Section 8.4: Compound Interest

§1 Simple Interest vs. Compound Interest

Compound interest is different than simple interest in that in compound interest, the interest is calculated on the principal AND on any accumulated interest as well! Many savings accounts pay compound interest – they calculate interest every month.

In reality, we can calculate interest how every many times we want to. Hence, for compound interest another variable is introduced – it represented by the letter $n$. It represents the number of times that interest is calculated per year. Hence if interest is calculated monthly, $n = 12$. If it is calculated quarterly, $n = 4$. The formula then is given as $A = P \left(1 + \frac{r}{n}\right)^{nt}$. As you can see, you need a scientific calculator to arrive at the answer!

For example, let’s say Ann deposits $5000 into a savings account that has a rate of 3%. The interest is calculated monthly. What is the future value in the account after one year?

We use $P = 5000$, $r = 0.03$, $t = 1$, and $n = 12$. Plug these into the formula to get $A = 5000 \left(1 + \frac{0.03}{12}\right)^{12 \times 1}$. Using the order of operations, we want to calculate what’s inside the parentheses first. We get $A = 5000 \left(1.0025\right)^{12}$. Next we do the exponent to get $A = 5000 \left(1.0304...\right)$. Multiply these together to get the final answer of $5152.08$. Note that we do not round the numbers, so don’t clear out the values once you get them in your calculator. We don’t round because then we lose precious cents! So just do each step in your calculator without clearing anything. You should try it a couple times so that you can get the correct answer!

PRACTICE

1) Find the future value if $100,000 is compounded quarterly at 5% for 3 years.
2) Find the future value if $1500 is compounded daily at 2% for three years.
3) Find the future value if $1500 is compounded monthly at 2% for three years.

§2 Continuous Compounding

Continuous compounding is so called because the interest is being calculated continuously – its an ongoing process. Look at the formula again, $A = P \left(1 + \frac{r}{n}\right)^{nt}$. We see that $n$ represents the number of times the interest is being calculated per year. What if we want to calculate interest every day? Or every second? Or every millisecond? We see that the value of $n$ gets larger and larger. As $n$ gets really large, the formula changes to $A = Pe^{rt}$, where $e$ is the natural exponential number which is approximately 2.71828.

It actually turn out that continuous compounding doesn’t produce enormous amounts of interest compared to other type of compounding. For example, say in the above practice problem 2, the interest is compounded continuously. Then we use the future value formula to get that $A = 1500e^{0.06}$, so the answer is $1592.75481,
which rounded gives $1592.75. We saw that the answer to practice problem 2 was $1592.7522, which rounded is the same $1592.75! The answer to practice problem 3 was $1592.6752, or rounded to $1592.68. Hence we see that the values aren't significantly different. The only time we would see a significant difference is if the principal was really large AND the interest rate was high AND the time was very long!

PRACTICE

4) Find the better deal - $5000 invested at 6% for 6 years compounded continuously or $5000 invested at 8% for 5 years compounded monthly.