%. To get the amount invested at 8%, just subtract this from $11,000$. The amount invested at 8% is $\$6000$.

Step 5: The interest at 5% for one year with a $5000 principle is: $I = 5000(0.05)(1) = 250$. The interest at 8% for one year with a $6000 principle is: $I = 6000(0.08)(1) = 480$. The total of these two interests is $730$. Therefore our answers are correct.
Example: The length of a rectangular pool is 6 meters less than twice the width. If the pool’s perimeter is 126 meters, what are its dimensions?

Step 1: We will let $x$ represent the width of the pool.

Step 2: We know the length is 6 less than twice the width. This expression is $2x - 6$.

Step 3: The word ‘perimeter’ means the sum of all the sides. In a rectangular there are two lengths and two widths. Therefore $P = 2L + 2W$. We can now plug in our expressions for the length and the width. We also are given that the perimeter is 126. You will get $126 = 2(2x - 6) + 2x$.

Step 4: Now we solve this. First I will distribute the 2. The result is: $126 = 4x - 12 + 2x$. Now I can add like terms on the right side. I will get $126 = 6x - 12$. Now add 12 to both sides and the result is $138 = 6x$. Divide both sides by 6 to get $x = 23$. This is width. To find the length, we will plug 23 into the expression $2x - 6$. This results in $2(23) - 6 = 40$.

Step 5: So the length is 40 and the width is 23. To check this, we will use the formula $P = 2L + 2W$. Therefore, $126 = 2(40) + 2(23)$. Simplifying gives: $126 = 80 + 46$. This is a true statement so our answers are correct.

Solving a Formula for One of Its Variables

These last few problems involve solving an equation in terms of the other variables. When solving for one variable, pretend that the other variables are numbers. Then isolate the variable the problem is asking for.

Example: Solve for $h$ in the following formula: $V = \pi r^2 h$.

$V = \pi r^2 h$ To solve this one, pretend the other variables are numbers. Isolate the $h$ term by dividing

$\frac{V}{\pi r^2} = h$ We are dividing by everything in front of the $h$ since division is the opposite of multiplication.

This is the answer. You answer will be in terms of letters.

Example: Solve for $b$ in the following formula: $A = \frac{1}{2} h(a + b)$.

$2\left(A = \frac{1}{2} h(a + b)\right)$ First multiply both sides by 2 to clear the fractions.

$2A = h(a + b)$ Now distribute the $h$.

$2A = ha + hb$ Isolate the term that has $b$ in it by subtracting $ha$ from both sides.

$2A - ha = hb$ Now divide both sides by $h$.

$\frac{2A - ha}{h} = b$ Just leave your answer in this form.