Math 126: Exam 2 Study Guide

1) Sketch the graph of the following polynomials. Plot the x-intercepts and y-intercepts, and use the end behavior to help (4.1)

a) \( f(x) = x^2(x+3)(x-2)^2 \)

b) \( f(x) = 2x^3 - 4x \)

c) \( f(x) = -2(x+2)^2(x-2)^2(x+4)^2 \)

2) Graph the following rational functions. Make sure you state any x-intercepts, y-intercepts, and the vertical and horizontal asymptotes (4.3)

a) \( f(x) = \frac{x+2}{x^2 + x - 6} \)

b) \( f(x) = \frac{x^2 - 4x + 3}{(x-1)^2} \)

3) Solve the following inequalities (3.5 and 4.4)

a) \( 3x^2 + 10x \leq 8 \)

b) \( (x+3)(x-1)(3x+1) > 0 \)

e) \( \frac{2x-1}{x-1} \leq 3 \)

4) Divide the following using synthetic division (4.5)

a) \( (2x^3 + 7x^2 + 9x - 20) \div (x + 3) \)

b) \( (3x^4 - 2x^2 + 5x - 1) \div (x - 2) \)

5) What is the remainder when \( f(x) = 2x^3 - x^2 + 6x - 4 \) is divided by \( x - 2 \)? Use the remainder theorem. (4.5)

6) Find all the zeros of the following polynomials. Give the final answer in factored form. (4.5 and 4.6)

a) \( 2x^3 - 5x^2 - 6x + 4 = 0 \)

b) \( x^3 - 2x^2 - 7x - 4 = 0 \)

7) Find an n-th degree polynomial function with real coefficients satisfying the given conditions (4.5 and 4.6)

a) \( n = 3; -3, 2, \text{ and } 1 \text{ are zeros} \)

b) \( n = 3; 2 \text{ and } 1 + i \text{ are zeros} \)

c) \( n = 4; \sqrt{5} \text{ and } 2i \text{ are zeros} \)

8) Find \( (f \circ g)(x) \) and give the domain of \( f \circ g \) (5.1)

a. \( f(x) = x^2 - 1; g(x) = \sqrt{2x - 5} \)

b. \( f(x) = \frac{3}{x-1}; g(x) = x^2 \)

9) Find an equation for the inverse function, and verify that your equation is correct by taking the composite function. You must show all your work! (5.2)

a. \( f(x) = \frac{2}{x} - 1 \)

b. \( f(x) = \frac{1}{2}x - 3 \)

10) Find the inverse of \( f(x) = \sqrt{x + 2} \) and find the domain of each. Sketch \( f(x) \) and \( f^{-1}(x) \) on the same coordinate axes. (5.2)

11) Graph the following. Make sure you plot and give the coordinates of at least three points. (5.3)

a) \( f(x) = 2x^2 - 1 \)

b) \( g(x) = 2^{-x} + 1 \)

c) \( h(x) = \left(\frac{1}{3}\right)^x \)

12) Evaluate without using a calculator (5.5)

a) \( \log_5 \frac{1}{\sqrt{5}} \)

b) \( \log_7 1 \)

c) \( 8^{\log_{12}} \)
d) \( \log_3 9^4 \)

13) Evaluate without using a calculator: (5.5)

a) \( 10^{\log_8 4} \)

b) \( e^{\ln 3x} \)

c) \( \ln e^{x^2} \)

d) \( 10^{\log_{10}\sqrt{x}} \)

14) Use properties of logs to expand as much as possible. (5.5)

a) \( \log_2 \sqrt[3]{3x^3} \)

b) \( \log_7 \left( \frac{x^2}{49 \sqrt{y+1}} \right) \)

c) \( \log_3 \frac{\sqrt[3]{xy^3}}{25} \)

15) Use properties of logs to condense into a single log with a coefficient of 1. (5.5)

a) \( \frac{1}{3} (\log_3 x - \log_3 y) \)

b) \( \frac{2}{3} \log x - 2 \log y + 4 \log z \)

c) \( 3 \ln (x + y) - 3 \ln x \)

16) Solve the following exponential equations by expressing each side as power of the same base and then equating exponents. (5.6)

a) \( 4^{x+1} = 32 \)

b) \( 3^{1-x} = 27 \)

c) \( 6^{\frac{x+1}{2}} = \sqrt{6} \)

d) \( e^{2x-1} = \frac{1}{e^{x+3}} \)

17) Solve the exponential equations. Express the final solution in terms of common or natural logs. You do not need to find the decimal answer. (5.6)

a) \( 4^{2x+1} = 20 \)

b) \( 7e^{x-1} = 35 \)

c) \( 2^{\frac{1}{5}} = 12 \)

18) Solve the following log equations. Be sure to state if any proposed solutions are extraneous. Give the exact answer. (5.6)

a) \( \log_5 (2x - 5) = 2 \)

b) \( 5 \ln (9x) = 20 \)

c) \( \log_5 (x + 5) - \log_5 (x - 3) = 1 \)

d) \( \log(3 + x) - \log(x - 2) = \log 2 \)