Instructions:
1. This assignment is ultimately an independent activity; however, if you wish to discuss the problems with peers or tutors, that is acceptable. All of the final answers (including any work) should be completed individually. DO NOT COPY!
2. The problems in this assignment are meant to be completed using the online graphing calculator found at www.desmos.com.
3. When a graph is required as part of an answer, a screen shot would be preferred over a hand-drawn sketch, but you may sketch the graph if you absolutely cannot print out a screen shot.

Questions:

1. Implicit Differentiation:
   Consider the equation $y^2 = x^6 (5 - x^2)$.
   a. Graph the equation in Desmos. Highlight the 5 points of horizontal tangency by clicking on the graph at those points. Print or sketch the resulting graph.
   b. Use implicit differentiation to find the exact $x$-values where the graph has a horizontal tangent line.

2. Integration and Riemann Sums:
   Go to: https://www.desmos.com/calculator/rfix3pe1oh
   An example about estimating area using rectangles will open. In this example, the area between the graph of a function and the $x$-axis from $x = 0$ to $x = 1$ is approximated using rectangles. The height of each rectangle is calculated by plugging the right $x$-value of the rectangle’s base into the original function. This method for calculating area is called a Right Riemann Sum.
   a. For the given function, $f(x) = 1 - x^2$, choose $n = 5$. Print or sketch the resulting graph.
   b. $n = 5$ is supposed to indicate that there are 5 rectangles under the graph. Explain why only 4 rectangles are shown.
   c. Calculate the area of the 4 rectangles by hand and compare this to the answer given in the box containing $\sum_{k=1}^{n} \left( \frac{1}{n} f \left( \frac{k}{n} \right) \right)$.
   d. Change the function at the top to $f(x) = \sqrt{1 - x^2}$ and set $n = 20$. Print or sketch the resulting graph.
   e. What is the given approximation for the area under the curve? How good is this approximation? (Subtract the approximation given from the actual area.)
   f. For the same function, set $n$ to 300. What approximation for the area is given? How good is this approximation?
3. **OPTIMIZATION:**  
A cylindrical metal can with height $h$ and radius $r$ is to be made to hold 900 cm$^3$ (0.9 liters) of liquid.

a. Using the figures below as reference, find a formula for the surface area of the can, $S$.

![Diagram of a cylinder with dimensions labeled](image)

b. Using the formula for the volume of a cylinder and the fact that the metal can only holds 900 cm$^3$ of liquid, solve for $h$ in terms of $r$.

c. Write the surface area of the can, $S$, as a function of $r$ only.

d. Graph the function $S$ in Desmos. Find the minimum point on the graph and click it. Change the domain (not range) of your graph to $-10 < r < 10$, and then **print or sketch the given graph**.

e. What are the *approximate* $r$ and $h$ values that minimize the surface area of the metal can?

f. Confirm your results from parts d and e by using the first derivative test to find the *exact* values of $r$ and $h$ that minimize the surface area of the metal can.

g. In fact, when cans are mass produced at a plant, the circular metal lids are cut from metal sheets that have been divided into regular hexagons like so:

![Image of hexagons](image)

The area of one such hexagon is given by $A = 2\sqrt{3} \cdot r^2$. Therefore, the total area of metal used to produce a single can is more accurately given by:

$$T = 4\sqrt{3} \cdot r^2 + 2\pi rh.$$

Write the total area of metal used to produce a 900 cm$^3$ metal can, $T$, as a function of $r$ only.

h. Graph the function $T$ in Desmos. Find the minimum point on the graph and click it. Change the domain (not range) of your graph to $-10 < r < 10$, and then **print or sketch the given graph**.

i. What are the *approximate* $r$ and $h$ values that minimize the total area of metal used?
Bonus (not required) questions for problem 3

j. Find a metal can in your house or at the store with volume near 900 cm$^3$ (for example, I used a 28 oz. net weight can of tomatoes which had a volume of about 903 cm$^3$). Take a picture of your can. Measure the radius and height of the can and record them. Then calculate the volume using $v = \pi r^2 h$.

k. Your measurements should be slightly different than the radius and height found in part i. This is for two main reasons. First, the volume of your can will be slightly more or less than 900 cm$^3$. Second, the joining process used to construct metal cans changes the area of material needed. When cans are produced, the circular base and top are made purposely large so that the edges can be folded over and connected to the side of the can. The amount of metal used to produce a metal can in this case is given by:

$$J = 4\sqrt{3} \cdot r^2 + 2\pi rh + k(4\pi r + h).$$

Where $k$ is a constant that changes depending on the exact joining process and costs. Using $k = 0.33$ and the volume of the can you found in part j above, write $J$ as a function of $r$ only.

l. In Desmos, graph the function $J$. Click on the minimum point. Print or sketch the given graph. Compare your answer to the dimensions of the can measured in part j.