Section 2.4: Set Operations With Three Sets

§1 Three Sets

Hopefully by this point you are comfortable being able to find the union, intersection and complement when dealing with two sets. In this section we simply extend the ideas into three sets. When drawing the Venn diagram, you will draw three intersecting circles. You will need to know what each of the regions in the Venn diagram using the proper notation.

Example: Say that we have the following sets:

\[ U = \{a, b, c, d, e, f, g, h\} \]
\[ A = \{a, b, c, d\} \]
\[ B = \{b, d, f, h\} \]
\[ C = \{c, d\} \]

You should be able to answer questions such as finding \( A \cap B \cup C \) and \( A \cap (B \cup C)' \). It’s best to draw a Venn diagram, but can we answer the questions without it? Sure we can! Remember to keep track of the symbols and to always remember the order of operations.

For the first one, we want to find \( A \cap B \cup C \). \( A \cap B \) is asking for the elements that are not in the intersection of A and B. Since \( A \cap B \) is \{b,c\}, then \( A \cap B \) is everything else: \{a,c,e,f,g,h\}. Now we want to find the union of this set that we just found, with set C. Since set C is \{c,d\}, the union of \{a,c,e,f,g,h\} and \{c,d\} is simply \{c\}.

For the second one, we want to find \( A \cap (B \cup C)' \). Set A consists of \{a,b,c,d\}. \( B \cup C \)' consists of everything not in B or C. This set would include the elements \{a,e,g\}. Finally, to answer the question, we need to find the intersection of \{a,b,c,d\} with \{a,e,g\}. The answer is \{a\}.

§2 Venn Diagrams With Three Sets

When drawing a Venn diagram with three sets, we see that the three overlapping circles actually create 8 different ‘regions.’ Look at the diagram below:
Let’s list out what each region represents:

Region V: the elements that are common to sets A, B, and C: \( A \cap B \cap C \)

Region II: the elements in both sets A and B that are not in set C: \( (A \cap B) \cap C' \)

Region IV: the elements in both sets A and C that are not in set B: \( (A \cap C) \cap B' \)

Region VI: the elements in both sets B and C that are not in set A: \( (B \cap C) \cap A' \)

Region I: the elements in set A that are not in sets B or C: \( A \cap (B' \cap C') \)

Region III: the elements in set B that are not in sets A or C: \( B \cap (A' \cap C') \)

Region VII: the elements in set C that are not in sets A or B: \( C \cap (A' \cap B') \)

Region VIII: the elements in the Universal set U that are not in sets A, B, or C: \( A' \cap B' \cap C' \)

NOTE: there are other ways to express each region but there are the ones used most often.

Let’s look at the previous example and draw the Venn diagram for that problem.

When placing the elements within the circles, make sure you always fill in region V first – the elements that are common to all three sets. Next, fill in regions II, IV, and VI – the intersection of A and B, A and C, and B and C. Lastly, fill in the remaining three regions – the elements that are only in A, only in B, and only in C.

Notice that using the Venn diagram, we can see that the element b is in set A and set B but not in set C. Similarly, we can see that the elements f and h are in set B but not in sets A or B.
§3 DeMorgan’s Laws

As stated previously, there are different ways that we can express the regions in the Venn diagram. The two most common are known as DeMorgan’s Laws. They are as follows:

\[
( A \cap B )' = A' \cup B'
\]
\[
( A \cup B )' = A' \cap B'
\]

Look at the Venn diagram. You will notice that if I asked you to shade \(( A \cap B )'\), and then I asked you to shade \( A' \cup B'\), the part that you shaded would be exactly the same! Hence some questions will ask you to prove the equality of sets by shading.

PRACTICE

1) Given that \( U = \{1,2,3,4,5,6,7\} \), \( A = \{1,3,5,7\} \), \( B = \{1,2,3\} \) and \( C = \{2,3,4,5,6\} \), draw the Venn diagram for this problem and find \( A \cup (B \cap C); A' \cap (B \cup C'); (B \cup C)' \cup A \)

2) Use the following Venn diagram to represent each set in roster form:

- a) \( A \cap (B \cap C) \)
- b) \( (A \cup B)' \cup C \)
- c) \( (B \cap C)' \cap A' \)
3) Use the symbols given in this chapter to represent the shaded region for each Venn diagram.

a)

b)