Section 8.6: Cars

§1 The Mathematics Of Financing A Car

An installment loan is any loan that is paid off with fixed payments over a certain time period. These are also called fixed installment loans because the payments are made regularly. Car loans are one type of installment loan. Most car loans involve making a payment every month until the loan is paid off. Of course, we always have to add in the interest that needs to be paid! That’s the one negative with installment loans – the amount of interest can get very high.

Most car loans have fixed terms – the length of the loan is given in years and is denoted by the letter \( t \); the interest rate of the loan is denoted by the letter \( r \); the number of payments made per year (usually 12) is denoted by the letter \( n \); the principal (the amount financed) is denoted by the letter \( P \). Then we can use the formula to calculate the monthly payment:

\[
PMT = \frac{P \left( \frac{r}{n} \right)}{1 - \left(1 + \frac{r}{n}\right)^{-nt}}
\]

This formula does look a little intimidating, but as long as you can plug in the values and do the calculations correctly, this is pretty much what this section is dealing with!

Let’s try an example. Say we want to finance \$20000 for 5 years at a rate of 4\%. Remember, we always assume that the payments are monthly, so \( n =12 \), \( r = 0.04 \), \( P = 20,000 \) and \( t = 5 \). Just by plugging these values into the formula we end up with \( \frac{20000 \left( \frac{0.04}{12} \right)}{1 - \left(1 + \frac{0.04}{12}\right)^{-125}} \). It’s best to do the numerator first, then the denominator. From the numerator you should end up with 66.6666… You have to be a little careful when calculating the denominator. First do the inner parentheses. Then when you do the exponent, make sure you make it negative! Hence the denominator should become 0.180996… As the last step, we divide these two numbers together to get the final answer, the monthly payment is \$368.33.

How much do you end up paying total for the car? Since we are making 60 equal payments of \$368.33, we simply multiply these numbers together. The result is \$22099.80. This represents the total payments made. Hence we can determine how much was paid in interest by subtracting the original amount financed! We see that the interest itself was \$2099.80.

PRACTICE

1) Find the monthly payment and the interest paid for the following two loans. Round to the nearest dollar:

A: \$30,000 financed at 6.5\% for 45 years

B: \$30,000 financed at 4\% for 6 years
§2 The True Cost Of Owning A Car

There are various other factors when determining the true cost of owning a car. Maintenance is a huge factor. Insurance, registration, parking and cleaning also factor in. We will focus on one aspect – the cost of gasoline. We can estimate the annual fuel expense by using three variables – the number of miles driven per year, the miles per gallon of the vehicle, and the price per gallon of gas. The formula is as follows:

\[
\text{Annual Fuel Expense} = \frac{\text{Annual Miles Driven}}{\text{Miles Per Gallon}} \times \text{Price Per Gallon}.
\]

Let’s say Alice drives her hybrid car on average for 18,000 miles per year. Her hybrid gets 40 miles per gallon and the average cost of gas per year is $3.50. Then her Annual Fuel Expense will be \(\frac{18000}{40} \times 3.50 = $1575\), or an average of $131.25 per month.

Let’s compare this to Abby who drives the same distance every year but has an SUV that gets 12 miles per gallon. Then her annual fuel expense will be $5250, which comes to $437.50 per month, which is $306.25 per month extra! The savings are huge!

PRACTICE

2) Brie is deciding between a hybrid model and a regular model of the same car. The hybrid model gets 30 miles per gallon while the regular model gets 20 miles per gallon. If Brie drives an average of 30,000 miles per year and gas costs $3.80 per gallon, how much will her annual fuel savings be? What will be her monthly fuel savings?