5.7 Financial Models

This section and the next section will look at some applications to exponential and logarithmic equations. First we will focus on an important application of exponential equations, which deal with calculating interest.

**Compound Interest**

The amount $A$ after $t$ years due to a principle $P$ invested at an annual interest rate $r$ (expressed as a decimal) compounded $n$ times per year is $A = P \cdot \left(1 + \frac{r}{n}\right)^{nt}$. The compound interest formula actually starts with the simple interest formula, $I = Prt$. Interest is calculated once, and then this interest is added to the principle, and the formula is repeated again. This process continues until it is compounded $n$ times. So instead of that long process, we have the compound interest formula.

In working with these kinds of problems, they will give you different payment periods as listed below, which also tell you what to enter for $n$:

- **Annually**: Once a year, $n = 1$
- **Semiannually**: Twice a year, $n = 2$
- **Quarterly**: Four times a year, $n = 4$
- **Monthly**: 12 times a year, $n = 12$
- **Daily**: 365 days per year, $n = 365$

EXAMPLE: $300 is invested at 12% compounded monthly for $1\frac{1}{2}$ years. Find the amount that results from this investment.

Here $n = 12$ since it is compounded monthly. The principle is $P = 300$, and $r = 0.12$ and $t = 1.5$. We will put these into the compound interest formula: $A = 300 \cdot \left(1 + \frac{0.12}{12}\right)^{12(1.5)}$. First simplify inside the parenthesis and also multiply the $n$ and $t$ together in the exponent position: $A = 300 \cdot (1.01)^{18}$. Now we will raise 1.01 to the power of 18. You will get $A = 300 \cdot 1.196147...$. Now multiply to get our answer: $A = 358.84$.

**Continuous Compounding**

If you take the same compound interest formula as above and have $n$ go to infinity, you will get $A = P \cdot e^{rt}$, which is the formula for compounding continuously.

EXAMPLE: $300 is invested at 12% compounded continuously for $1\frac{1}{2}$ years. Find the amount that results from this investment.

Since we are compounding continuously, we will use $A = P \cdot e^{rt}$, which $P = 300$, $r = 0.12$, and $t = 1.5$. Plug it in to get: $A = 300 \cdot e^{0.12(1.5)}$. Work the exponent part first: $A = 300 \cdot e^{0.18}$. Now we will raise $e$ to the power of 0.18. Use the $e^x$ key on your calculator: $A = 300 \cdot (1.197217...)$ . So $A = 359.17$. Notice that this yields a slightly higher investment when compared to compounding monthly.
EXAMPLE: Determine the rate that represents the better deal:

- 9% compounded quarterly or \(\frac{9}{4}\)% compounded annually

For this problem, they do not give you a principle or time. To make it easy, let’s assume \(P = 1000\) and \(t = 1\). Then we will apply the compound interest formula for each one separately. Note that we can use any value for \(P\) and \(t\) as long as the same ones are used for each rate. Now let’s calculate:

\[
9\% \text{ compounded quarterly (} n = 4, \ r = 0.09 \) \quad A = 1000 \cdot \left(1 + \frac{0.09}{4}\right)^{4(1)} = 1000(1.0225)^4 = \$1093.08
\]

\[
\frac{9}{4}\% \text{ compounded annually (} n = 1, \ r = 0.0925 \) \quad A = 1000 \cdot \left(1 + \frac{0.0925}{1}\right)^{1(1)} = 1000(1.0925) = \$1092.50
\]

The first option (9% compounded quarterly) is the better deal. It may not seem like much savings with such a small principle, however if you have a principle of, say, a million dollars, then the savings become more significant.

EXAMPLE: What rate of interest compounded annually is required to triple an investment in 10 years? (Round your answer to the nearest thousandth.)

For this one, our initial principle is \(P\). If we want this to triple after ten years, then \(A = 3P\). Let’s put this into our compound interest formula. Here \(n = 1\) and \(t = 10\).

\[
A = P \cdot \left(1 + \frac{r}{n}\right)^{nt}
\]

\[
3P = P \cdot \left(1 + \frac{r}{1}\right)^{1(10)} \quad \text{Divide both sides by } P \text{ and simplify.}
\]

\[
3 = (1 + r)^{10}
\]

Now take the 10\(^{th}\) root of both sides.

\[
\sqrt[10]{3} = 1 + r
\]

Subtract 1 from both sides

\[
\sqrt[10]{3} - 1 = r
\]

Now convert this into a decimal. You may want to write \(\frac{1}{10} - 1 = r\) if your calculator cannot do this kind of root.

\[
1.116123 - 1 = r
\]

\[
0.116123\ldots = r \quad \text{Now convert into a percent}
\]

\[
r = 11.612\%
\]

So the annual interest rate required to triple the principle in 10 years is 11.612%.

NOTE: If the problem asked for the interest rate to double the investment, then \(A = 2P\).