§1 Polygons

A polygon is any closed shape with 3 or more sides. They are named according to the number of sides. Note that the sides and angles do not necessarily all have to be the same length. If it does turn out that the polygon has all equal sides and equal angles, then it is called a regular polygon.

A quadrilateral is any four-sided closed shape. It turns out that there are different kinds of quadrilaterals, based on the relationship of the sides and the angles. Please see the table below for some of the different types.

<table>
<thead>
<tr>
<th>Name</th>
<th>Characteristics</th>
<th>Representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parallelogram</td>
<td>Quadrilateral in which both pairs of opposite sides are parallel and have the same measure. Opposite angles have the same measure.</td>
<td><img src="image" alt="Parallelogram" /></td>
</tr>
<tr>
<td>Rhombus</td>
<td>Parallelogram with all sides having equal length.</td>
<td><img src="image" alt="Rhombus" /></td>
</tr>
<tr>
<td>Rectangle</td>
<td>Parallelogram with four right angles. Because a rectangle is a parallelogram, opposite sides are parallel and have the same measure.</td>
<td><img src="image" alt="Rectangle" /></td>
</tr>
<tr>
<td>Square</td>
<td>A rectangle with all sides having equal length. Each angle measures 90°, and the square is a regular quadrilateral.</td>
<td><img src="image" alt="Square" /></td>
</tr>
<tr>
<td>Trapezoid</td>
<td>A quadrilateral with exactly one pair of parallel sides.</td>
<td><img src="image" alt="Trapezoid" /></td>
</tr>
</tbody>
</table>

§2 Perimeter

The perimeter of a polygon is simply the sum of the lengths of its sides. Remember, in a regular polygon, all the sides have the same length and all the angles have the same measure.

For example, let’s say that the length of the side of regular octagon is 6 inches. Then what is the perimeter of the octagon?

Well, we know that a octagon has eight sides. Since its regular, we also know that all the sides have the same length. So in this case, the perimeter is going to by $8 \times 6 = 48$ inches squared.

We can do some applications of perimeter as well. For example, let’s say a rectangular field has a length of 28 feet and a width of 16 feet. If fencing costs $4 per foot, how much will it cost to enclose the field?

To find the answer, we first need to find the perimeter of the field. Then, we need to multiply the perimeter by the price of fencing per foot. This result will give us the total cost.

The perimeter of the field then becomes $28 + 28 + 16 + 16 = 88$ feet. Since fencing costs $4 per foot, the total cost will be $88 \times 4 = 352$. 
PRACTICE

1) Find the perimeter of an equilateral triangle whose side measures 8 inches.

2) If the length of a rectangular field is 20 meters and the width of the field is 26 meters, how much will it cost to enclose the field with fencing if the fence costs $2.50 per meter?

3) Find the perimeter of the following figure:

![Image of a figure with sides measuring 3 ft, 3 ft, 6 ft, and 4 ft.]

§3 The Sum Of The Measures Of A Polygon’s Angles

We know that a triangle has three sides and three angles and that the sum of the angles in a triangle is 180 degrees. Is there a formula we can use to find the sum of the angles of any polygon? The key is to divide the polygon into as many triangles as we can, without overlapping any of the sides. The easiest way to do this is to pick a single point where two sides meet and draw line segments to the other points.

What you should notice that the number of triangles formed is ALWAYS equal to 2 less than the number of sides of the polygon. Remember, the sum of the angles in a triangle is 180 degrees. Hence, any 5-sided shape can be divided into three triangles, which means the sum of all the angles in a pentagon is 540 degrees! Similarly, any 6 sided shape can be divided into four triangles, which means the sum of the angles in a hexagon is 720 degrees.

The general formula is as follows: For any n-sided closed polygon, the sum of the angles of the polygon is equal to \((n - 2)180^\circ\)

One important thing to note – if the shape is a regular polygon, then all the angles have the same measure. So in a regular pentagon, we know that there are five angles. The sum of all five angles is 540 degrees, but because its regular we know that all the angles have the same measure! Hence we find the measure of each angle by dividing the sum of the angles by the number of angles. In a regular pentagon, the measure of each angle is 108 degrees. Hence we can conclude that in a regular hexagon, each angle going to be 120 degrees.

PRACTICE

3) Find the measure of each angle of a regular octagon.