Section 2.1: The Properties of Equality

We start with Chapter 2. Make sure you go back and review Chapter 1. Chapter 1 is basically a review of Math 93 – concepts include fractions, exponents, integers and inequalities.

§1 Linear Equations

An equation is a statement asserting that two algebraic expressions are equal. We start off this class with linear equations. A linear equation is of the form \( Ax + B = C \), where \( A \), \( B \) and \( C \) are real numbers and \( A \) is not equal to zero. You may have seen this before called ‘two-step equations’ in your previous math class. If the value of \( A = 1 \), then this is called a ‘one-step equation’. Some examples are \( 3x - 5 = 7 \) or \( -2x + 4 = 10 \). The goal is to find the solution – the value of \( x \) that makes the equation true when it replaces the variable.

So how do we go about doing this? We need to isolate the variable. The variable can be given by any letter, but for the most part we usually use the letter \( x \). We use the two properties of equality to isolate the variable.

§2 Addition Property of Equality

We always use the Addition Property of Equality first. It states that if \( A = B \), then \( A + C = B + C \). This simply means that we can add (or subtract) the same quantity to both sides of an equation without changing the solution. When solving one or two-step equations, we use the Addition Property of Equality to isolate the variable term. Look at the following examples.

Example 1: Solve \( 5x - 7 = 0 \). Note that the coefficient of the variable term is 1. This is a one-step equation. We simply add 5 to both sides. We end up with \( 5x - 7 + 5 = 0 + 5 \). This becomes \( 5x = 5 \). One we have isolated the variable term, the value on the other side should be the solution (assuming we did everything correctly!)

Example 2: Solve \( x + 4 = -3 \). Similarly, we need to bring the 4 to the other side. We do so by subtracting the 4 from both sides. We end up with \( x + 4 - 4 = -3 - 4 \). We end up with \( x = -7 \). If you plug this value back into the original equation, we see that the two sides end up equaling each other!

Remember, it doesn’t matter which side the variable is on. For example, \( x + 16 = -22 \) is the same as \( -22 = x + 16 \). How would you solve this one?

Also, you may end up working with fractions and decimals, and in some cases you may have to use the Addition Property of Equality twice.

Example 3: Solve \( 6x - 8 = 12 + 5x \). Note that here there is a variable and constant term on each side. We want to combine like terms first. Let’s move the 5x to the left side. We end up with \( 6x - 8 - 5x = 12 + 5x - 5x \). This becomes \( x - 8 = 12 \). Now we can add the 8 to both sides to get \( x = 20 \). Could you have done this another way?

PRACTICE

1) Solve \( \frac{7}{2} p + 4 = \frac{9}{2} p \)
2) \(4(x + 1) - (3x + 5) = 1\)

3) \(9x + 4x + 6 - 2 = 9x + 4 + 3x\)

§2 Multiplication Property of Equality

For the most part, we will be dealing with equations where the coefficient of the variable is not equal to 1. In this case, we need to use the Multiplication Property of Equality to solve. Remember, we do this step after using the Addition Property of Equality. That means we leave the coefficient of the variable for last. Make sure you use the correct order of operations to combine all like terms first!

The Multiplication Property of Equality states that if \(A = B\), then \(AC = BC\). This says that we can multiply both sides of an equation by the same non-zero number, and it will not change the solution. Note that this also means we can divide both sides by the same number as well. You want to remember the following rule:

- If the coefficient of the variable is a whole number, then divide both sides by that coefficient.
- If the coefficient of the variable is a fraction, then multiply both sides by the reciprocal of the coefficient.

Example 4: Solve \(3x = 12\). Note that the coefficient of the variable is not equal to 1. In this case, it’s a whole number. Hence we divide both sides by 3. We end up with \(\frac{3x}{3} = \frac{12}{3}\). The answer is \(x = 4\).

Example 5: Solve \(\frac{3}{4}x = -6\). Note that the coefficient of the variable is a fraction. In this case, multiply both sides by the reciprocal. We end up with \(\left(\frac{4}{3}\right)\frac{3}{4}x = -6\left(\frac{4}{3}\right)\). The answer is \(x = -8\).

In some cases, the coefficient of the variable may be a negative number. Make sure when you use the Multiplication Property of Equality, you include the negative sign in the algebra.

Example 6: Solve \(-6x = -10\). The coefficient here is -6. We can divide both sides by -6. We end up with \(\frac{-6x}{-6} = \frac{10}{-6}\). The final answer is \(x = \frac{-10}{6}\), or \(-\frac{5}{3}\).

PRACTICE

4) Solve \(6x - 4x = 13 - 21\)

5) Solve \(-3x - 4x - 5x = -36\)

6) \(\frac{3x}{5} - \frac{2x}{5} = -4\)