How to Find Solutions of Absolute Value Equations

If \( c \) is positive, \(|ax + b| = c\) is equivalent to \((ax + b) = c\) or \(-(ax + b) = c\).

You have to solve both equations!!! The word "or" is a part of the formula and must be there. It does not mean that you can use either one of the equations when you solve the variable.

- Isolate the absolute value on one side of the equation. Be sure the coefficient is 1.
- Solve both equations \((ax + b) = c\) or \(-(ax + b) = c\).
- Check the solutions in the original equation, rejecting any that do not satisfy it.

**Problem 1:**

Solve \(|4 - 5x| - 19 = 0\)

Let's isolate the absolute value as follows:

\[|4 - 5x| = 19\]

By definition, \((4 - 5x) = 19\)

or \(-(4 - 5x) = 19\)

\[-4 + 5x = 19\]

and \(-5x = 15\) or \(5x = 23\)

then \(x = \frac{23}{5}\) or \(x = -3\)
Checking the solutions in the original equation we find

\[ |4 - 5(-3)| = |4 + 15| = |19| = 19 \]

\[ |4 - 5\left(\frac{23}{5}\right)| = |4 - 23| = |-19| = 19 \]

Problem 2:

Solve \(|3x| - 6 = 0\)

Let's isolate the absolute value as follows:

\[ |3x| = 6 \]

By definition, \(3x = 6\) or \(-3x = 6\)

and \(x = 2\) or \(x = -2\)

Checking the solutions in the original equation we find

\[ |3(2)| = |6| = 6 \]

\[ |3(-2)| = |-6| = 6 \]

Problem 3:

Solve \(|x + 2| + 1 = 0\)

Let's isolate the absolute value as follows:

\[ |x + 2| = -1 \]

Please note that an absolute value is never equal to a negative number. There is actually NO solution to this problem. However, let's just go ahead and pretend that we did not notice.

By definition, \(x + 2 = -1\)

or \(-(x + 2) = -1\)

\[-x - 2 = -1\]

and \(x = -3\) or \(x = -1\)
Checking the solutions in the original equation we find

\[ |-3 + 2| + 1 = |-1| + 1 = 1 + 1 = 2 \neq 0 \]
\[ |-1 + 2| + 1 = |1| + 1 = 1 + 1 = 2 \neq 0 \]

We find that there is NO solution.

**Problem 4:**

Solve \(|x - 4| = 7\)

By definition, \(x - 4 = 7\)

or \(-(x - 4) = 7\)

\[-x + 4 = 7\]

and \(x = 11\) or \(x = -3\)

Checking the solutions in the original equation we find

\[ |11 - 4| = |7| = 7 \]
\[ |-3 - 4| = |-7| = 7 \]

**Problem 5:**

Solve \(|3x + 2| - 1 = 4\)

Let's isolate the absolute value as follows:

\[ |3x + 2| = 5 \]

By definition, \(3x + 2 = 5\)

or \(-(3x + 2) = 5\)

\[-3x - 2 = 5\]

\[-3x = 7\]

\[x = -\frac{7}{3}\]

and \(x = 1\) or \(x = -3\)

**Problem 6:**

Solve \(|x| = 15\)

By definition, \(x = 15\) or \(-x = 15\)

and \(x = 15\) or \(x = -15\)