2.2 The Graph of a Function

This section will review function notation used in graphs in addition to domain, range, min, max.

EXAMPLE: Use the graph below to answer the following:

a.) Find the domain
Since we don’t include the endpoints we have (-2, 2) (x values)

b.) Find the range
The answer is (-1, 1] (y-values)

c.) Indicate the intercepts
x-int: (-1, 0) (1, 0)   y-int: (0, 1)

d.) Indicate any symmetry this graph has.
You can fold this in half over the y-axis, so it has y-axis symmetry.

EXAMPLE: Use the graph below to answer the following:

a.) Find \( f(-2) \):
This is asking you for the y value when x is -2.
The answer is \( f(-2) = 1 \).

b.) Find all x such that \( f(x) = 3 \)
This is asking you to find all x that give a y value of 3.
This happens at the point (5, 3), so x = 5.

c.) Is \( f(3) \) positive or negative?
This is asking you if the y value at x = 3 is above or below the x axis. To find this go over to x = 3. We notice the graph is below the x-axis, so answer is neg.

d.) What is the domain?
This is asking you for all the x values the graph uses.
This would be [-4, 6]. (lowest x to highest x)

e.) What is the range?
This is asking you for all the y values the graph uses.
The answer is [-2, 3]. (lowest y to highest y).

f.) For which values is \( f(x) > 0 \)?
This is asking you which part of the graph has positive y values. In other words, what part of the graph is above the x-axis, but not on the x-axis. We have two places this occurs. [-4, 0) or (4, 6) Notice the values I gave in the interval notation are x values. We include the -4 because it is not on the x-axis.
EXAMPLE: Use the graph below to answer the following:

![Graph with labeled points and positive x-intercepts]

a.) Find \( f(-1) \):
This is asking you for the y value when x is -1.
The answer is \( f(-1) = 2 \).
It does not matter if the x-value has a dot or not.

b.) Find all x such that \( f(x) = 0 \)
This is asking you to find all x that give a y value of 0.
This happens at \( x = -2, 3, \) and \( 5 \).

c.) Is \( f\left(-\frac{3}{2}\right) \) positive or negative?
This fraction is the same as -1.5. When you go to this x value the graph is above the x-axis here, so positive.

d.) What is the domain?
The domain is referring to the x-values the graph uses.
Since there is an open circle at \(-3\), this x-value is not included. So the domain is: \((-3, 6]\).

e.) What is the range?
The range is referring to the y-values the graph uses.
Again since there is an open circle at \(-3\), this y-value is not included. So the range is \((-4, 4]\).

f.) Indicate the x and y intercepts.
y-int: \((0, 3)\)  x-int: \((-2, 0), (3, 0), (5, 0)\).

g.) Indicate what kind of symmetry, if any, this graph has.
This graph does not have any symmetry.

EXAMPLE: Use the equation \( f(x) = -2x^2 + 3x \) to answer the following:

a.) Is the point \((-3, 27)\) on the graph?
To answer this, substitute a -3 for x and a 27 for f(x) in the equation.
\[
27 = -2(-3)^2 + 3(-3)
\]
\[
27 = -2(9) - 9
\]
This last statement is not true so we know \((-3, 27)\) is not on the graph.
b.) If \( x = -1 \) then what is \( f(x) \)? What is the point on the graph?

For this one we just put in a -1 for \( x \). You will get:

\[
\begin{align*}
  f(-1) &= -2(-1)^2 + 3(-1) \\
  f(-1) &= -2(1) - 3 \\
  f(-1) &= -5
\end{align*}
\]

As a point you would write (-1, -5).

\[
\begin{align*}
  f(-1) &= -5
\end{align*}
\]

c.) If \( f(x) = 0 \) then what is \( x \)?

This is the same as asking what the x-intercept is. \( F(x) \) is the same as \( y \). So put a zero in for \( y \).

\[
\begin{align*}
  0 &= -2x^2 + 3x \\
  0 &= x(-2x + 3)
\end{align*}
\]

When we set these equal to zero we get \( x = 0 \) and \( x = \frac{3}{2} \). However we need to write these in the proper form for x-intercepts: \( (0,0) \) and \( \left( \frac{3}{2},0 \right) \).

d.) What is the domain?

For this one there are no square roots or fractions in \( f(x) \). Therefore there will be no conditions that will make the function undefined, so we are allowed to use any number for \( x \). So the domain is all reals. This is the same as \( (-\infty, \infty) \).