Section 2.1: Basic Set Concepts

§1 Sets

The topic of this chapter is set theory. A set can be thought of as a collection of objects. That seems simple enough. The days of the week can be thought of as a set. There are 7 objects in that set. Similarly, the months of the year can be a set. There are 12 objects in that set. In a set, the objects must be clearly determined. For example, I couldn’t ask you for the set consisting of the five best basketball players of all time. This set is not clearly determined, since the objects would be subjective.

We generally use capital letters to name sets. For example, we could call the set \( W = \{ \text{Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday} \} \). A couple things to note about how we express sets:

i) The order that the objects in the set are listed does not matter.

ii) We always use braces to represent the objects in the set. These objects are also called elements. We never use parentheses or brackets.

We can describe a set one of three ways. The first is using a description. The second is called set-builder notation. The third is called the roster method because we simply list the elements of the set.

For example, we can say that the set \( A \) is the set of states that begin with the letter C. Using a roster, we can express the set \( A \) as \( \{ \text{California, Colorado, Connecticut} \} \).

Set-builder notation is a little different. We would express the set \( A \) as \( A = \{ x | x \text{ is state whose name begins with the letter } C \} \). The format is always the same. We use the letter \( x \) to represent the elements that are to be determined. The vertical line means ‘such that.’ So we can literally read this as ‘the set of all elements \( x \) such that \( x \) is a state whose name begins with the letter \( C \).’

PRACTICE

1) If the set \( B \) consists of the elements \( \{1, 3, 5, 7\} \), then represent the set using a description and also using set-builder notation.

2) If the set \( A \) consists of the last five letters of the alphabet, then represent the set using set-builder notation and the roster method.

§2 The Empty Set

Not all sets need to have at least one element. These sets have no objects that satisfy the conditions necessary to be an element of the set. For example, let’s say the set \( P \) is the set of all female U.S. Presidents. This set has no elements! Hence this set is called the empty set or the null set. We can express this one of two ways. The first is using the braces but with nothing in between, like \( \{} \). The other is to use the symbol for the null set, \( \emptyset \).

Hence, in the above example, we would say that the set \( P \) is the empty set, or we can express it as \( \{} \) or as \( \emptyset \).

IMPORTANT!

\( \{} \) and \( \emptyset \) have the same meaning – they both represent the empty set. What does \( \{ \emptyset \} \) represent?
§3 Notations For Set Membership

There are two ways to denote membership of a set. Either an object is an element of a set or it is not. The symbol for ‘is an element of’ is ∈, which is like an uppercase E but with rounded edges. Similarly, the symbol for ‘is not an element of’ is ∉.

For example, if we call set $O$ the set of all odd numbers, then we can say that $1 \in O$ but $8 \notin O$, since 8 is not an odd number.

If we call the set $A$ the set of letters in the alphabet, then we can see that $a \in A$ but $7 \notin A$.

PRACTICE

3) True or false, $7 \notin \{x \mid x \text{ is an even number}\}$

4) True or false, $\{j\} \in \{a, b, c...z\}$

§4 Sets of Natural Numbers

The natural numbers is actually a set itself! Remember, a set is any collection of objects. Then what are the natural numbers? If we call the set of natural numbers $N$, then $N = \{1, 2, 3,...\}$. Note that the three dots means that there is no final element so the set continues on. Other sets that we deal with are the set of whole numbers and the set of integers.

The set of whole numbers $W = \{0,1, 2,3,...\}$. The set of integers $I = \{..., -2, -1,0,1, 2,...\}$.

§5 Cardinality and Equivalent Sets

The cardinality of a set is simply the number of distinct elements in set. We can express this as $n(A)$, or ‘n of A’. Remember, the cardinality refers to the distinct elements in the set, so we do not count any repeating elements in the set. For example, if the set $V$ represents the vowels of the alphabet, then $n(V) = 5$. If the set $C$ represents the natural numbers less than 8, then $n(C) = 7$. However, although the set $D = \{1,1, 2,3, 4,5, 6, 6\}$ has 8 elements, there are 6 distinct elements, hence $n(D) = 6$.

Two sets are called equivalent if they have the same cardinality, i.e. they have the same number of elements. Hence if two sets $A$ and $B$ are equivalent, we can say that $n(A) = n(B)$.

Two sets are called equal if they have the same exact elements, regardless of order or possible repetition of elements. Hence if two sets $A$ and $B$ are equal, then we can say that $A = B$.

PRACTICE

5) True or false: $\{2, 4, 6, 8\} = \{8, 6, 4, 4, 2\}$

6) True or false: $\emptyset \in \{\emptyset\}$

7) True or false: $\{1, 2\} = \{0,1, 2\}$