Below are problems to help you review common algebra, precalculus, and trig concepts that give students trouble in calculus.

These concepts are YOUR responsibility to thoroughly understand. You will be tested over these concepts during the second week of class. You will NOT be allowed to use a calculator.

Here are some resources to help you review:

- Josh’s Intermediate Algebra Videos: http://sites.csn.edu/jmartin/math96videos.html
- Josh’s Precalculus Resources: http://sites.csn.edu/jmartin/math126videos.html
- Josh Trigonometry Resources: http://sites.csn.edu/jmartin/math127videos.html
- Professor Hutchinson’s Precalc Resources: http://sites.csn.edu/ehutchinson/MATH128NOTES.html
- Many times, a quick internet search will turn up helpful videos on the topic. (For example, try to search “precalculus videos about radicals and exponents” for help with the first question.)

1. Radicals and Exponents
   (i) Write each expression using radicals. Simplify when possible.
   a. \(x^{1/2}\)  
b. \(x^{1/3}\)  
c. \(x^{3/4}\)  
d. \(x^{-4/5}\)  
e. \(\frac{1}{x^{3/2}}\)  
f. \(\frac{1}{x-\sqrt{3}}\)  
g. \(\frac{5}{7x^{3/5}}\)
   h. \(\frac{x^{2/3}}{x^{3/2}}\)  
i. \(\frac{x^0}{x^{7/5+1}}\)

   (ii) Write each radical using nonnegative exponents.
   a. \(\sqrt[3]{x^5}\)  
b. \(\frac{1}{\sqrt{x^2}}\)  
c. \(\frac{x}{\sqrt{x^3}}\)  
d. \(\frac{x^{-3}}{\sqrt{x-6}}\)

2. Factoring with negative and rational exponents
   Factor each expression completely. Write your answer without negative exponent
   a. \(3x^5 + 5x^4\)  
b. \(-45x^2 + 15x\)  
c. \(4x^3 + 12x^{-1}\)  
d. \(54x^{-3} + 18x^2\)
   e. \(4x^{-7} + 44x^{-2}\)  
f. \(4x^{3/2} + 12x^{1/2}\)  
g. \(4x^{3/2} + 12x^{-1/2}\)
   h. \(36x^{-5/2} + 12x^{1/2}\)  
i. \(4x^{-3/2} + 64x^{-5/2}\)  
j. \(x^{7/3} + 5x^{4/3} + 6x^{1/3}\)
   k. \(3x^{-2/3} - 11x^{1/3} + 6x^{4/3}\)  
l. \(3x^{2/3} - 22x^{1/3} + 7\)  
m. \(6e^{2x} + 15e^x - 9\)
3. Lines: graphing
Graph each line
a. \( y = x \)  
   b. \( y = 3x \)  
   c. \( y = -2x \)  
   d. \( y = -\frac{1}{3}x \)  
   e. \( y = -\frac{3}{2}x + 1 \)
   f. \( y = -2x - \frac{1}{2} \)  
   g. \( 2y - 3x = -6 \)  
   h. \( y = 3 \)  
   i. \( x = 3 \)
   j. Graph the three given lines, find the points of intersection, and find the area of the resulting triangle.
   \( x = 1, \ y = 2x, \ y + 2x = 7 \)

4. Lines: writing equations
Write the equation of the line that fits the given information
a. The line has slope \(-\frac{1}{3}\) and passes through the point \((1, -4)\).
   b. The line passes through the points \((2, 4)\) and \((-2, \frac{1}{2})\).
   c. The line hits the graph of \( y = x^2 \) when \( x = -1 \) and \( x = 2 \).
   d. The line is parallel to \( y = \frac{1}{5}x + 2 \) and passes through the point \((-1, 5)\).
   e. The line is perpendicular to the line \( 2x + 5y = 3 \) and passes through the point \((-3, 6)\).
   f. The line is perpendicular to the line that intersects the graph of \( y = x^3 \) when \( x = 2 \) and \( x = 5 \) and passes through the origin.

5. Common graphs
Simplify each function and then sketch the function. What is the pattern?
(i) Power functions
   a. \( y = x \)  
   b. \( y = x^2 \)  
   c. \( y = x^3 \)  
   d. \( y = x^4 \)  
   e. \( y = x^5 \)
   (ii) Square root functions
   a. \( y = x^{1/2} \)  
   b. \( y = x^{2/2} \)  
   c. \( y = x^{3/2} \)  
   d. \( y = x^{4/2} \)  
   e. \( y = x^{5/2} \)
   (iii) Cube root functions
   a. \( y = x^{1/3} \)  
   b. \( y = x^{2/3} \)  
   c. \( y = x^{3/3} \)  
   d. \( y = x^{4/3} \)  
   e. \( y = x^{5/3} \)
   f. \( y = x^{6/3} \)

6. Zeros (a.k.a. \( x \)-intercepts)
Find all zeros of each function.
   a. \( f(x) = x^2 + 5x \)  
   b. \( f(x) = x^2 - 16 \)  
   c. \( f(x) = x^2 - 10 \)  
   d. \( f(x) = x^3 - 7x \)
   e. \( f(x) = 4x^2 - 3x - 10 \)  
   f. \( f(x) = x^2 - 10x + 7 \)  
   g. \( f(x) = \frac{3x + 1}{x^2 - 5} \)
   h. \( f(x) = \frac{1}{x^3 - 5x + 1} \)  
   i. \( f(x) = x^{3/2} + 5x^{1/2} \)  
   j. \( f(x) = 3x^{5/3} - x^{-4/3} \)
   k. \( f(x) = \frac{1}{x^{3/4} + 5x^{-1/4}} \)  
   l. \( f(x) = \sin x \)  
   m. \( f(x) = \cos 2x \)
7. Vertical asymptotes

Find ALL vertical asymptotes of each function.

a. \( f(x) = \frac{1}{x} \) 
   b. \( f(x) = \frac{1}{x^2 - 9} \) 
   c. \( f(x) = \frac{3x}{x^5 - 3} \) 
   d. \( f(x) = \frac{x^2 + x}{x^2 - 3x - 4} \)
   e. \( \frac{x^{2/3}}{x^{-5/3} - x^{1/3}} \)
   f. \( f(x) = \frac{\cos x}{\sin x} \) 
   g. \( y = \tan (17x) \) 
   h. \( y = \csc \left(3x - \frac{\pi}{2}\right) \)

8. Arrow notation

Fill in the missing information.

a. Let \( f(x) = 3x \). As \( x \to \infty \), \( f(x) \to ? \)
   b. Let \( f(x) = 3x \). As \( x \to -\infty \), \( f(x) \to ? \)
   c. Let \( f(x) = \frac{1}{x} \). As \( x \to \infty \), \( f(x) \to ? \)
   d. Let \( f(x) = \frac{1}{x} \). As \( x \to -\infty \), \( f(x) \to ? \)
   e. Let \( f(x) = \frac{1}{x} \). As \( x \to 0^+ \), \( f(x) \to ? \)
   f. Let \( f(x) = \frac{1}{x} \). As \( x \to 0^- \), \( f(x) \to ? \)
   g. Let \( f(x) = \frac{5x}{4x^2 - 3} \). As \( x \to \infty \), \( f(x) \to ? \)
   h. Let \( f(x) = \frac{5x^2}{4x^2 - 3} \). As \( x \to \infty \), \( f(x) \to ? \)
   i. Let \( f(x) = \frac{5x^3}{4x^2 - 3} \). As \( x \to \infty \), \( f(x) \to ? \)
   j. Let \( f(x) = \frac{5x}{4x^2 - 3} \). As \( x \to -\infty \), \( f(x) \to ? \)
   k. Let \( f(x) = \frac{5x}{4x^2 - 3} \). As \( x \to \frac{\sqrt{3}}{2} \), \( f(x) \to ? \)

9. Graphing with transformations

Use transformations to sketch each graph. It should take you less than 1 minute to sketch each curve.

a. \( y = x^2 - 9 \) 
   b. \( y = \sqrt{x + 3} \) 
   c. \( y = x^{1/3} + 1 \) 
   d. \( y = \frac{1}{2} x^3 \)
   e. \( y = (x - 1)^{2/5} + 2 \)

10. Graphing with zeros

Use end behavior, zeros, and multiplicity to sketch each graph. It should take you less than 2 minutes to sketch each curve.

a. \( y = x^3 - 2x \) 
   b. \( y = -x^4 + 10x^3 \) 
   c. \( y = -\frac{1}{4} (x + 1)^2 (x + 2)^3 \)

11. Graphing with trig functions

Use period and amplitude to sketch each trig function

a. \( y = \sin x \) 
   b. \( y = \cos x \) 
   c. \( y = \tan x \) 
   d. \( y = \sin (2x) \) 
   e. \( y = \cos(\pi x) \)
   f. \( y = \tan \frac{1}{3} x \)
12. Evaluating trig functions
Evaluate each expression. It should take you less than 20 seconds to evaluate each simple expression (the longer expression may take a minute or so).

a. \( \sin \frac{\pi}{2} \)  

b. \( \cos \pi \)  

c. \( \tan \frac{\pi}{6} \)  

d. \( \csc \frac{3\pi}{4} \)  

e. \( \sec \frac{5\pi}{6} \)  

f. \( \cot \frac{5\pi}{3} \)  

g. \( 2 \sin \frac{3\pi}{2} \cos \frac{4\pi}{3} - 3 \tan \frac{5\pi}{4} \sec \pi \)

13. Graphing exponential and log functions
Sketch a quick graph of each function. What is the pattern?

* Remember, \( e \) is a transcendent number, like \( \pi \) and \( e \approx 2.718 \)

(i) Exponential functions \( (b > 1) \)
a. \( y = 2^x \)  

b. \( y = e^x \)  

c. \( y = 3^x \)  

d. \( y = 4^x \)  

e. \( y = 10^x \)

(ii) Log functions \( (b > 1) \)
a. \( y = \log_2 x \)  

b. \( y = \ln x \)  

c. \( y = \log_3 x \)  

d. \( y = \log_4 x \)  

e. \( y = \log x \)

(iii) Exponential functions \( (b < 1) \)
a. \( y = (1/2)^x \)  

b. \( y = (1/e)^x \)  

c. \( y = (1/3)^x \)  

d. \( y = (1/4)^x \)

(iv) Log functions \( (b < 1) \)
a. \( y = \log_{1/2} x \)  

b. \( y = \log_{1/e} x \)  

c. \( y = \log_{1/3} x \)  

d. \( y = \log_{1/4} x \)

14. Simplifying exponential and log functions
Simplify each expression completely.
a. \( 2^5 \)  

b. \( \log_2 32 \)  

c. \( \ln (e) \)  

d. \( \ln(\sqrt{e}) \)  

e. \( \log(10^x) \)  

f. \( \ln(e^x) \)  

g. \( 3^{\log_3 x} \)

h. \( \ln(e^{\log_4 x}) \)

15. Log rules
(i) Expand each expression, and then simplify each term if possible.
a. \( \log_5(25x^2) \)  

b. \( \log_2 \left( \frac{\sqrt{x-5}}{16(x+1)^5} \right) \)  

h. \( \ln \left( \frac{e^{x^2-2x^3}}{3y} \right) \)

(ii) Condense each expression into a single term with a coefficient of 1.
a. \( \frac{1}{2} \ln x + \frac{1}{3} \ln y - 2 \ln z \)  

b. \( \log x^2 - \log x^5 \)  

c. \( \ln x - \ln y + \ln z - \ln a + \ln b \)

(iii) Let \( \ln a = x, \ln b = y, \) and \( \ln c = z. \) Write the expression in terms of \( x, y, \) and \( z. \)
\( \ln \left( \frac{a^3b^6}{\sqrt{c}} \right) \)

16. Trig Identities
Use identities to simplify each expression.
a. \( \cos x \text{ sec } x \)  

b. \( \sin^2 \theta + \cos^2 \theta \)  

c. \( \csc^2 y - \cot^2 y \)  

d. \( \tan x \cos x \text{ csc } x - \frac{\sec x}{\cos x} \)

b. \( \sin 2x \cdot \frac{1}{2} \tan x + \cos 2x + \sin^2 x \)  

f. \( 2\sin^2 x + 2\cos^2 x + \tan^2 x + \cot^2 x - \sec^2 x - \csc^2 x \)