1. Find a set of parametric equations and also a set of symmetric equations of the line through the given point parallel to the indicated line or vector. For each line, express the direction numbers as integers:

(a) \((0,0,0)\) \(v = (1, 2, 3)\)

(b) \((-2,0,3)\) \(v = 2i + 4j - 2k\)

(c) \((1,0,1)\) \(\frac{x-3}{3} = \frac{y}{5 - 2t}, \quad \frac{z}{-7} + t\)

(d) \((-3,5,4)\) \(\frac{x-1}{3} = \frac{y+1}{-2} = z-3\)

2. Find a set of parametric equations and also a set of symmetric equations of the line through the two points (express the direction numbers as integers):

(a) \((5,-3,-2), \ (-2/3, 2/3, 1)\)

(b) \((1,0,1), \ (1, 3, -2)\)

3. Find a set of parametric equations of the line described:

(a) The line passes through \((2,3,4)\) and is parallel to the \(xz\)-plane and the \(yz\)-plane.

(b) The line passes through \((2,3,4)\) and is perpendicular to the plane given by \(3x + 2y - z = 6\).

4. Determine whether the following pairs of lines intersect, and if so, find the point of intersection and the cosine of the acute angle between the planes:

(a) Line #1 is: \(x = 4t + 2, \ y = 3, \ z = -t + 1\)
   Line #2 is: \(x = 2s + 2, \ y = 2s + 3, \ z = s + 1\)

(b) Line #1 is: \(\frac{x}{3} = \frac{y-2}{-1} = z+1\).
   Line #2 is: \(\frac{x-1}{4} = y+2 = \frac{z+3}{-3}\)
5. Find an equation of the plane passing through the point that is perpendicular to the given vector or line:
   (a) (2,1,2) \quad n = i
   (b) (3,2,2) \quad n = 2i+3j-k
   (c) (0,0,6) \quad x=1-t, \ y = 2+t, \ z = 4-2t.

6. Find an equation of the plane passing through (0,0,0), (1,2,3) and (-2,3,3).

7. Find an equation of the plane containing the lines given by:
   Line #1: \( (x-1)y-2 = y-4 \) and Line #2: \( (x-2)y-3 = (y-1)/4 = (z-2)/-1 \).

8. Find an equation of the plane passing through the points (2,2,1) and (-1,1,-1) that is perpendicular to the plane \( 2x-3y+z = 3 \).

9. Determine the angle of intersection of the following pairs of planes:
   (a) \( 5x-3y+z = 4 \) and \( x+4y+7z = 1 \).
   (b) \( x-3y+6z = 4 \) and \( 5x+y-z = 4 \).
   (c) \( x-5y-z = 1 \) and \( 5x-25y-5z = -3 \).

10. Mark the intercepts and make a rough sketch of the graph of each of the following planes:
    (a) \( 4x+2y+6z = 12 \)
    (b) \( 2x-y+3z = 4 \)
    (c) \( y+z = 5 \)

11. Find the intersection point, if any, of the point and the line. Also determine whether the line lies in the plane:
    Plane: \( 2x-2y+z = 12 \)
    line: \( x-1/2 = (y+3/2)/-1 = (z+1)/2 \)

12. Find the distance between the point (1,2,3) and the plane \( 2x+3y+z = 4 \).

13. Find the distance between the planes \( x-3y+4z = 10 \) and \( x-3y+4z = 6 \).
Solutions for Lines and Planes

(a) \[ \frac{x-o}{1} = \frac{y-o}{2} = \frac{z-o}{3} \]
\[ x = \frac{y+2}{2} = \frac{z+3}{3} \text{ symmetric form} \]
\[ \begin{array}{l}
  x = t \\
  y = 2t \\
  z = 3t
\end{array} \text{ parametric form} \]

(b) \[ \frac{x+2}{2} = \frac{y-o}{4} = \frac{z-3}{-2} \text{ symmetric form} \]
\[ \begin{array}{l}
  x = 2t - 2 \\
  y = 4t \\
  z = -2t + 3
\end{array} \text{ parametric form} \]

(c) \[ \frac{x-1}{3} = \frac{y-o}{-2} = \frac{z-1}{1} \text{ symmetric form} \]
\[ \begin{array}{l}
  x = 3t + 1 \\
  y = -2t \\
  z = t + 1
\end{array} \text{ parametric form} \]

(d) \[ \frac{x+3}{3} = \frac{y-5}{-2} = \frac{z-4}{1} \text{ symmetric form} \]
\[ \begin{array}{l}
  x = 3t - 3 \\
  y = -2t + 5 \\
  z = t + 4
\end{array} \text{ parametric form} \]
(2) (a) \[ \vec{v} = \left[ 5 - \left( -\frac{4}{3} \right) \right] \hat{i} + \left( -3 - \frac{2}{3} \right) \hat{j} + (-2 - 1) \hat{k} \]
\[ = \frac{17}{3} \hat{i} - \frac{11}{3} \hat{j} - 3 \hat{k} \]
\[ \frac{x-5}{17/3} = \frac{y+3}{-11/3} = \frac{z+2}{-3} \]

Symmetric form:
\[ \frac{x-5}{17/3} = \frac{y+3}{-11/3} = \frac{z+2}{-3} \]

Parametric form:
\[ x = \frac{17t + 5}{17} \]
\[ y = \frac{-11t - 3}{-11} \]
\[ z = \frac{-3t - 2}{-3} \]

(b) \[ \vec{v} = (0, 3, -3) \] so line is parallel to yz-plane, so

No symmetric form. Parametric form is:
\[ x = 1 + 3t \]
\[ y = 3 + 0t \]
\[ z = -3t + 1 \]

(b) \[ \begin{cases} x = 2 + 3t \\ y = 3 + 2t \\ z = 4 - t \end{cases} \]
\( 4 \) (a) \[ y_1 + 2 = 2x + 2 \implies x = 2t \]

So \[ 3 = 2x + 3 \implies x = t \]

\[ \therefore t = \frac{1}{2} \implies \frac{1}{2} (0) = 0 \]

So for Line 1: \[ x = y(0)+2 = 2 \]

\[ y = \frac{3}{2} \]

\[ 3 = -0+1 = 1 \]

for Line 2: \[ x = 2y+2 = 2 \]

\[ y = 2(0)+3 = 3 \]

\[ 3 = 0+1 = 1 \]

\[ \therefore (2,3) \text{ is point of intersection} \]

\[ \mathbf{V}_1 = (4, 1, -1), \quad \mathbf{V}_2 = (2, 3, 1) \]

\[ \mathbf{V}_1 \cdot \mathbf{V}_2 = 7 = \sqrt{7} \sqrt{9} \]

\[ \therefore \theta = \frac{7}{3\sqrt{7}}, \text{ etc.} \]

(b) \[ x = 3t, \quad y = -t+2, \quad z = t-1 \text{ for Line 1} \]

\[ x = 4s+1, \quad y = s-2, \quad z = -3s-3 \text{ for Line 2} \]

\[ 3t = 4s + 1 \implies 3t - 4s = 1 \]

\[ -t + 2 = s - 2 \implies t + s = 4 \]

\[ \therefore \frac{3t - 4s = 1}{4t + 4s = 16} \]

\[ \frac{7t = 17}{t = 17/7} \]

\[ \therefore \text{Lines don't intersect} \]

Then for Line 1: \[ x = 3(17/7) = 51/7 \]

for Line 2: \[ x = 4(17/7) = 75/7 \] (Contradiction!!!)
(a) \(1(x-2) + 0(y-1) + 0(z-2) = 0\)

\[x = 2\]

(b) \(2(x-3) + 3(y-2) + 1(z-2) = 0\), \(e + c\)

(c) \(-1(x-0) + 1(y-0) + 2(z-0) = 0\), \(e + c\)

(6) \(A(0,0,0)\)
\(B(1,1,3)\)
\(C(-2,3,3)\)

\[\overrightarrow{N} = \overrightarrow{AB} \times \overrightarrow{AC} = (1,3) \times (-2,3,2)\]

\[= (3,-9,7)\]

So equation of plane is

\[-3(x-0) - 9(y-0) + 7(z-0) = 0\]

or simply

\[3x + 9y - 7z = 0\]

(7) Let \(\overrightarrow{N} = (-3,1,4) \times (-2,1,1) = (5, 5, 5)\)

Then \(\overrightarrow{N}\) is normal to desired plane. Note \((1,4,9)\) lies on a line in desired plane, so \((1,4,9)\) lies in desired plane; so its equation is:

\[5(x-1) + 5(y-4) + 5(z-0) = 0\]

or simply

\[x + y + z = 5\]

(8) \(\overrightarrow{V}_1 = (2,-3,1)\) is parallel to desired plane
\(\overrightarrow{V}_2 = (3,4,-1,1)\) is also parallel to desired plane.

Let \(\overrightarrow{N} = \overrightarrow{V}_1 \times \overrightarrow{V}_2 = (-7,-1,11)\); normal to desired plane.

So eq of desired plane is

\[-7(x+1) - 1(y-1) + 11(z+1) = 0\]

or simply

\[-7x - y + 11z = -5\]
(a) \( \mathbf{v}_1 \cdot \mathbf{v}_2 = (5,-3,1) \cdot (1,4,7) = 0 \): \( \theta = 90^\circ \)

(b) \( \mathbf{v}_1 \cdot \mathbf{v}_2 = (1,3,6) \cdot (5,1,-1) = -4 \)
\[
-4 = \sqrt{1+9+36} \cdot \sqrt{25+1+1} 
\theta = \cos^{-1}\left(\frac{-4}{\sqrt{42} \cdot \sqrt{27}}\right) = 96.52^\circ 
\]

(c) \( \mathbf{v}_1 \cdot \mathbf{v}_2 = (1,5,-1) \cdot (5,2,5,5) = 135 \)
\[
135 = \sqrt{1+25+1} \cdot \sqrt{25+62.5+25} 
\theta = \cos^{-1}\left(\frac{135}{\sqrt{452.5}}\right) = \cos^{-1}(1) = 0 
\]

(iso planes are parallel.)
parametric form of line: \[
\begin{align*}
\gamma &= t + \frac{1}{2} \\
y &= -t - \frac{3}{2} \\
z &= 2t - 1
\end{align*}
\]

\[2(t + \frac{1}{2}) - 2(-t - \frac{3}{2}) + 2t - 1 = 12\]

\[t = 3\]

then \[
\begin{align*}
\gamma &= \frac{3}{2} + \frac{1}{2} = 2 \\
y &= -\frac{3}{2} - \frac{3}{2} = -3 \\
z &= 2(3) - 1 = 5
\end{align*}
\]

so \((2, -3, 2)\) is point of intersection. The line does not lie in the given plane.

\[P(1, 2, 3). \text{ Note } \vec{u}(2, 1, 0) \text{ lies in given plane and } \vec{N} = (2, 3, 1) \text{ is perp. to given plane. So...}\]

\[D = \|\text{proj}_{\vec{N}} \vec{PA}\| = \frac{\vec{PA} \cdot \vec{N}}{\|\vec{N}\|} = \frac{7}{\sqrt{14}}, \text{ etc.}\]

\[A(3, 9, 10) \text{ is in one plane and } B(2, 3, 4) \text{ lies in the other, } \vec{N} = (1, -3, 4) \text{ is perp. to both planes.}\]

\[D = \|\text{proj}_{\vec{N}} \vec{AB}\| = \frac{\vec{AB} \cdot \vec{N}}{\|\vec{N}\|} = \frac{7}{\sqrt{26}}, \text{ etc.}\]