Section 11.7: Events Involving And; Conditional Probability

§1 Independent Events

Two events $A$ and $B$ are called independent if the occurrence of either of them has no effect on the probability of the other. A common example is the experiment of flipping a fair coin two times in succession. Does what you get on the first toss have any effect on the probability of what will happen on the second toss? We can see that it does not. Hence we would call this event independent.

Hence if we know what the probability of $A$ is, and we know what the probability of $B$ is, then the probability of $A$ and $B$ occurring is simply the product of the individual probabilities. We can write this as $P(A \text{ and } B) = P(A) \cdot P(B)$.

When dealing with questions that deal with multiple events, you should always first ask the question, ‘Are these events independent of each other?’ If you know the probability of each of the events, then you can find the probability that both events will occur.

For example, the probability of flipping a heads is $1/2$. What is the probability of flipping two heads in a row? Since we know that these are independent events, the probability of flipping two heads is $\frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$.

We can use this same rule for as many events in the experiment. For example, what is the probability of flipping 5 heads in a row? We know that each flip is independent of the next, and the probability of getting a heads on any one flip is $1/2$.

Hence, the probability of flipping five heads in a row is $\left(\frac{1}{2}\right)^5 = \frac{1}{32}$.

PRACTICE

1) A card is drawn from a standard 52-card deck, placed back into the deck, and then another card is drawn. What is the probability that both cards are spades? What is the probability that both cards are aces?

2) A fair die is rolled three times in succession. What is the probability that an even number is rolled all three times? What is the probability that a two is rolled all three times?

3) The probability that a certain town will get hit by a tornado in any single year is $\frac{1}{8}$. What is the probability that the town will get by a tornado for four straight years? What is the probability that the town will NOT get hit by a tornado for 5 straight years?

§2 The Probability Of ‘At Least Once’

This concept gets a little confusing. Let’s look at practice problem 3 dealing with the tornado. What is the probability that the town will get hit by a tornado at least once in the next 4 years?

Note that at least once means that the town could get hit once, or twice, or three times, or four times. We can find all these probabilities and add them together but that takes too much work. However, we know from the previous section that $P(\text{not } E) + P(E) = 1$. Hence what is the complement of at least one tornado in the next 5 years? The complement is no tornado for the next five years. Hence the probability of at least one tornado in the next 5 years is equal to $1 - P(E) = 1 - \left(\frac{1}{8}\right)^5$, which is 0.9999694.

In general, we can say that $P(\text{event happening at least once}) = 1 - P(\text{event does not happen})$. 
4) A committee of 5 people is to be selected from a group of 8 males and 6 females. What is the probability that the committee consists of no females? What is the probability that the committee will consist of at least one female?

§3 Dependent Events

In probability, two events are said to be dependent if the occurrence of one of them has an effect on the probability of the other. We see this quite often when we repeat an experiment without replacing items. For example, let’s say we draw two cards from a deck, without replacement. This means that once you draw the first card, you don’t put it back in the deck. You proceed with drawing the second card. What is the probability that both cards are spades?

Note that the probability depends on the first card being a spade. We know that his probability is $\frac{1}{4}$. But about for the second card? We need to find the probability that the second card is a spade GIVEN that the first card was a spade. If the first card was a spade, then that means there are 12 spades left in the deck, and also there are 51 cards left in the deck. Hence the probability of drawing two spades is

$$\frac{13}{52} \times \frac{12}{51} = \frac{1}{17} = 5.88\%.$$ Whenever we are dealing with dependent events, you need to calculate the probability of the first event, and then calculate the probability of the second event knowing that the first event has already occurred.

Try this one. Say a certain jar has 4 red, 5 white, and 8 blue balls. Two balls are drawn without replacement. What is the probability the both balls are blue? We need to find the probability that the first ball is blue, times the conditional probability that the second ball is blue GIVEN that the first ball was blue. The probability that the first ball is blue is $\frac{8}{17}$, since there are 8 blue balls and 17 balls total. The probability that the second ball is blue is then $\frac{7}{16}$, because once a blue ball is drawn first, there are 7 blue balls left and 17 balls overall left. Hence the answer is

$$\frac{8}{17} \times \frac{7}{16} = \frac{7}{34} = 20.59\%.$$ 

§4 Conditional Probability

Conditional probability is similar to dependent events. The probability of event B, given that the event A has already occurred, is denoted by $P(B|A)$. Basically, conditional probability is helpful because it provides a little more information regarding the event. For example, we know if we draw one card from a standard 52-card deck, the probability of drawing a king is $\frac{1}{13}$. Now try to answer this question – what is the probability of drawing a king if we know that the card is a picture card?

We know that there are 4 kings. We also know that there are 12 picture cards. Hence the probability of drawing a king, given that the card is a picture card, is $\frac{4}{12}$, or $\frac{1}{3}$.

In general, the formula is

$$P(B|A) = \frac{P(B \text{ and } A)}{P(A)}.$$

PRACTICE

5) A panel consists of 10 football players, 8 basketball players, and 5 baseball players. Suppose that two people are selected at random. What is the probability that two baseball players are selected? What is the probability that a football player then a basketball player is selected?
6) Use the following table to answer the questions.

<table>
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<tr>
<th></th>
<th>Owns A Smartphone</th>
<th>Owns A Regular Cell Phone</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>40</td>
<td>24</td>
<td>64</td>
</tr>
<tr>
<td>Female</td>
<td>60</td>
<td>36</td>
<td>96</td>
</tr>
<tr>
<td>Total</td>
<td>100</td>
<td>60</td>
<td>160</td>
</tr>
</tbody>
</table>

If one person is selected at random from this population, find the probability that the person:

a) is male

b) owns a smartphone

c) is male and owns a smartphone

d) owns a smartphone, given that the person is male

e) is male, given that the person owns a smartphone