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Letter to Students (and/or Instructors)

Dear Prealgebra Student (and/or Instructor):

We are very excited about your using this Prealgebra book! Prealgebra is a wonderful course because it gives you the chance to perfect your prealgebra skills as you get ready to take more algebra classes. Be sure to attend all classes, read the book, take good notes, do the homework, and ask questions when you don’t understand something. This is your chance to really learn and lock in the skills you need to make progress in math.

If you need practice with your multiplication facts, there is a multiplication table at the end of the book. We highly recommend you practice if you are a little rusty.

This textbook is designed to bridge the gap between arithmetic and elementary algebra. We made the assumption that you were already familiar with most arithmetic ideas; we took what you should know about arithmetic and applied it to algebra. The committee took about two years to write, edit, revise, and refine the first edition. It also took a year to gather comments and make revisions for the second edition. Additional changes have been made over the years so now I am on the fifth edition.

Our goal when writing the book was to present certain concepts time and again, giving you the opportunity to learn the concepts with new numbering systems. We wanted you to simplify (combine like terms, distribute, use order of operations), evaluate, and solve equations using whole numbers, integers, fractions, and decimals. We also wanted to briefly introduce formulas, polynomials, and graphing in two variables. Therefore, we organized the topics the way we wanted to teach them.

Please be aware the committee deliberately left out some information that is thoroughly covered in elementary algebra. For example, we did not introduce the quotient rule of exponents in Chapter 4, Section 1. For another example, we also did not introduce FOIL in Chapter 6, Section 3. We didn’t talk about the Cartesian coordinate system in Chapter 7, Section 1. There are probably other areas we didn’t cover. If you already know about those ideas, great! If your instructor decides to cover those ideas, great! Just consider anything additional your instructor does include as “helpful hints for your future.”

If you have any comments you wish to share (or if you find any errors), please feel free to send those to michelle.wyatt@csn.edu

All comments will be carefully considered as I get ready to make revisions for the next edition of the prealgebra book. Thank you for your time and consideration!
Chapter 1: Whole Numbers

In Chapter 1, you will be studying how algebra is used with whole numbers. Whole numbers are the numbers 0, 1, 2, 3, 4, 5, and so on (the numbers “build up” by one at a time). Within this chapter, you will first learn about the language of algebra so everyone will be able to talk about the concepts using the same vocabulary. Then you will learn about simplifying algebraically. Essentially, simplifying is a way to manipulate what you’re working with and make it look “shorter.” Simplifying doesn’t change the numerical value of what you’re working with; simplifying changes the looks. At the end of the chapter, you’ll look at solving equations. In this chapter, all the algebra concepts will be taught with whole numbers in mind.

In Chapter 1 on whole numbers, you will learn how to combine like terms, how to distribute, how to distribute and combine like terms, and how to use order of operations. All of these are considered avenues to simplifying. Additionally, you will learn how to solve equations. Solving equations uses simplifying, as well as rules of add, subtract, multiply, and divide, in order to isolate the variable. Don’t worry if these ideas are not familiar; you’ll learn about these ideas in this course. While you are learning the various techniques, keep in mind that these techniques will be used again and again as you progress through the course. You want to make every effort to learn thoroughly so that you are comfortable as you progress through the book.

What happens after you’ve learned about simplifying and solving with whole numbers? The rest of the textbook will expand on these concepts and will cycle back the main ideas of Chapter 1. The concepts of Chapter 1, simplifying and solving, are the core of the course.

You will have plenty of opportunity to recall the main concepts, to learn new ideas, to “meld together” the new with the old, and to build a strong foundation to help you succeed in the next levels of algebra.
Section 1: Symbol Notation and Vocabulary

Symbol Notation

In algebra, multiplying is often shown the following ways. If the problem was
2 times 5, you would show that one of several ways:
2 \cdot 5 using a raised times dot
2(5) using parentheses with no symbol
(2)(5) using parentheses with no symbol

In algebra, division is often shown the following way. If the problem was
15 divided by 3, you would show that as
\[ \frac{15}{3} \] using a fraction bar
15 ÷ 3 using division symbol
3 \overline{15} using long division symbol

Recall, 0 divided by a number is zero. For example, \( \frac{0}{2} = 0 \). When you think
about long division, you can see why: \( \frac{20}{0} = 0 \) because \( 0(2) = 0 \). However, division
by zero is undefined: \( \frac{18}{0} \) is undefined. Again, think about long division to see why:
\( 0 \overline{18} \) and nothing works because 0 times any number you try equals zero, not 18.

Vocabulary

Learning algebra is like learning a new language—there is a math vocabulary.

An **expression** is a collection of letters, numbers, and arithmetic operations
but not including an equal sign. An **equation** has two equal expressions so does
include an equal sign.

Which one of these is an expression, and which one is an equation?

a. \( 3x + 4 \)  
   b. \( 5x + 10 = 30 \)

The answers:

a. expression; no equal sign  
   b. equation; has an equal sign

In the expression \( 5x + 4 \), each part has a name. First, there are two terms in
\( 5x + 4 \). One term is \( 5x \), and the other term is 4. **Terms** are separated from each
other with add or subtract symbols.

The \( x \) part is called the **variable**. The variable is usually a letter that stands
for a number we don’t know.
The 5 in front of the variable is called the **coefficient**. A coefficient is located in front of a variable, and the coefficient and variable are being multiplied. Therefore, if you see the term 8z, it is understood that 8 and z are being multiplied.

The 4 is called the **constant**. A constant is a number without a variable.

Even when the expression is in a different order, the coefficient is still in front of the variable and is a multiplier of the variable; the constant is still a number without a variable. In the expression $8 + 7y$, the variable is $y$, the coefficient is 7, and the constant is 8.

Before you go on, answer the following questions:

- c. In the expression $3z + 9$, how many terms are there?
- d. In the expression $4a + 3b + 8c$, how many terms are there?
- e. In the expression $14 + 7m$, what is the variable? What is the coefficient? What is the constant?

The answers:  
- c. 2 terms
- d. 3 terms
- e. variable is $m$, coefficient is 7, constant is 14

In the term $5y$, the 5 is the coefficient that is multiplied by the variable $y$. The 5 and the $y$ are said to be **factors**. Factors are numbers or variables that are being multiplied. However, remember that **terms** are added or subtracted. Therefore, in the expression $41k + 13$, there are two terms. One term is $41k$. In that term, 41 is the coefficient while $k$ is the variable. At the same time, 41 and $k$ are called factors of $41k$ (because 41 and $k$ are being multiplied). In the expression $41k + 13$, the other term is 13. This term is called the constant. 13 is also considered a whole number, where **whole numbers** are $\{0, 1, 2, 3, \ldots\}$. 


Chapter 1 Section 1  
Symbol notation and vocabulary  

Homework for Chapter 1 Section 1  

1. In $7x + 8$, what is the variable? The coefficient? The constant?  
2. In $9y + 12$, what is the variable? The coefficient? The constant?  
3. In $4z + 7$, what is the variable? The coefficient? The constant?  
4. In $9 + 8a$, what is the variable? The coefficient? The constant?  
5. In $6 + 4m$, what is the variable? The coefficient? The constant?  
6. In $22 + 7y$, how many terms are there?  
7. In $11m$, what are the factors?  
8. What operation does $13y$ demonstrate?  
9. In $4x + 18y + 23z + 7$, how many terms are there?  
10. In $13z + 24$, what is the constant?  
11. What are the first five whole numbers?  
12. Is $5z + 18 = 38$ an expression or an equation? Why?  

Evaluate.  

13. a. \[
\begin{array}{c}
\frac{9}{0} \\
\end{array}
\]  
b. \[
\begin{array}{c}
\frac{0}{9} \\
\end{array}
\]
Answer Key for Chapter 1 Section 1.

1. variable is \( x \); coefficient is 7; constant is 8
2. variable is \( y \); coefficient is 9; constant is 12
3. variable is \( z \); coefficient is 4; constant is 7
4. variable is \( a \); coefficient is 8; constant is 9
5. variable is \( m \); coefficient is 4; constant is 6
6. 2 terms
7. factors are 11 and \( m \)
8. 13\( y \) demonstrates multiplication
9. 4 terms
10. constant is 24
11. The first five whole numbers are 0, 1, 2, 3, and 4
12. \( 5z + 18 = 38 \) is an equation; it contains an equal sign.
13. a. undefined
    b. 0 or zero
Section 2: Combine Like Terms

In algebra, combining like terms is an important concept. You’ll approach this first by considering “everyday” ideas and then connecting those ideas to algebraic concepts.

If you saw the words “3 couches + 5 stars + 8 couches + 2 stars,” you would probably say you had “11 couches + 7 stars.” You have just combined like terms in a natural way. You added the couches with the couches and the stars with the stars. You did this because those items are alike. Algebra works the same way, except algebra uses letters and numbers instead of words:

a. \[ 3x + 5 + 8x + 2 \]
   \[ = 3x + 8x + 5 + 2 \] line 1 rearranged
   \[ = 11x + 7 \]

You have just combined like terms. Algebraically, you added together the variable terms (you added the coefficients) and you added together the constant terms. Combine like terms means to add similar items.

b. \[ 2 \text{ dogs} + 3 \text{ cars} + 5 \text{ ants} + 6 \text{ cars} + 2 \text{ ants} + 8 \text{ dogs} \]
   \[ = 2 \text{ dogs} + 8 \text{ dogs} + 3 \text{ cars} + 6 \text{ cars} + 5 \text{ ants} + 2 \text{ ants} \] line 1 rearranged
   \[ = 10 \text{ dogs} + 9 \text{ cars} + 7 \text{ ants} \]

You would naturally add together those items that were alike. Even though the original problem “scrambled” the order of the words, you still added dogs with dogs, cars with cars, and ants with ants. Algebra works similarly:

c. \[ 2x + 3y + 5 + 6y + 2 + 8x \]
   \[ = 2x + 8x + 3y + 6y + 5 + 2 \] line 1 rearranged
   \[ = 10x + 9y + 7 \]

Notice that this result is written in a certain way. The variable terms are written first, in alphabetical order, and the constant is written last. This is a typical, standard way of writing this expression. Also notice that you don’t try to combine further than this. With adding, the terms must be exactly alike to be combined. As an analogy, if you saw 8 feet + 3 hours, you know you would not combine further whereas if you saw 8 hours + 3 hours you would combine to get 11 hours.
To summarize, **combining like terms algebraically involves adding together those terms that look alike.** There is an algebraic reason this works, but for now you’ll approach this concept informally. At the end of this section, you will learn some new vocabulary to help describe what is occurring with combining like terms.

A few more words about combining before you practice. If you see x with no number in front, it is understood that this means 1x. If it makes you more comfortable, go ahead and write the 1 in front whenever you see a variable without a number.

d. \[ k + 9 + 3k + 6 \]
   \[ 1k + 9 + 3k + 6 \] you can think or write this line
   \[ 1k + 3k + 9 + 6 \] you can think or write this line
   \[ 4k + 15 \] the result after combining like terms


e. \[ 9 + 6m + 5 + 4m + m \]
   \[ 9 + 5 + 6m + 4m + 1m \] thinking step!
   \[ 14 + 11m \] result after combining like terms
   \[ 11m + 14 \] another way to write the result after combining like terms

Both \[ 14 + 11m \] and \[ 11m + 14 \] are correct, but it is usually preferred to write the result as \[ 11m + 14 \]. This is more of a standard order, to write the variable terms first and then write the constant last.

Additionally, the instructions for the problems would probably say **“Simplify.”** In this situation, you are expected to know by looking that you should combine like terms.

Addition is considered to be **commutative** and **associative.**

**Commutative** describes the condition of being able to reverse the order of adding two numbers and still getting the same result. For example,

f. \[ 3 + 4 = 7 \]
   \[ 4 + 3 = 7 \]

Notice that the order of the addition of the two numbers didn’t affect the result.

**Associative** describes the condition of being able to change the grouping of the adding and still getting the same result. For example, compare the following:

g. \[ 3 + (7 + 8) \] compared to \[ (3 + 7) + 8 \]
   \[ 3 + 15 \] \[ 10 + 8 \]
   \[ 18 \] \[ 18 \]
Notice that changing the grouping (what was added first as indicated by the parentheses) didn’t affect the result.

**Commutative** changes the order of an addition of two numbers without changing the result while **associative** changes the grouping of the numbers of an addition problem without changing the result.

Because addition is both commutative and associative, parentheses are usually not used with addition problems. In fact, you can add numbers in any order:

h. \[18 + 33 + 2 + 41 + 7\]
   \[= 18 + 2 + 33 + 7 + 41\] line 1 rearranged
   \[= 20 + 40 + 41\] add quickly \((18 + 2; 33 + 7)\)
   \[= 101\]

How do commutative and associative apply to combining like terms? When we have the expression

i. \[3m + 8 + 24m + 82\] we use commutative and associative to rearrange the problem to
   \[= 3m + 24m + 8 + 82\] so we can “group” together similar terms and find the result
   \[= 27m + 90\]

Also, commutative is used to reverse the order of an ending result. If an expression is written as \(17 + 5m\), the commutative property allows us to re-write that expression as \(5m + 17\).

Multiplication is also considered to be commutative and associative. If we have the factors \(3(5)\), we can reverse those factors to \(5(3)\) and still obtain the same result 15. If we have the factors \(2(5\cdot7)\), we can change the grouping to \((2\cdot5)7\) and still obtain the same result of 70. Because multiplication is both commutative and associative, many times a multiplication problem will not contain parentheses. This allows you to multiply in order from left to right OR this allows you to multiply in any order that is more convenient.

j. \[2(7)(5)\] you might think \(2(5)(7)\) to quickly get 70.

One final thought about multiplying. We already know that \(7z\) means 7 times \(z\). Now we also know that \(z\) times 7 is the same problem in reverse order. However, we want to write the final version of \(z\) times 7 as \(7z\). This way we will all have similar looking results. Observe the following examples:

k. \[
m(8) \quad y(10) \quad 9(a) \quad k \cdot 12
\]
   \[
   8m \quad 10y \quad 9a \quad 12k
   \]

When you simplify, write the result with the coefficient written before the variable.
Homework for Chapter 1 Section 2

Simplify the following.

1. $2m + 3 + 5m + 12$
2. $4z + 11 + 9z + 18$

3. $26 + 14y + 30 + 17y$
4. $a + 12 + 7a + 18$

5. $7 + 2x + 3 + 6x$
6. $13 + 20b + 8 + 15b$

7. $4k + 12 + 13 + k + 2k + 7$
8. $9x + 17y + 11 + 16 + 3y + 6x$

9. In $9m + 19$, how many terms are there? What are the terms?
10. In $32 + 17y$, what is the coefficient? the constant? the variable?
11. In $17z$, what are the factors?
12. In $f + 19$, what is the coefficient?
13. Is $11a + 13$ considered to be an equation or an expression?
14. Does this problem show that addition is commutative or associative?
   $15 + (5 + 9) = (15 + 5) + 9$
15. Does this problem show that addition is commutative or associative?
   $25 + 19z = 19z + 25$

Simplify.

16. $14(m)$
17. $y(11)$
18. $b \cdot 7$

19. $(13)(z)$
20. $(22) \cdot m$
21. $d(1)$

22. Is multiplication associative? Give an example.
23. Is multiplication commutative? Give an example.
24. If you saw $13(25)(81)(0)(9)$, what would be the advantage of looking at the entire problem first?
Chapter 1 Section 2

Combine like terms

Answer Key for Chapter 1 Section 2.

1. $7m + 15$
2. $13z + 29$
3. $31y + 56$
4. $8a + 30$
5. $8x + 10$
6. $35B + 21$
7. $7k + 32$
8. $15x + 20y + 27$
9. 2 terms; one term is $9m$; the other term is 19
10. coefficient 17; constant 32; variable $y$
11. factors 17 and $z$
12. coefficient is 1
13. expression; no equal sign
14. associative
15. commutative
16. $14m$
17. $11y$
18. $7b$
19. $13z$
20. $22m$
21. $d$
22. yes; for example $8(5 \cdot 11) = (8 \cdot 5)11$ because they both result in 440
23. yes; for example $9(10) = 10(9)$ because they both result in 90
24. The advantage to looking at the entire problem is you may find a “quick” multiplication fact that will speed you up in your work. In this example, anytime you multiply by zero the result is zero. If you look at the entire problem first and notice this, you save yourself some time.
Section 3: Distributive Property

In English, you have a way to indicate that you’re going to multiply several items by the same number. For example, if you read “2 baskets, each with 5 marbles and 3 cookies,” you would say that you had 10 marbles and 6 cookies. You multiplied both items by 2. In algebra, you would indicate this problem by using parentheses:

a. \[2(5 \text{ marbles} + 3 \text{ cookies})\]
   \[10 \text{ marbles} + 6 \text{ cookies}\]

This shows the **distributive property of multiplication over addition**. Most of the time, we’ll shorten that and say we’re using the distributive property. The full name is important, though, because it does indicate that you need to have an addition (or subtraction) symbol on the inside of the parentheses. It also indicates each term inside the parentheses is being multiplied by the number in front of the parentheses. See the examples.

b. \[2(3y + 9)\]
   \[6y + 18\]

c. \[6(5z + 8)\]
   \[30z + 48\]

d. \[9(a + 3)\]
   \[9a + 27\]

Examples b, c, and d are all the distributive property because all had an addition symbol on the inside of the parentheses. In examples b, c, and d, each term inside the parentheses was multiplied by the number in front of the parentheses. Once you multiply, you remove the parentheses.

Look at example b once more, in more detail.

\[
2(3y + 9) \text{ means } \quad 2(3y) + 2(9) \text{ to get } \quad 6y + 18
\]

Usually, you don’t show the middle step; you multiply each term inside the parentheses by the number in front of the parentheses and write the result.

The distributive property also works when you have subtraction symbols on the inside of the parentheses. You still multiply each term inside the parentheses by the number in front of the parentheses; remove the parentheses.

e. \[8(7m - 5)\]
   \[56m - 40\]

f. \[4(x - 3)\]
   \[4x - 12\]

g. \[5(5k - 12)\]
   \[25k - 60\]
The distributive property can be extended to include the following. Do you see why?

h. \(4(2x + 3y + 8)\)  
i. \(9(4x - 2y + 8)\)

\[8x + 12y + 32\]  
\[36x - 18y + 72\]

Again, the word “Simplify” will be used in the instructions. You will be expected to be able to tell by looking at the problem that the distributive property is to be used.

NOTE: How would you show in algebra the concept of 8 apples and 20 bagels being shared among 4 people? One way to show that would be to write

\[(8 \text{ apples and 20 bagels}) \div 4\]

Another way to show the same idea would be to write

\[
\frac{8 \text{ apples and 20 bagels}}{4}
\]

No matter which way you write this, the result would be 2 apples and 5 bagels for each person. In essence, when you divide an entire expression by a number, each separate term in the expression is divided by the same number. This is a type of distributive thinking. Observe the following for more on this idea.

j. \[\frac{18x + 9}{3}\]

means

\[\frac{18x}{3} + \frac{9}{3}\]

\[6x + 3\]

k. \[\frac{28m - 63}{7}\]

means

\[\frac{28m}{7} - \frac{63}{7}\]

\[4m - 9\]
Homework for Chapter 1 Section 3

Simplify the following.

1. $2(5x + 3)$
2. $3(7m + 1)$
3. $4(z + 2)$
4. $5(k - 6)$
5. $9(3y - 12)$
6. $10(5a - 6)$
7. $7(3x + 9y - 11)$
8. $6(9v - w - 1)$
9. $3(a + 3b + c)$
10. $\frac{40y + 16}{8}$
11. $\frac{72a - 45}{9}$
12. $6y + 3y + 9 + 20$
13. $9a + 13 + 10a + 17$
14. $15 + 6m + 19 + m$
15. $a + 3 + a + 1$
16. $z \cdot 7$
17. $a(8)$
18. $6(2)(k)$

19. Is $9k + 11 = 92$ an equation or an expression? Why?
20. In $91p + 16$, what are the terms? What is the constant? What is the coefficient? What is the variable? Which term has factors in it? What are those factors?
21. Does this problem show that addition is commutative or associative?
   $8 + 11 = 11 + 8$
22. Give an example of the distributive property.
23. Does this problem show that addition is commutative or associative?
   $5 + (25 + 8) = (5 + 25) + 8$
Chapter 1 Section 3

Distributive property

Answer Key for Chapter 1 Section 3.

1. 10x + 6
2. 21m + 3

3. 4z + 8
4. 5k – 30

5. 27y – 108
6. 50a – 60

7. 21x + 63y – 77
8. 54v – 6w – 6

9. 3a + 9b + 3c
10. 5y + 2

11. 8a – 5
12. 9y + 29

13. 19a + 30
14. 7m + 34

15. 2a + 4
16. 7z

17. 8a
18. 12k

19. 9k + 11 = 92 is an equation; it has an equal sign

20. terms are 91p and 16; constant is 16; coefficient is 91; variable is p; 91p has the factors in it; factors are 91 and p

21. commutative

22. example: 
   \[
   5(3m + 9) \\
   15m + 45
   \]

23. associative
**Section 4: Distribute and Combine**

You are ready to distribute AND combine like terms in one problem.

**Distribute and combine like terms** will mean the following. Distribute means to multiply each term inside the parentheses by the number in front of the parentheses AND remove the parentheses. Combine like terms means to mentally rearrange the problem so like terms are together and then to add those like terms.

See if you can follow the steps on these examples.

a. \[5(3k + 6) + 9(4k + 7)\]
   \[15k + 30 + 36k + 63\]
   \[51k + 93\]

b. \[4(3x + 7) + 5(x + 2)\]
   \[12x + 28 + 5x + 10\]
   \[17x + 38\]

What do you think you would do with a problem that looked like this?

c. \[7(a + 9) + (5a + 8)\]
   \[7(a + 9) + 1(5a + 8)\]
   \[7a + 63 + 5a + 8\]
   \[12a + 71\]
   you might think: \[1(5a + 8)\]
   rewrite showing 1 in front of \((5a + 8)\)
   think: \[7a + 5a + 63 + 8\]
   combine like terms

How would you approach the following? See if you can follow the steps.

d. \[18 + 10(3z + 7) + (z + 11) + 4(5z + 6)\]
   \[18 + 10(3z + 7) + 1(z + 11) + 4(5z + 6)\]
   \[18 + 30z + 70 + 1z + 11 + 20z + 24\]
   \[30z + 1z + 20z + 18 + 70 + 11 + 24\]
   \[51z + 123\]

Again, the instructions will say “Simplify.” You will be expected to know by looking at this type of problem that you are to distribute (multiply) to remove the parentheses and then to combine like terms.
Homework for Chapter 1 Section 4

Simplify the following.

1. $5(x + 9) + 8(2x + 7)$
2. $6(2y + 8) + 3(4y + 9)$
3. $2(4m + 1) + 3(5m + 2)$
4. $8(7a + 10) + 9(a + 6)$
5. $2(4k + 3) + (9k + 10)$
6. $(5A + 3) + (6A + 2)$
7. $3(6m + 3) + 8(m + 6) + 9(4m + 5)$
8. $9(2z + 6) + (z + 3) + 3(10z + 12)$
9. $5(2x + 3y + 8) + 7(4x + 9y + 2)$
10. $4 + 3(6k + 5)$
11. $5A + 3 + 7(2A + 8)$
12. $16 + 4C + 8(5 + 2C) + 10$

13. Give an example of a two-term expression.
14. Give an example of a one-term expression that contains factors.
15. In $6g + 93$, what is the constant? What is the variable? What is the coefficient?
16. When you rewrite $3(8y + 9)$ as $24y + 27$, what property have you used?
17. When you rewrite $51 + 13y$ as $13y + 51$, what property have you used?

Simplify.

18. $\frac{4y + 20}{4}$
19. $\frac{45a - 30}{5}$
20. $\frac{14a + 20b + 8c}{2}$
21. $\frac{33x - 6y - 21z}{3}$
Chapter 1 Section 4

Distribute and combine

Answer Key for Chapter 1 Section 4.

1. $21x + 101$
2. $24y + 75$
3. $23m + 8$
4. $65a + 134$
5. $17k + 16$
6. $11A + 5$
7. $62m + 102$
8. $49z + 93$
9. $38x + 78y + 54$
10. $18k + 19$
11. $19A + 59$
12. $20C + 66$
13. example: $45z + 16$
14. example: $19a$
15. constant is 93
   variable is g
   coefficient is 6
16. distributive
17. commutative
18. $y + 5$
19. $9a - 6$
20. $7a + 10b + 4c$
21. $11x - 2y - 7z$
Section 5: Exponents, Order of Operations, Evaluate

Exponents

When you see $5^3$ (read as “5 to the third power”), you are looking at an exponential expression. This expression is a short way of writing a repeated multiplication. The number 5 is the base, and the number 3 is the exponent.

$$5^3 = 5 \cdot 5 \cdot 5 = 125$$

When you see $8 \cdot 8 \cdot 8 \cdot 8$ (or if you see $8^1 \cdot 8^1 \cdot 8^1 \cdot 8^1$), you can rewrite it as $8^4$ (read as “8 to the fourth power”), where 8 is the base and 4 is the exponent.

You want to be able to quickly expand and find the result for exponential expressions. Observe the following.

<table>
<thead>
<tr>
<th>a. $6^2$</th>
<th>b. $3^4$</th>
<th>c. $2^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$6 \cdot 6$</td>
<td>$3 \cdot 3 \cdot 3 \cdot 3$</td>
<td>$2 \cdot 2 \cdot 2$</td>
</tr>
<tr>
<td>$36$</td>
<td>$81$</td>
<td>$8$</td>
</tr>
</tbody>
</table>

Order of Operations

When you work with an expression that has several operations (for example, multiply, subtract, exponents), there is an order to simplifying so that everyone gets the same result. Here is the order of operations:

1. Simplify inside the grouping symbols first. Grouping symbols include parentheses ( ), brackets [ ], and braces { }. A grouping symbol has an operation inside that needs to occur.
2. Simplify exponents second.
3. From left to right, simplify all divisions and/or multiplications, whichever one comes first. Since divide and multiply are “co-equal” operations in math, neither one takes precedence over the other.
4. From left to right, simplify all subtractions and/or additions, whichever one comes first. Since subtract and add are “co-equal” operations in math, neither one takes precedence over the other. Note: addition and/or subtraction (not inside grouping symbols) is done last after exponents, multiplication, division.

The easiest way to become comfortable with simplifying with order of operations is to work several examples. Observe the following.

d. $8 - 5 + 3$
   $3 + 3$
   $6$
   subtract first since subtraction is on left
   add second since addition is on right
Chapter 1 Section 5  

**Exponents, order of operations, evaluate**

e. \[24 \div 2 \cdot 3\]
   12 \cdot 3
   divide first since division is on left
   36
   multiply second since multiplication is on right

f. \[5 + 8(9 - 5)\]
   5 + 8(4)
   simplify inside the parentheses first
   5 + 32
   multiply
   37
   add

g. \[19 - 2(5 + 1)\]
   19 - 2(6)
   simplify inside the parentheses first
   19 - 12
   multiply
   7
   subtract

h. \[16 \div 2 \cdot 7 - 3(7 - 5)\]
   16 \div 2 \cdot 7 - 3(2)
   simplify inside the parentheses first
   8 \cdot 7 - 3(2)
   divide (divide was left of multiply)
   56 - 6
   multiply
   50
   subtract

i. \[(3 + 4)^2 - 2^2\]
   \((7)^2 - 2^2\)
   simplify inside the parentheses first
   49 - 8
   exponents
   41
   subtract

Even within a grouping symbol, you still follow order of operations.

j. \[18 + 3(70 - 3 \cdot 5)\]
   18 + 3(70 - 15)
   inside parentheses, multiply first
   18 + 3(55)
   simplify inside parentheses
   18 + 165
   multiply
   183
   add

k. \[18 + 3[70 - 3(5)]\]
   18 + 3[70 - 15]
   inside brackets, multiply first
   18 + 3[55]
   simplify inside brackets
   18 + 165
   multiply
   183
   add
Exponents, order of operations, evaluate

l. 
\[ 7(9) - 5^2 + 24 ÷ 3 \cdot 7 \]
\[ 7(9) - 25 + 24 ÷ 3 \cdot 7 \]  
- exponent
\[ 63 - 25 + 8 \cdot 7 \]  
- multiply/divide (left to right)
\[ 63 - 25 + 56 \]  
- multiply
\[ 94 \]  
- subtract/add (left to right)

m. 
\[ \left( \frac{10}{2} \right)^2 \cdot (4) + (7 - 3)^2 \]
\[ (5)(4) + (4)^2 \]  
- inside parentheses (both sets)
\[ (5)(4) + 16 \]  
- exponent
\[ 20 + 16 \]  
- multiply
\[ 36 \]  
- add

These examples give you an idea of how to approach an order of operations problem. You may find that you need to work the above examples several times before you become comfortable with the concept. Also, you will probably encounter problems that are not exactly like the above; remember order of operations will guide you to the answer.

In summary, order of operations is a method of simplifying so everyone who works an expression will get the same result. You should simplify inside grouping symbols first, simplify exponents second, simplify all multiplication and division in order from left to right, and finally simplify all addition and subtraction in order from left to right. The instructions would say “Simplify.” You would be expected to know that order of operations is the appropriate approach to simplifying expressions.

Evaluate

To evaluate an expression means to replace a variable with a number and simplify using order of operations. When you substitute the number for the variable, it is a good practice to use parentheses around the number. When you evaluate, the variable is removed and replaced by the given number. Observe.

m. Evaluate \( 3x + 8 \) when \( x = 12 \)
\[ 3(12) + 8 \]
\[ 36 + 8 \]
\[ 44 \]

n. Evaluate \( 5y - 7 \) for \( y = 8 \)
\[ 5(8) - 7 \]
\[ 40 - 7 \]
\[ 33 \]
Chapter 1 Section 5

Exponents, order of operations, evaluate

Homework for Chapter 1 Section 5

Simplify.

1. $2^5$
2. $3^4$
3. $6^2$
4. $4^3$

Simplify.

5. $18 - 11 + 7$
6. $36 \div 2 \cdot 3$
7. $11 + 7(2 + 5)$
8. $8(3) + 20 \div 2$
9. $24 - 5(1 + 2)$
10. $18 \div 3 \cdot 10 - 7^2$
11. $9^2 - 27 \div 3$
12. $10 \cdot 6 \div 5 + 2^4$
13. $(9^2 - 27) \div 3$
14. $(2 + 8^2)(7 - 4)^2$
15. $18 + 3 \div 4(6 + 1)$
16. $\frac{20}{4} + 5(8 - 6)$
17. $\left(\frac{30}{5}\right)(3) - 2(1 + 4)$
18. $\left(\frac{45}{9}\right)(4) + 6(8 - 2)$
19. $5[7 + 3(8)] + 5^3$
20. $24 + (13 - 5)^2$
21. $35 - 2(16 - 4^2)$
22. $29 - 3^3 + 36 \div (7 + 2)$

Evaluate.

23. $5z + 20$ when $z = 6$
24. $8k - 2$ when $k = 6$
25. $14a - 11$ for $a = 3$
26. $10m + 7$ for $m = 2$

Simplify.

27. $9m + 15 + 12m + 19$
28. $7(3y + 8)$
29. $2(3a + 4) + 7(5a + 9)$
30. $6 + 7M + 3(4 + 8M)$
31. $\frac{18k + 12}{3}$
32. $\frac{20c - 32}{4}$
33. Is $5m + 8$ an expression or an equation? Why?

34. In $7y$, what is the coefficient? What are the factors?

35. When you rewrite $3(4m + 11)$ as $12m + 33$, what property have you used?

### Additional Practice with Order of Operations

Simplify.

36. $7 + 8(12 - 10)$  
37. $34 - 5(2 + 1)$

38. $5 + 6(11 + 3^2)$  
39. $2(7^2 - 2^3)$

40. $2(7 - 2)^2$  
41. $2(7 - 2)^3$

42. $100 ÷ (3 + 2)^2$  
43. $(8 + 2 \cdot 4^2) ÷ 5$

44. $24 ÷ 2 + (8 + 1)^2 - \frac{28}{4}$
### Answer Key for Chapter 1 Section 5.

1. 32  
2. 81  
3. 36  
4. 64  
5. 14  
6. 54  
7. 60  
8. 34  
9. 9  
10. 11  
11. 72  
12. 28  
13. 18  
14. 594  
15. 34  
16. 15  
17. 8  
18. 56  
19. 280  
20. 88  
21. 35  
22. 6  
23. 50  
24. 46  
25. 31  
26. 27  
27. $21m + 34$  
28. $21y + 56$  
29. $41a + 71$  
30. $31M + 18$  
31. $6k + 4$  
32. $5c - 8$  
33. an expression; it contains no equal sign  
34. coefficient is 7; factors are 7 and y  
35. distributive  
36. 23  
37. 19  
38. 125  
39. 82  
40. 50  
41. 250  
42. 4  
43. 8  
44. 86
Section 6: Solve Equations Using Whole Numbers

Recall that $5x + 18 = 93$ is considered an equation because it contains an equal sign. Also, recall the two sides of the equation represent the same numerical value. Since one of the major goals in algebra is to solve equations, this section will present how to solve equations using whole numbers.

**Solving equations** means to perform a series of manipulations so the variable is isolated. In general, you try to “undo” what is written in the equation so the variable is alone on one side of the equal sign. While you are manipulating the equation, what you do to one side of the equation you do to the other side of the equation, in order to keep the equation “balanced.”

To solve an equation with ADDITION, you would SUBTRACT.
To solve an equation with SUBTRACTION, you would ADD.
To solve an equation with MULTIPLICATION, you would DIVIDE.

Look at the following examples and the explanations. When you look at an equation, you should keep in mind that you are trying to get the variable isolated by “undoing” the operations in the equation.

a. $m + 12 = 17$

\[
\begin{align*}
\text{To “undo” addition, subtract 12} \\
m & = 5
\end{align*}
\]

b. $y - 18 = 30$

\[
\begin{align*}
\text{To “undo” subtraction, add 18} \\
y & = 48
\end{align*}
\]

c. $5m = 30$

\[
\begin{align*}
\text{To “undo” multiplication, divide by 5} \\
m & = 6
\end{align*}
\]

The equations above are all considered one-step equations, because it took only one operation to isolate the variable. Equations do become more complicated. The second type of equation you’ll solve will be two-step equations, where you have two operations to “undo.” When that occurs, you usually “undo” addition and subtraction first; then you “undo” multiplication. You want variable terms on one side of the equation and constant terms on the other side. Observe the steps in the following examples.
Chapter 1 Section 6  Solve equations using whole numbers

d. $5a = 18 + 2a$
   $-2a -2a$
   \[3a = 18\]  Subtract 2a to “undo” addition

   $\frac{3a}{3} = \frac{18}{3}$  Divide by 3 to “undo” multiplication

   a = 6

e. $2z + 17 = 35$
   $-17 -17$
   \[2z = 18\]  Subtract 17 to “undo” addition

   $\frac{2z}{2} = \frac{18}{2}$  Divide by 2 to “undo” multiplication

   z = 9

f. $7k = 36 - 2k$
   $+2k +2k$
   \[9k = 36\]  Add 2k to “undo” subtraction

   $\frac{9k}{9} = \frac{36}{9}$  Divide by 9 to “undo” multiplication

   k = 4

g. $3a - 18 = 39$
   $+18 +18$
   \[3a = 57\]  Add 18 to “undo” subtraction

   $\frac{3a}{3} = \frac{57}{3}$  Divide by 3 to “undo” multiplication

   a = 19

Another way to talk about solving equations is to talk about manipulating the equation to get all variable terms on one side of the equal sign and to get all constant terms on the other side of the equal sign. Then you continue until the variable is completely isolated. Observe the following examples and explanations.
Chapter 1 Section 6

Solve equations using whole numbers

h. \[ 5k + 11 = 81 \]
\[
\begin{array}{c}
\underline{-11} \\
5k = 70
\end{array}
\]
\[
\begin{array}{c}
5k = 70 \\
5
\end{array}
\]
\[
\frac{5k}{5} = \frac{70}{5}
\]
\[
k = 14
\]

Subtract 11 to “undo” addition--this gets all constant terms on right side of equals

Variable terms on left, constant terms on right of equals

Divide by coefficient 5 to “undo” multiplication

Variable completely isolated on left of equals

i. \[ 11y - 15 = 62 \]
\[
\begin{array}{c}
\underline{+15} \\
11y = 77
\end{array}
\]
\[
\begin{array}{c}
11y = 77 \\
11
\end{array}
\]
\[
\frac{11y}{11} = \frac{77}{11}
\]
\[
y = 7
\]

Add 15 to “undo” subtraction--this gets all constant terms on right side of equals

Variable terms on left, constant terms on right of equals

Divide by coefficient 11 to “undo” multiplication

Variable completely isolated on left of equals

NOTE: Notice these problems have been worked in a vertical fashion. You can also work the problems in a horizontal fashion. Observe.

j. \[ 40z = 56 + 33z \]
\[
40z - 33z = 56 + 33z - 33z
\]
\[
7z = 56 + 0
\]
\[
7z = 56
\]
\[
\frac{7z}{7} = \frac{56}{7}
\]
\[
z = 8
\]

When you solve equations, it is usually easier if you simplify each side of the equation first. Observe.

k. \[ 4m + 8 + 2m + 4 = 30 \]
\[
4m + 2m + 8 + 4 = 30
\]
\[
6m + 12 = 30
\]
\[
6m + 12 - 12 = 30 - 12
\]
\[
6m = 18
\]
\[
\frac{6m}{6} = \frac{18}{6}
\]
\[
m = 3
\]
Suppose you wanted to check your results. To check, you replace the variable in the original equation with the result you obtained, follow order of operations to simplify each side, and see if the numbers on both sides of the equal sign are the same. If the numbers are the same, then you know the result you obtained is the solution. If the numbers are not the same, you have made an error somewhere.

Observe the steps to check the result from example i.

\[ 11y - 15 = 62; \quad \text{check } y = 7 \]
\[ 11(7) - 15 = 62 \quad \text{Replace } y \text{ with } 7 \text{ in parentheses} \]
\[ 77 - 15 = 62 \]
\[ 62 = 62 \quad \text{This checks.} \]

Therefore, \( y = 7 \) is the solution for the equation \( 11y - 15 = 62 \).

The instructions for solving equations would be “Solve.” It is implied that you should check your result, even when that isn’t stated explicitly.

NOTE: If you are wondering why this works, here is an analogy that may demonstrate. For the equation \( 4m + 5 = 17 \), imagine a videotape made of someone starting with a number, multiplying it by 4, then adding 5, getting an answer 17. Then imagine playing the tape backwards in order to get back to the number \( m \). You would see the opposite operation of the last thing that was done, then the opposite operation of the first thing that was done.
Chapter 1 Section 6  Solve equations using whole numbers

Homework for Chapter 1 Section 6

Solve.

1. \( z + 12 = 34 \)  
2. \( z + 48 = 50 \)
3. \( k - 17 = 39 \)  
4. \( y - 28 = 37 \)
5. \( 4m = 28 \)  
6. \( 8a = 96 \)
7. \( x - 19 = 41 \)  
8. \( y + 32 = 80 \)
9. \( 8a = 24 + 5a \)  
10. \( 2m = 30 - 3m \)
11. \( 2c + 13 = 19 \)  
12. \( 6w - 13 = 71 \)
13. \( 5y + 3y + 16 = 48 \)  
14. \( 16k = 18 + 20 - 3k \)
15. \( 12v = 60 \)  
16. \( d + 48 = 83 \)
17. \( 7k - 21 = 35 \)  
18. \( 3z + 14 = 104 \)
19. \( 10A = 60 - 2A \)  
20. \( 29x = 42 + 23x \)
21. \( 4 + 5Z + 10 = 29 \)  
22. \( 2X + 18 + 9X + 12 = 63 \)
23. \( 9m + 18 = 36 \)  
24. \( 5a - 38 = 17 \)
25. \( 4x + 30 = 30 \)  
26. \( 5y + 13 = 18 \)

Simplify.

27. \( 2y + 8 + 3y + 20 \)  
28. \( 5(3z - 8) \)
29. \( (8x + 11) \)  
30. \( (2x + 3) + 7(4x + 1) \)
31. \( 3(m + 4) + 6(2m + 3) + 18 + 9m + m + 2 \)
32. \( 15 + 5 \cdot 9 + 6(13 - 10) - 4^2 \)
In the expression $18 + 11m$,

33. What is the coefficient?  
34. What is the constant?  
35. What is the variable?  
36. Which term has factors? What are the factors?

Try these.

37. $14 + x = 20$  
38. $28 + y = 35$

39. $39 = 5 + m$  
40. $42 = z + 19$
Chapter 1 Section 6

Answer Key for Chapter 1 Section 6.

1. \( z = 22 \) 2. \( z = 2 \) 3. \( k = 56 \)
4. \( y = 65 \) 5. \( m = 7 \) 6. \( a = 12 \)
7. \( x = 60 \) 8. \( y = 48 \) 9. \( a = 8 \)
10. \( m = 6 \) 11. \( c = 3 \) 12. \( w = 14 \)
13. \( y = 4 \) 14. \( k = 2 \) 15. \( v = 5 \)
16. \( d = 35 \) 17. \( k = 8 \) 18. \( z = 30 \)
19. \( A = 5 \) 20. \( x = 7 \) 21. \( Z = 3 \)
22. \( X = 3 \) 23. \( m = 2 \) 24. \( a = 11 \)
25. \( x = 0 \) 26. \( y = 1 \) 27. \( 5y + 28 \)
28. \( 15z - 40 \) 29. \( 8x + 11 \) 30. \( 30x + 10 \)
31. \( 25m + 50 \) 32. \( 29 \)
33. The coefficient is 11.
34. The constant is 18.
35. The variable is \( m \).
36. \( 11m \) is the term with factors; 11 and \( m \) are the factors.
37. \( x = 6 \) 38. \( y = 7 \)
39. \( 34 = m \) or \( m = 34 \)
40. \( 23 = z \) or \( z = 23 \)
### Chapter 1 Summary

<table>
<thead>
<tr>
<th>Expression</th>
<th>$2x + 7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equation</td>
<td>$2x + 7 = 19$</td>
</tr>
<tr>
<td>Terms</td>
<td>In $2x + 7$, $2x$ is one term; 7 is another term</td>
</tr>
<tr>
<td>Coefficient</td>
<td>In $2x$, 2 is the coefficient</td>
</tr>
<tr>
<td>Variable</td>
<td>In $2x$, $x$ is the variable</td>
</tr>
<tr>
<td>Constant</td>
<td>In $2x + 7$, 7 is the constant</td>
</tr>
<tr>
<td>Factors</td>
<td>In $2x$, 2 and $x$ are factors (they are multiplied)</td>
</tr>
<tr>
<td>Whole Numbers</td>
<td>${0, 1, 2, 3, \ldots }$</td>
</tr>
<tr>
<td>Combine Like Terms</td>
<td>$5m + 12 + 3m + 17 \rightarrow 8m + 29$</td>
</tr>
<tr>
<td>Distribute</td>
<td>$5(4y + 2) \rightarrow 20y + 10$</td>
</tr>
<tr>
<td></td>
<td>$4(6y - 3) \rightarrow 24y - 12$</td>
</tr>
<tr>
<td></td>
<td>$\frac{16y + 20}{2} \rightarrow 8y + 10$</td>
</tr>
<tr>
<td>Distribute and Combine</td>
<td>$2(k + 7) + 10(3k + 4)$</td>
</tr>
<tr>
<td></td>
<td>$2k + 14 + 30k + 40$</td>
</tr>
<tr>
<td></td>
<td>$32k + 54$</td>
</tr>
<tr>
<td>Order of Operations</td>
<td>Perform operations inside grouping symbols; simplify with exponents; multiply or divide from left to right; add or subtract from left to right</td>
</tr>
<tr>
<td></td>
<td>$90 - 2(5 + 4) - 4^2 + 20 ÷ 2 \cdot 5$</td>
</tr>
<tr>
<td></td>
<td>$90 - 2(9) - 4^2 + 20 ÷ 2 \cdot 5$</td>
</tr>
<tr>
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<td>$90 - 2(9) - 16 + 20 ÷ 2 \cdot 5$</td>
</tr>
<tr>
<td></td>
<td>$90 - 18 - 16 + 10 \cdot 5$</td>
</tr>
<tr>
<td></td>
<td>$90 - 18 - 16 + 50$</td>
</tr>
<tr>
<td></td>
<td>$72 - 16 + 50$</td>
</tr>
<tr>
<td></td>
<td>$56 + 50$</td>
</tr>
<tr>
<td></td>
<td>$106$</td>
</tr>
</tbody>
</table>
Evaluate

4a + 7 for a = 3

\[ 4(3) + 7 \]
\[ 12 + 7 \]
\[ 19 \]

Solve

a. \[ x + 9 = 14 \]
   \[ -9 \]
   \[ x = 5 \]

b. \[ y - 12 = 20 \]
   \[ + 12 \]
   \[ y = 32 \]

c. \[ 5k = 30 \]
   \[ 5 \]
   \[ k = 6 \]

d. \[ 6y = 36 - 3y \]
   \[ 6y + 3y = 36 - 3y + 3y \]
   \[ 9y = 36 + 0 \]
   \[ 9y = 36 \]
   \[ 9y ÷ 9 = 36 ÷ 9 \]
   \[ y = 4 \]

e. \[ 4z + 12 = 96 \]
   \[ -12 \]
   \[ 4z = 84 \]
   \[ 4 \]
   \[ z = 21 \]

Check Your Solution

a. Check the result to example e above.
   \[ 4z + 12 = 96; z = 21 \]
   \[ 4(21) + 12 = 96 \]
   \[ 84 + 12 = 96 \]
   \[ 96 = 96 \]
   This checks; z = 21 is the solution
Chapter 1 Review

1. In $3x + 14$, what is the variable? the coefficient? the constant?

2. In $3A + 4B + 5C + 6$, how many terms are there? What are the terms?

3. What operation does $28Z$ demonstrate? What are the factors?

4. What are the first four whole numbers?

5. Is $3A + 4B + 5C + 6$ an expression or an equation? Why?

6. Does this show that multiplication is commutative or associative?
   
   $(13 \cdot 8) \cdot 2 = 2 \cdot (13 \cdot 8)$

7. Is addition associative? Give an example.

8. Give an example of the distributive property.

9. Is $2^3$ equal to $3^2$? Explain your answer.

Simplify.

10. $4A + 3B + 2 + 5B + 3 + 3A + B$

11. $x(1)$

12. a. $4(2X - Y + 9Z - 4)$

   b. $\frac{8y + 32z + 28}{4}$

13. $19 + 4(3M + 2N) + 3(3M + 2) + 5(M + N + 3)$

14. $5^3$

15. $24 - 7 + 5 \cdot 2$

16. $5(3 + 2^2) - 2^3$

17. $(7 - 4)^3 + 9 \cdot 3$

18. $(7^2 - 3^3) \div 2 - 1$

19. $3^2 \cdot 2^3 \cdot (1 - 1)^3$
Chapter 1

Evaluate.

20. \(15B + 18\) for \(B = 3\)

21. \(8y - 9\) when \(y = 7\)

Solve.

22. \(x - 15 = 27\)

23. \(3A = 30\)

24. \(13a = 40 + 5a\)

25. \(4k - 18 = 14\)

26. \(3B + 5 + 4B = 47\)

27. \(11y + 14 = 40 + 18\)

28. \(16V = 28 - 12V\)

29. \(7m = 13 + 6m\)

30. Is \(Q = 12\) the solution to \(5Q - 18 = 42\)?
Answer Key for Chapter 1 Review.

1. variable is x; coefficient is 3; constant is 14 1
2. 4 terms; terms are 3A, 4B, 5C, and 6 1
3. 28Z demonstrates multiplication; factors are 28 and Z 1
4. The first four whole numbers are 0, 1, 2, and 3. 1
5. 3A + 4B + 5C + 6 is an expression; it contains no equal sign 1
6. Commutative (note only order is switched) 2
7. yes; for example (2 + 9) + 1 = 2 + (9 + 1) because they both result in 12 2
8. answers will vary; for example 8(3R - 2S + T) 24R - 16S + 8T 3
9. no; 2³ = 2(2)(2) = 8 , 3² = 3(3) = 9 5
10. 7A + 9B + 5 2
11. x 2
12. a. 8X - 4Y + 36Z - 16 3
    b. 2y + 8z + 7 3
13. 26M + 13N + 40 4
14. 125 5
15. 27 5
16. 31 5
17. 9
18. 10
19. 0
20. 63
21. 47
22. \(x = 42\)
23. \(A = 10\)
24. \(a = 5\)
25. \(k = 8\)
26. \(B = 6\)
27. \(y = 4\)
28. \(V = 1\)
29. \(m = 13\)
30. yes; since \(5(12) - 18 = 42\)
Chapter 1 Test

1. For $2x + 17$, name
   a. the variable
   b. the coefficient
   c. the constant

2. How many terms are there in the expression $7x + 2y + 3$?

3. Give an example of
   a. an expression
   b. an equation

4. List the factors in $5x$.

5. Does the following demonstrate the associative or commutative property?
   $7x + 2 = 2 + 7x$

Simplify.

6. a. $z \cdot 6$
   b. $3(4)(y)$

7. $3x + 4 + 5x + 9$

8. $4 + 2y + 6 + y$

9. $3(2y + 7)$

10. $4(5x - 3)$

11. $2(3x + 4y - 5)$

12. $\frac{304 - 15}{5}$

13. $3(2x + 1) + 5(3 + 2x)$

14. $10 + 4(x + 3) + 9(x + 7)$

15. $3^3$

16. $5^2$

17. $17 - 10 + 4$

18. $12 \div 3 \cdot 5 - 2^2$

19. $11 + (8 - 5)^3$

20. $\left(\frac{12}{2}\right)^2 - 3(5 - 1)$
Evaluate.

21. a. \(6y - 9\) when \(y = 12\)
   b. \(4x + 3\) for \(x = 7\)

Solve.

22. \(x + 7 = 13\)

23. \(y - 5 = 15\)

24. \(4a = 24\)

25. \(9m + 3m = 16 + 4m\)

26. \(3k = 18 - 3k\)

27. \(19 + 3x + 2 = 33\)

28. \(4y - 5 = 15\)
Chapter 1 Test

Answer Key for Chapter 1 Test.

1. a. $x$  
   b. 2  
   c. 17  

2. 3  

3. a. answers will vary; $x + 5$ is an example  
   b. answers will vary; $2x + 3 = 7$ is an example  

4. 5 and $x$  

5. commutative  

6. a. $6z$  
   b. $12y$  

7. $8x + 13$  

8. $3y + 10$  

9. $6y + 21$  

10. $20x - 12$  

11. $6x + 8y - 10$  

12. $6A - 3$  

13. $16x + 18$  

14. $13x + 85$  

15. 27  

16. 25  

17. 11  

18. 16  

19. 38  

20. 24  

21. a. 63  
   b. 31  

22. $x = 6$  

23. $y = 20$  

24. $a = 6$  

25. $m = 2$  

26. $k = 3$  

27. $x = 4$  

28. $y = 5$
Chapter 2a: Integers and Combining

In Chapter 1, you learned the core concepts of how to simplify and how to solve with whole numbers. You learned about the vocabulary of algebra, how to combine like terms, how to distribute, how to distribute and combine like terms, and how to solve equations. In Chapter 2, you will extend these concepts.

In Chapter 2, you will be studying how algebra is used with integers. **Integers** are the whole numbers and their opposites. Within this chapter, you will learn about integers and the meaning of absolute value. Then you will learn how to combine integers. After that, you will expand on the concepts of how to simplify and how to solve using integers. Combining like terms with integers and solving equations with integers will give you another opportunity to truly learn the basic concepts needed to be successful in algebra.

One of the main ideas in this chapter will be to learn a new way of thinking about addition and subtraction symbols. As you continue to study algebra, you will find that addition and subtraction symbols are **combining** symbols.

This chapter on integers is extremely important to your long-term success in algebra. When you learn to combine integers, you need to learn thoroughly so that you become fast and accurate with any problem you are given.
Section 1: Integers, Absolute Value, and Comparison

**Integers**
- What is the opposite of winning $100? Losing $100.
- What is the opposite of losing 5 pounds? Gaining 5 pounds.
- What is the opposite of positive 8? Negative 8.
- What is the opposite of negative 12? Positive 12.

When you want to talk about the concept of “opposites” in algebra, you need a way to symbolize that. When you see the words “negative 8,” you know the symbol would be $-8$.

The set of whole numbers and their opposites is called the set of **integers**. **Integers** include {..., $-3$, $-2$, $-1$, 0, 1, 2, 3,...}. If you move from left to right in this set, the numbers are getting larger.

<table>
<thead>
<tr>
<th>When you see this:</th>
<th>Write this:</th>
</tr>
</thead>
<tbody>
<tr>
<td>a. Opposite of 6</td>
<td>$-6$</td>
</tr>
<tr>
<td>b. Opposite of negative 12</td>
<td>$12$</td>
</tr>
<tr>
<td>c. Opposite of +15</td>
<td>$-15$</td>
</tr>
<tr>
<td>d. Opposite of $-25$</td>
<td>$25$</td>
</tr>
</tbody>
</table>

In English, you would see the words “Opposite of 5 is negative 5.” In math symbols, this would be seen as “$-5 = -5$.” In English, you would see the words “Opposite of negative 11 is 11.” In math symbols, this would be seen as “$-(-11) = 11$."

<table>
<thead>
<tr>
<th>When you see this:</th>
<th>Write this:</th>
</tr>
</thead>
<tbody>
<tr>
<td>e. $-(23)$</td>
<td>$-23$</td>
</tr>
<tr>
<td>f. $-(-15)$</td>
<td>$15$</td>
</tr>
</tbody>
</table>

By removing the parentheses on examples e and f, you have simplified the expression.
**Absolute Value**

On a number line, what is the distance from the number 5 to zero? 5 units.

On a number line, what is the distance from the number -3 to zero? 3 units.

**Absolute Value** is the distance from the number to zero (Note: the result is never negative). Absolute value is symbolized with straight vertical lines:

- g. \(|7|\) means the absolute value (the distance) of 7 which is 7 units.
- h. \(|-44|\) means the absolute value (the distance) of -44 which is 44 units.

**Comparison**

To compare 4 to 10, you would say “4 is less than 10.” In math symbols, you would see “4 < 10.” To compare 11 to 5, you would say “11 is greater than 5.” In math symbols, you would see “11 > 5.” Observe the examples.

<table>
<thead>
<tr>
<th>When you see this:</th>
<th>Write this:</th>
</tr>
</thead>
<tbody>
<tr>
<td>i. 5 is greater than -2</td>
<td>5 &gt; -2</td>
</tr>
<tr>
<td>j. -9 is greater than -13</td>
<td>-9 &gt; -13</td>
</tr>
<tr>
<td>k. -3 is less than 6</td>
<td>-3 &lt; 6</td>
</tr>
<tr>
<td>l. -12 is less than 0</td>
<td>-12 &lt; 0</td>
</tr>
<tr>
<td>m. -20 is less than -19</td>
<td>-20 &lt; -19</td>
</tr>
</tbody>
</table>
Chapter 2a Section 1  Integers, absolute value, comparison

Homework for Chapter 2a Section 1

Write in symbols and simplify.

1. Opposite of 18  
2. Opposite of −9

Simplify.

3. −(27)  
4. −(−32)  
5. −(−6)

6. −(19)  
7. |29|  
8. |−52|

9. a. Is 24 an integer?  
b. Is 24 a whole number?

10. a. Is −15 an integer?  
b. Is −15 a whole number?

Compare, using the appropriate symbol ( <  or > ).

11. −8  5  
12. −7  −1

13. 0  −4  
14. −3  −5

Simplify.

15. 4z + 19 + 8z + 15  
16. 24 + 11m + m + 5

17. 3(a + 8)  
18. 4(2y − 7)

19. 2(3x + 2) + 5(x + 3)  
20. 13y + 2(8 + 9y)

21. 2(4k + 2) + (7k + 11)  
22. (7B + 3) + (B + 6)

23. 73 − (9 − 7)^4  
24. 51 + \left(\frac{15}{3}\right)(7 + 3 · 2)

25. Is 9z + 18 = 27 an expression or an equation? Why?

26. In 49a + 33, what is the constant?

Solve.

27. m + 12 = 35  
28. y − 43 = 18

29. 6a + 2a = 136  
30. 3z + 21 = 96
Chapter 2a Section 1
Integers, absolute value, comparison

Answer Key for Chapter 2a Section 1.

1. \(-(18)\) is \(-18\)  
2. \(-(-9)\) is 9  
3. \(-27\)
4. 32  
5. 6  
6. \(-19\)
7. 29  
8. 52  

9a. 24 is an integer  
9b. 24 is a whole number

10a. \(-15\) is an integer  
10b. \(-15\) is not a whole number

11. <  
12. <
13. >  
14. >
15. \(12z + 34\)  
16. \(12m + 29\)  
17. \(3a + 24\)
18. \(8y - 28\)  
19. \(11x + 19\)  
20. \(31y + 16\)
21. \(15k + 15\)  
22. \(8B + 9\)
23. 57  
24. 116
25. \(9z + 18 = 27\) is an equation; it has an equal sign

26. 33 is the constant

27. \(m = 23\)  
28. \(y = 61\)  
29. \(a = 17\)
30. \(z = 25\)
Section 2: Combine Integers

When you begin to work with positive and negative number manipulation, it is sometimes easier to connect the problem with money. You can connect positive numbers with winning money; you can connect negative numbers with losing money. Observe the following examples.

Expression: Think:

a. 39 + 28
   Win $39 and win $28. The result is winning $67.

b. -30 + (-21)
   Lose $30 and lose $21. The result is losing $51.

c. 15 + (-20)
   Win $15 and lose $20. The result is losing $5.

d. 40 + (-35)
   Win $40 and lose $35. The result is winning $5.

e. -50 + 16
   Lose $50 and win $16. The result is losing $34.

f. -70 + 95
   Lose $70 and win $95. The result is winning $25.

g. -18 + 18
   Lose $18 and win $18. The result is zero.
   Notice that opposites combine to zero.

You can actually think of any gain/loss situation to give you an idea of what to do with your numbers (for example, gaining or losing weight, gaining or losing yards in a football game, gaining or losing altitude). If visualizing a situation helps you, then visualize!

Notice that the addition symbol is now a combining symbol. So what are the rules that guide how to combine positive and negative numbers? To combine integers, you find a result by the following:

- If the signs of the numbers are the same, find the sum and keep the sign.
- If the signs of the numbers are different, find the difference and keep the sign of the “bigger” number. “Bigger” number technically means the number with the larger absolute value.
You can also take certain word problems and turn them into math problems by reversing the above thinking.

h. Lose $18 and lose $42. Think: Lose $18 becomes neg 18; lose $42 becomes neg 42
   \[-18 + (-42)\]  Think: same signs means find sum and keep the sign
   \[-60\]

i. Win $30 and lose $17. Think: Win $30 becomes pos 30; lose $17 becomes neg 17
   \[30 + (-17)\]  Think: different signs means find difference and keep “bigger” sign
   \[13\]

When you have an expression with integers and addition symbols (combining symbols), you want to simplify by combining integers; to combine integers, you need to know whether the signs of the numbers are the same or different. If the signs of the numbers are the same, you find the sum and keep the common sign. If the signs of the numbers are different, you find the difference of the numbers and keep the sign of the “bigger” number.

Expression: Think:

j. \[-14 + (-88)\]  same signs means find sum and keep sign
   \[-102\]

k. \[-25 + 79\]  different signs means find difference and keep “bigger” sign
   \[54\]

l. \[39 + (-98)\]  different signs means find difference and keep “bigger” sign
   \[-59\]

m. \[-108 + (-47)\]  same signs means find sum and keep sign
   \[-155\]

n. \[49 + (-49)\]  different signs means find difference and keep “bigger” sign
   \[0\]
You may also find that you need to combine a “string” of integers. You can proceed from left to right. However, you can also use the commutative and associative properties to group together all the positive integers and to group together all the negative integers. Find the result for each. Then combine further to find the ending result. Observe.

\( o. \quad -28 + (-35) + 17 + (-13) + 18 + 26 \)

<table>
<thead>
<tr>
<th>negatives</th>
<th>positives</th>
</tr>
</thead>
<tbody>
<tr>
<td>-28</td>
<td>+ 17</td>
</tr>
<tr>
<td>-35</td>
<td>+ 18</td>
</tr>
<tr>
<td>-13</td>
<td>+ 26</td>
</tr>
<tr>
<td>-76</td>
<td>+ 61</td>
</tr>
</tbody>
</table>

-76 + 61  different signs means find difference; keep “bigger” sign
-15

OR

\( -28 + (-35) + 17 + (-13) + 18 + 26 \)

-63 + 17 + (-13) + 18 + 26 combine left to right
-46 + (-13) + 18 + 26 combine left to right
-59 + 18 + 26 combine left to right
-41 + 26 combine left to right
-15 combine left to right

\( p. \quad 52 + (-12) + 82 + (-105) + (-3) + 294 \)

<table>
<thead>
<tr>
<th>negatives</th>
<th>positives</th>
</tr>
</thead>
<tbody>
<tr>
<td>-12</td>
<td>+ 52</td>
</tr>
<tr>
<td>-105</td>
<td>+ 82</td>
</tr>
<tr>
<td>-3</td>
<td>+ 294</td>
</tr>
<tr>
<td>-120</td>
<td>+ 428</td>
</tr>
</tbody>
</table>

-120 + 428 different signs means find difference; keep “bigger” sign
308

You need to become fast and accurate with combining integers. Remember, if the signs of the numbers are the same, find the sum and keep the sign; if the signs of the numbers are different, find the difference and keep the “bigger” sign. The instructions will say “Simplify.” You will be expected to know that you combine according to what you have learned above.
Homework for Chapter 2a Section 2

Simplify.

1. $29 + 82$
2. $-34 + (-43)$
3. $-17 + (-9)$
4. $12 + (-12)$
5. $16 + (-20)$
6. $19 + (-53)$
7. $24 + (-91)$
8. $39 + (-100)$
9. $35 + (-24)$
10. $14 + (-11)$
11. $58 + (-22)$
12. $105 + (-74)$
13. $-41 + 57$
14. $-23 + 30$
15. $-16 + 42$
16. $-35 + 48$
17. $-47 + 23$
18. $-99 + 25$
19. $-34 + 32$
20. $-12 + 7$
21. $29 + (-16)$
22. $-30 + (-49)$
23. $24 + 18 + (-11)$
24. $-41 + 58 + (-16)$
25. $19 + (-35) + 61 + (-39) + (-54) + 12$
26. $-44 + 31 + 72 + (-57) + 19 + (-28)$

Simplify.

27. $24 + 19k + 22k + 8$
28. $9x + 31 + x$
29. $7 + 8(3z + 12)$
30. $2(5k + 9) + 3(4k + 6)$
31. $2m + 6 + (4 + 8m) + 3(7 + m)$
32. $\frac{42a + 18}{6}$

Solve.

33. $13y = 39$
34. $5a = 7 - 2a$
35. $c - 19 = 0$
36. $2d - 20 + 3d = 35$
**Answer Key for Chapter 2a Section 2.**

1. 111  
2. $-77$  
3. $-26$  
4. 0  
5. $-4$  
6. $-34$  
7. $-67$  
8. $-61$  
9. 11  
10. 3  
11. 36  
12. 31  
13. 16  
14. 7  
15. 26  
16. 13  
17. $-24$  
18. $-74$  
19. $-2$  
20. $-5$  
21. 13  
22. $-79$  
23. 31  
24. 1  
25. $-36$  
26. $-7$  
27. $41k + 32$  
28. $10x + 31$  
29. $24z + 103$  
30. $22k + 36$  
31. $13m + 31$  
32. $7a + 3$  
33. $y = 3$  
34. $a = 1$  
35. $c = 19$  
36. $d = 11$
Section 3: Combine Integers; Evaluate

Subtraction of integers is a little more detailed. Observe.

\[ a. \quad 8 - 6 \quad b. \quad 8 + (-6) \]

Notice both expressions simplified to 2; \( 8 - 6 \) and \( 8 + (-6) \) are equivalent expressions.

One way to approach subtraction of integers is to say “Subtraction means add the opposite.” Follow the examples.

\[ c. \quad 9 - 12 \quad d. \quad -8 - 14 \]
\[ 9 + (-12) \quad -8 + (-14) \]
\[ -3 \quad -22 \]

\[ e. \quad -2 - (-5) \quad f. \quad -24 - (-20) \]
\[ -2 + 5 \quad -24 + 20 \]
\[ 3 \quad -4 \]

\[ g. \quad -8 - (-8) \quad h. \quad 8 - (-8) \]
\[ -8 + 8 \quad 8 + 8 \]
\[ 0 \quad 16 \]

As you become more comfortable with subtraction of integers, you will find that you begin to think like this. When you see a single subtract symbol, link that symbol to the number to its right and think of it as a negative number. When you see subtract a negative, immediately change that to a positive. Therefore, a single subtract = add a negative number. However, subtract a negative = add a positive number. Then use the combining rules for combining integers. Therefore, the rules that govern both addition and subtraction symbols are exactly the same:

**Combining Rules for Addition/Subtraction Symbols**

Numbers have same sign means find the sum and keep the sign
Numbers have different signs means find the difference and keep the sign of the “bigger” number

The following examples will help clarify.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Think</th>
</tr>
</thead>
<tbody>
<tr>
<td>i. ( 5 - 20 )</td>
<td>pos 5 and neg 20</td>
</tr>
<tr>
<td>( -15 )</td>
<td>different signs means difference, keep “bigger” sign</td>
</tr>
<tr>
<td>j. ( -11 - 8 )</td>
<td>neg 11 and neg 8</td>
</tr>
<tr>
<td>( -19 )</td>
<td>same signs means find sum, keep sign</td>
</tr>
</tbody>
</table>
Chapter 2a Section 3  

Combine integers: evaluate

k. \(-6 - (-2)\)
   \(-6 + 2\)
   neg 6 and pos 2 (because of subtract neg)
   different signs means find difference, keep “bigger” sign
   \(-4\)

l. \(-14 - (-20)\)
   \(-14 + 20\)
   neg 14 and pos 20 (because of subtract neg)
   different signs means find difference, keep “bigger” sign
   \(6\)

m. \(25 - (-25)\)
   \(25 + 25\)
   pos 25 and pos 25 (because of subtract neg)
   same signs means find sum, keep sign
   \(50\)

n. \(-18 - (-18)\)
   \(-18 + 18\)
   neg 18 and pos 18 (because of subtract neg)
   different signs means find difference, keep “bigger” sign
   \(0\) opposites combine to zero

How do you approach a “string” of combining symbols (addition or subtraction symbols)? There are several approaches.

One approach is to change all subtraction to add the opposite.

o. \(29 + (-32) - 24 + 58 - 90 - (-14) + (-12) - 18 + 7 - (-25)\)
   \(29 + (-32) + (-24) + 58 + (-90) + 14 + (-12) + (-18) + 7 + 25\)

<table>
<thead>
<tr>
<th>negatives</th>
<th>positives</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-32)</td>
<td>+29</td>
</tr>
<tr>
<td>(-24)</td>
<td>+58</td>
</tr>
<tr>
<td>(-90)</td>
<td>+14</td>
</tr>
<tr>
<td>(-12)</td>
<td>+7</td>
</tr>
<tr>
<td>(-18)</td>
<td>+25</td>
</tr>
<tr>
<td>(-176)</td>
<td>+133</td>
</tr>
</tbody>
</table>

\(-176 + 133\) different signs means find difference, keep “bigger” sign
\(-43\)
A second approach is to immediately change all “double” signs to a single sign. Observe.

\[ p \]
\[ 29 + (-32) - 24 + 58 - 90 - (-14) + (-12) - 18 + 7 - (-25) \]
\[ 29 - 32 - 24 + 58 - 90 + 14 - 12 - 18 + 7 + 25 \]

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>-32</td>
<td>+ 29</td>
</tr>
<tr>
<td>-24</td>
<td>+ 58</td>
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<td>+ 14</td>
</tr>
<tr>
<td>-12</td>
<td>+ 7</td>
</tr>
<tr>
<td>-18</td>
<td>+ 25</td>
</tr>
<tr>
<td>-176</td>
<td>+ 133</td>
</tr>
</tbody>
</table>

\[ -176 + 133 \]
\[ \text{different signs means find the difference, keep “bigger” sign} \]
\[ -43 \]

A third approach is to change all subtract a negative to add a positive. All other signs stay the same, and you think of a single subtraction symbol as a negative number. Observe.

\[ q \]
\[ 29 + (-32) - 24 + 58 - 90 - (-14) + (-12) - 18 + 7 - (-25) \]
\[ 29 + (-32) - 24 + 58 - 90 + 14 + (-12) - 18 + 7 + 25 \]

<table>
<thead>
<tr>
<th>negatives</th>
<th>positives</th>
</tr>
</thead>
<tbody>
<tr>
<td>-32</td>
<td>+ 29</td>
</tr>
<tr>
<td>-24</td>
<td>+ 58</td>
</tr>
<tr>
<td>-90</td>
<td>+ 14</td>
</tr>
<tr>
<td>-12</td>
<td>+ 7</td>
</tr>
<tr>
<td>-18</td>
<td>+ 25</td>
</tr>
<tr>
<td>-176</td>
<td>+ 133</td>
</tr>
</tbody>
</table>

\[ -176 + 133 \]
\[ \text{different signs means find the difference, keep “bigger” sign} \]
\[ -43 \]

What you have observed with these examples is that addition and subtraction symbols are just combining symbols. All three approaches give you the same result. You need to decide on the approach that makes the most sense to you. Then practice until you are comfortable with combining integers.

The instructions would again say “Simplify.” You would be expected to know how to proceed.
Evaluate

In Chapter 1, you learned to evaluate expressions by substituting a number for a variable and simplifying by order of operations. You can also evaluate expressions using integers. Observe.

r. \( m + 9 \) when \( m = -14 \)
   \[
   (-14) + 9 \\
   -14 + 9 \\
   -5
   \]

s. \( y + 17 \) for \( y = -11 \)
   \[
   (-11) + 17 \\
   -11 + 17 \\
   6
   \]

t. \( k - 12 \) for \( k = -19 \)
   \[
   (-19) - 12 \\
   -19 - 12 \\
   -31
   \]

u. \( -z - 3 \) when \( z = -14 \)
   \[
   -(-14) - 3 \\
   14 - 3 \\
   11
   \]

v. \( -a + 12 \) when \( a = 1 \)
   \[
   -(1) + 12 \\
   -1 + 12 \\
   11
   \]
Homework for Chapter 2a Section 3

Simplify.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>28 - 34</td>
</tr>
<tr>
<td>3.</td>
<td>32 - 40</td>
</tr>
<tr>
<td>5.</td>
<td>24 - 21</td>
</tr>
<tr>
<td>7.</td>
<td>-18 - 22</td>
</tr>
<tr>
<td>9.</td>
<td>-18 - 46</td>
</tr>
<tr>
<td>13.</td>
<td>-24 - (-38)</td>
</tr>
<tr>
<td>15.</td>
<td>-77 - (-90)</td>
</tr>
<tr>
<td>17.</td>
<td>46 - (-98)</td>
</tr>
<tr>
<td>19.</td>
<td>-21 - (-21)</td>
</tr>
<tr>
<td>21.</td>
<td>29 - 42</td>
</tr>
<tr>
<td>23.</td>
<td>20 - 35 + (-18) - (-4)</td>
</tr>
</tbody>
</table>

Evaluate.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>25.</td>
<td>( z + 6 ) for ( z = -14 )</td>
</tr>
<tr>
<td>27.</td>
<td>( y + 39 ) when ( y = -20 )</td>
</tr>
<tr>
<td>29.</td>
<td>(-x + 5) when ( x = 3)</td>
</tr>
</tbody>
</table>
Simplify.

31. \(\frac{18}{3} + 6(3 + 2)^2\)
32. \(24 \div 3 + 8 \cdot 2 - 2^3\)
33. \(4y + 12 + 3y + 18\)
34. \(5m + 13 + m + 8 + 2m\)
35. \(6y + 3(2 + 7y) + 24\)

Solve.

36. \(z + 35 = 81\)
37. \(y - 13 = 49\)
38. \(4y = 30 - 2y\)
39. \(7m + 21 = 49\)
40. \(x + 2x + 18 = 33\)
41. \(2k + 17 + 5k + 4 = 77\)

42. In the expression \(5k + 19\), state the coefficient.
43. In the expression \(14 + 19x\), what are the terms; what are the factors?
44. Is \(\{..., -3, -2, -1, 0, 1, 2, 3, ...\}\) the set of whole numbers or the set of integers?
Additional Practice with Combining Integers.

Simplify.

45. $25 + (-10)$  
46. $38 + (-40)$
47. $-39 + 17$  
48. $-45 + 53$
49. $-16 + (-14)$  
50. $-29 + (-28)$
51. $19 - 20$  
52. $44 - 40$
53. $-12 - 7$  
54. $-1 - 35$
55. $-16 - (-2)$  
56. $-52 - (-60)$
57. $5 - (-5)$  
58. $-17 - (-17)$
59. $-16 + 14$  
60. $-50 - 12$
61. $18 - 30$  
62. $36 + (-20)$
63. $-37 + (-40)$  
64. $-80 - (-70)$
65. $-75 + 10$  
66. $15 - (-16)$
67. $18 + (-35) + 29 + (-40) + 7 + (-12)$
68. $-28 - 31 - (-17) + 20 + (-50) + (-19) - (-8)$
Chapter 2a Section 3  

Combine integers: evaluate

Answer Key for Chapter 2a Section 3.

1. −6  
2. −4  
3. −8  
4. 6  
5. 3  
6. 17  
7. −40  
8. −49  
9. −64  
10. −8  
11. −17  
12. −35  
13. 14  
14. 24  
15. 13  
16. 40  
17. 144  
18. 39  
19. 0  
20. −46  
21. −13  
22. 8  
23. −29  
24. −74  
25. −8  
26. −29  
27. 19  
28. 0  
29. 2  
30. −1  
31. 156  
32. 16  
33. 7y + 30  
34. 8m + 21  
35. 27y + 30  
36. z = 46  
37. y = 62  
38. y = 5  
39. m = 4  
40. x = 5  
41. k = 8  
42. coefficient is 5  
43. The terms are 14 and 19x (separated by addition symbol); the factors are 19 and x (they are being multiplied)  
44. set of integers  
45. 15  
46. −2  
47. −22  
48. 8  
49. −30  
50. −57  
51. −1  
52. 4  
53. −19  
54. −36  
55. −14  
56. 8  
57. 10  
58. 0  
59. −2  
60. −62  
61. −12
<table>
<thead>
<tr>
<th>Chapter 2a Section 3</th>
<th>Combine integers: evaluate</th>
</tr>
</thead>
<tbody>
<tr>
<td>62. 16</td>
<td>63. -77</td>
</tr>
<tr>
<td>65. -65</td>
<td>66. 31</td>
</tr>
<tr>
<td>68. -83</td>
<td>64. -10</td>
</tr>
<tr>
<td></td>
<td>67. -33</td>
</tr>
</tbody>
</table>
Section 4: Combine Like Terms Using Integers

In this section, you will be combining like terms using integers. Before you begin, review the vocabulary in example a and then notice how that vocabulary applies to example b.

a. In the expression $2z + 11$, one term is $2z$
   another term is $11$
   the variable is $z$
   the coefficient is $2$
   the constant is $11$

b. In the expression $-4m - 23$, one term is $-4m$
   another term is $-23$
   the variable is $m$
   the coefficient is $-4$
   the constant is $-23$

Note: on example b, the expression $-4m - 23$ means $-4m + (-23)$.

Example c reviews combining like terms with whole numbers. Notice how the same concepts apply to example d, combining like terms with integers. In example d, you will need to use the rules for combining positive and negative numbers. Then the other examples continue to combine like terms with integers. Therefore, combining like terms with integers means to combine those terms that are the same and use the rules for signs (same signs means find the sum and keep sign while different signs means find the difference and keep “bigger” sign).

c. Simplify:
   $4y + 18 + 9y + 11 + 17 + y$ original problem
   $4y + 18 + 9y + 11 + 17 + 1y$ show “understood” 1
   $4y + 9y + 1y + 18 + 11 + 17$ rearranged
   $14y + 46$ combine like terms

d. $8a + 12 - 14a + 20$ original problem
   $8a - 14a + 12 + 20$ rearranged
   $-6a + 32$ combine like terms

Notice that when you rearranged the terms, the subtraction symbol was considered a negative and was linked to the $14a$; therefore, the $14a$ was considered $-14a$. Then since $8a$ was positive and $14a$ was negative, you found the difference and kept the “bigger” sign.

On these examples, see if you can follow the thinking to simplify.

e. $-24k + 8 - 17k - 19$
   $-24k - 17k + 8 - 19$
   $-41k - 11$
Chapter 2a Section 4
Combine like terms using integers

f. \( x + 19 + x - 10 + 3 \)
   \( 1x + 19 + 1x - 10 + 3 \)
   \( 1x + 1x + 19 - 10 + 3 \)
   \( 2x + 12 \)

\[ \begin{align*}
g. \quad 26 - 13x - (-5) + (-14) + 19x - 27x - 15 + x \\
&= 26 - 13x + 5 - 14 + 19x - 27x - 15 + 1x \\
&= -13x + 19x - 27x + 1x + 26 + 5 - 14 - 15 \\
&= -20x + 2
\end{align*} \]
Homework for Chapter 2a Section 4

1. In the expression $19x - 25$,
   a. what is the coefficient?
   b. what is the constant?
   c. How many terms are in this expression?

2. In the expression $-2m + 11$,
   a. what is the coefficient?
   b. what is the constant?
   c. What is the first term? What is the second term?

3. In the expression $39 - 17d$,
   a. what is the coefficient?
   b. what is the constant?

Simplify.

4. $12m + 18 - 14m + 2$
5. $6y - 12 + 7y - 15$

6. $-9c - 28 + 2c - 15$
7. $14d + 20 - 19d - 7$

8. $-5k - (-14) + 4k - 6$
9. $-19a - 16 - 4a - 12$

10. $x + 19 + x - 6 + 5$
11. $-(-7z) + 6 - 5z + 1$

12. $28 + (-2w) - 24 + 8w$
13. $-16 - 22a - 28 + a$

14. $31d - 25 + 3d + 19 - 6d - (-41) + 6 + (-14d)$

15. $| -8 |$
16. $7 + 3(8 + 4)$

17. $\frac{24}{8} + (1 + 4)^2$
18. $4m + 19 + 2(3m + 5)$

19. $4(3y + 8) + 5(y + 7) + 6(4y + 1)$
Chapter 2a Section 4  
Combine like terms using integers

Solve.

20. \( m + 24 = 40 \)  
21. \( y - 17 = 6 \)

22. \( 18 + 2z + 4z = 24 + 48 \)  
23. \( 7a = 27 - 2a \)

Evaluate.

24. \( 2k + 13 \) when \( k = 6 \)

25. When you re-write \( 3(2m + 8) \) as \( 6m + 24 \), what property have you used?

26. Describe your thinking to get the result for \( \frac{18m + 36}{9} \).
Answer Key for Chapter 2a Section 4.

1a. coefficient is 19
1b. constant is -25
1c. There are 2 terms

2a. coefficient is -2
2b. constant is 11
2c. The first term is -2m; the second term is 11

3a. coefficient is -17
3b. constant is 39

4. -2m + 20
5. 13y - 27
6. -7c - 43
7. -5d + 13
8. -k + 8
9. -23a - 28
10. 2x + 18
11. 2z + 7
12. 6w + 4
13. -21a - 44
14. 14d + 41
15. 8
16. 43
17. 28
18. 10m + 29
19. 41y + 73
20. m = 16
21. y = 23
22. z = 9
23. a = 3
24. 25
25. distributive property of multiplication over addition
26. Think: each term will be divided by 9 to get 2m + 4.
Section 5: Solve Equations Using Addition and Subtraction with Integers

In Chapter 1, you solved equations using whole numbers. In this section, you will learn to solve equations using integers. Before you begin, review examples a and b.

a. \[ m + 14 = 38 \]
\[
\begin{align*}
\text{Check:} \\
m + 14 &= 38 \\
24 + 14 &= 38 \\
m &= 24 \\
\text{m = 24 does check!}
\end{align*}
\]

b. \[ y - 31 = 57 \]
\[
\begin{align*}
\text{Check:} \\
y - 31 &= 57 \\
88 - 31 &= 57 \\
y &= 88 \\
y = 88 \text{ does check!}
\end{align*}
\]

Solving equations means to perform a series of manipulations so the variable is isolated. In general, you “undo” what is written in the equation so the variable is alone on one side of the equal sign. While you are manipulating the equation, what you do to one side of the equation you do to the other side of the equation, in order to keep the equation “balanced.”

To solve an equation with an ADDITION symbol, you would SUBTRACT. To solve an equation with a SUBTRACTION symbol, you would ADD.

When you solve equations with integers, you follow the same thinking as above. You still “undo” addition with subtraction; you still “undo” subtraction with addition. However, now you will incorporate the rules for combining integers. Observe.

c. \[ m + 28 = 17 \]
\[
\begin{align*}
\text{Check:} \\
m + 28 &= 17 \\
-11 + 28 &= 17 \\
m &= -11 \\
m = -11 \text{ does check!}
\end{align*}
\]

d. \[ y + 49 = -25 \]
\[
\begin{align*}
\text{Check:} \\
y + 49 &= -25 \\
-74 + 49 &= -25 \\
y &= -74 \\
y = -74 \text{ does check!}
\end{align*}
\]
Chapter 2a Section 5

Solve equations using add/subtract and integers

e. \[ a - 31 = -51 \]
\[ + 31 \]
\[ a = -20 \]

f. \[ k - 68 = -20 \]
\[ + 68 \]
\[ k = 48 \]

g. \[ x + 14 = -14 \]
\[ - 14 \]
\[ x = -28 \]

h. \[ z - 5 = -5 \]
\[ + 5 \]
\[ z = 0 \]

What you should notice is you use the same process to solve equations with both whole numbers and integers. You just need to remember the combining rules when you solve with integers!

NOTE: You can solve these equations in a horizontal, rather than vertical, fashion. Observe.

i. \[ z + 18 = 12 \]
\[ z + 18 - 18 = 12 - 18 \]
\[ z + 0 = -6 \]
\[ z = -6 \]

Check:
\[ z + 18 = 12 \]
\[ -6 + 18 = 12 \]
\[ 12 = 12 \]
\[ z = -6 \text{ is the solution.} \]

j. \[ m - 19 = -28 \]
\[ m - 19 + 19 = -28 + 19 \]
\[ m + 0 = -9 \]
\[ m = -9 \]

Check:
\[ m - 19 = -28 \]
\[ -9 - 19 = -28 \]
\[ -28 = -28 \]
\[ m = -9 \text{ is the solution.} \]
Homework for Chapter 2a Section 5

Solve.

1. \( a + 25 = 6 \)  
2. \( d + 19 = -7 \)
3. \( f + 28 = 28 \)  
4. \( m + 27 = -27 \)
5. \( w - 15 = 6 \)  
6. \( y - 38 = -12 \)
7. \( x - 44 = -10 \)  
8. \( z - 52 = -52 \)
9. \( q - 5 = 5 \)  
10. \( c - 18 = 0 \)
11. \( b + 14 = -29 \)  
12. \( e - 25 = -13 \)
13. \( v - 39 = -40 \)  
14. \( x + 29 = 45 \)

Simplify.

15. \( 8 + (-20) - 6 + 4 \)  
16. \( -22 + (-18) + 4 - (-11) \)
17. \( 3y + 16 - 7y + 9 \)  
18. \( -9m - 27 - m + 5 \)
19. \( 8(9 + 1) + 30 \div 2 \)  
20. \( 4(z + 8) + 2(5z + 1) \)
21. \( 6 + (7x + 9) \)  
22. \( (4 + 2y) + 3(5y + 6) \)

Evaluate.

23. \( 5m + 3 \) when \( m = 6 \)  
24. \( 7y - 11 \) for \( y = 5 \)
25. \( k + 9 \) for \( k = -16 \)  
26. \( z - 15 \) when \( z = -10 \)
27. In the expression \( 5m + 25 \), what are the terms; what are the factors? Explain the difference between terms and factors in this expression.
Chapter 2a Section 5

Solve equations using add/subtract and integers

Answer Key for Chapter 2a Section 5.

1. \( a = -19 \)  
2. \( d = -26 \)  
3. \( f = 0 \)

4. \( m = -54 \)  
5. \( w = 21 \)  
6. \( y = 26 \)

7. \( x = 34 \)  
8. \( z = 0 \)  
9. \( q = 10 \)

10. \( c = 18 \)  
11. \( b = -43 \)  
12. \( e = 12 \)

13. \( v = -1 \)  
14. \( x = 16 \)  
15. \(-14\)

16. \(-25\)  
17. \(-4y + 25\)  
18. \(-10m - 22\)

19. \(95\)  
20. \(14z + 34\)  
21. \(7x + 15\)

22. \(17y + 22\)  
23. \(33\)  
24. \(24\)

25. \(-7\)  
26. \(-25\)

27. The terms are 5m and 25; the factors are 5 and m. In this expression, the terms are separated by an addition symbol; the factors are being multiplied.
Chapter 2a Summary

Integers \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots \}

Absolute Value distance from zero on a number line
\[ |3| = 3 \quad |-8| = 8 \]

Add Integers like signs \rightarrow find the sum of the numbers; keep the sign
unlike signs \rightarrow find the difference of the numbers; keep sign of “larger” absolute value

\[
\begin{align*}
9 + 6 \text{ or } 6 + 9 & \rightarrow 15 \\
8 + (-14) \text{ or } -14 + 8 & \rightarrow -6 \\
-3 + (-16) \text{ or } -16 + (-3) & \rightarrow -19 \\
-9 + 12 \text{ or } 12 + (-9) & \rightarrow 3
\end{align*}
\]

Subtract Integers subtracting is the same as adding the opposite; then
like signs \rightarrow find the sum of the numbers; keep the sign
unlike signs \rightarrow find the difference of numbers; keep sign of “larger” absolute value

\[
\begin{align*}
12 - 8 & \rightarrow 4 \\
7 - 9 & \rightarrow -2 \\
-4 - 8 & \rightarrow -12 \\
1 - (-3) \text{ means } 1 + 3 & \rightarrow 4 \\
-5 - (-11) \text{ means } -5 + 11 & \rightarrow 6 \\
-8 - (-7) \text{ means } -8 + 7 & \rightarrow -1 \\
-3 - (-3) \text{ means } -3 + 3 & \rightarrow 0
\end{align*}
\]

Evaluate \[ x + 19 \text{ for } x = -17 \]
\[ (-17) + 19 \]
\[ 2 \]

Combine Like Terms \[-4w + 8 + 9w - 19 \rightarrow 5w - 11 \]
Solve

a. \[ x + 17 = 3 \]
\[ \quad - 17 - 17 \]
\[ x = -14 \]

b. \[ y + 6 = -9 \]
\[ \quad - 6 - 6 \]
\[ y = -15 \]

c. \[ m - 20 = -13 \]
\[ \quad + 20 + 20 \]
\[ m = 7 \]

d. \[ a - 35 = -12 \]
\[ \quad + 35 + 35 \]
\[ a = 23 \]

e. Recall, you should also check your solutions.
Chapter 2a Review

1. Write in symbols and simplify: opposite of $-8$

Simplify.

2. $-(18)$

3. $-(-21)$

4. $|14|$

5. $|-3|$

6. a. Is $-1$ an integer?
   b. Is $-1$ a whole number?

Compare, using the appropriate symbol ($<$ or $>$).

7. $-14 < -8$

8. $0 < -3$

9. $-10 < 2$

Simplify.

10. $29 + (-5)$

11. $17 + (-27)$

12. $(-18) + (-11)$

13. $(-32) + 15 + (-13) + 20 + (-10) + 20$

14. $14 - 32$

15. $-35 - 8$

16. $-14 - (-12)$

17. $-28 - (-22) + (-4) - (-13)$

18. $13 - 17 + (-2) - (-5) + 12 - (-3)$
Evaluate.

19. $y - 19$ for $y = -1$
20. $m + 16$ for $m = -6$

21. $x + 14$ when $x = -25$
22. $k - 20$ when $k = -20$

23. In the expression $-5x - 15$,
   a. what is the coefficient?
   b. what is the constant?
   c. what is the first term? what is the second term?
   d. what are the factors?

Simplify.

24. $2x - 17 - 5 - 12x$

25. $4a + (-3) - (2a) + (-3) - (2a) + 6$

Solve.

26. $v + 10 = -24$
27. $k - 18 = -3$

28. $w + 30 = 6$
29. $A + 8 = -8$

30. $Y - 10 = -10$
**Answer Key for Chapter 2a Review.**

1. \(-(-8)\) is 8
2. \(-18\)
3. 21
4. 14
5. 3
6. a. \(-1\) is an integer  
   b. \(-1\) is not a whole number
7. <
8. >
9. <
10. 24
11. \(-10\)
12. \(-29\)
13. 0
14. \(-18\)
15. \(-43\)
16. \(-2\)
17. 3
18. 14
<p>| | | |</p>
<table>
<thead>
<tr>
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<td>19.</td>
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<tr>
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<td>10</td>
<td>3</td>
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<td>3</td>
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<td>22.</td>
<td>−40</td>
<td>3</td>
</tr>
<tr>
<td>23. a.</td>
<td>coefficient is −5</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>b. constant is −15</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>c. first term is −5x; second term is −15</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>d. factors are −5 and x</td>
<td>4</td>
</tr>
<tr>
<td>24.</td>
<td>−10x − 22</td>
<td>4</td>
</tr>
<tr>
<td>25.</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>26.</td>
<td>v = −34</td>
<td>5</td>
</tr>
<tr>
<td>27.</td>
<td>k = 15</td>
<td>5</td>
</tr>
<tr>
<td>28.</td>
<td>w = −24</td>
<td>5</td>
</tr>
<tr>
<td>29.</td>
<td>A = −16</td>
<td>5</td>
</tr>
<tr>
<td>30.</td>
<td>Y = 0</td>
<td>5</td>
</tr>
</tbody>
</table>
Chapter 2a Test

Compare, using < or >.

1. \(-5\) \(-12\) 
2. \(0\) \(-2\)

3. Write in symbols and then simplify: \(\text{absolute value of } -3\)

Simplify.

4. \(|4|\) 
5. \(-(7)\)
6. \(-(-16)\) 
7. \(-4 + (-4)\)
8. \(13 + 5\) 
9. \(-5 + 7\)
10. \(-5 + (-8)\) 
11. \(-7 - 9\)
12. \(16 - 17\) 
13. \(8 + (-8)\)
14. \(-16 + 4 - 5\) 
15. \(14 + (-3) + 6\)

Evaluate.

16. \(x + 9\) when \(x = -5\) 
17. \(y - 7\) when \(y = -6\)

Simplify.

18. \(3x - 6 - 4x + 9\) 
19. \(-7y + 2(3y + 6)\)

Solve.

20. \(x + 7 = 5\) 
21. \(y - 4 = -10\)
22. \(a - 8 = -7\) 
23. \(b + 5 = 14\)
24. \(x - 12 = 6\) 
25. \(y + 7 = -21\)
Answer Key for Chapter 2a Test.

1. $-5 > -12$  
2. $0 > -2$  
3. $|3| = 3$  
4. $4$  
5. $-7$  
6. $16$  
7. $-8$  
8. $18$  
9. $2$  
10. $-13$  
11. $-16$  
12. $-1$  
13. $0$  
14. $-17$  
15. $17$  
16. $4$  
17. $-13$  
18. $-x + 3$  
19. $-y + 12$  
20. $x = -2$  
21. $y = -6$  
22. $a = 1$  
23. $b = 9$  
24. $x = 18$  
25. $y = -28$
Cumulative Review #1 Through Chapter 2a

1. Is $4x - 5 = 3x + 2$ an expression or an equation? Why?

2. Compare, using the appropriate symbol ($<$ or $>$): $-12$ $-$ $13$

3. Write in symbols and simplify: opposite of $-3$.

4. In the list of numbers $8$, $-3$, $0$, $-2$, $5$, which ones are
   a. whole numbers?
   b. integers?

5. Put the following in order, from smallest to largest:
   $0$, $-1$, $2$, $-3$, $-4$, $5$, $6$, $-7$

Simplify.

6. $3(A + 2B) + 2(A + 3B) + (2A + B)$

7. $(8 - 4)^2 + 4 \cdot 2$

8. $8 + 2 \cdot 4 + 6$

9. $10 + 2(3^3 - 4^2)$

10. $\left(5 \left( \frac{28}{4} \right) \right) + 6(4 - 1)^2$

11. $-(-3)$

12. $-| -2 |$

13. $-4 - 3 - 2$

14. $(-3) + 7$

15. $-8 - (-5)$

16. $14 + (-2) + (-3) + 8 + (-10)$
Simplify.

17. \(-12 + (-3) - (5) - (-7) + (-2)\)
18. \(1 + (-1) - (2) - (-2) + (-3) - (3) + 4 - (-4)\)
19. \(3x - (-2) + (2x) - 1\)
20. \(4 - 2p - (-3p) + 4 - 1 - p\)
21. \(4(3A - 7)\)
22. \(\frac{36k - 9}{9}\)
23. \(8 + 2k + 3(1 + 4k)\)
24. Evaluate \(9 - 2x\) when \(x = 3\).
25. Is \(B = 4\) the solution to \(5B - 8 = 28\)?

Solve.

26. \(5k = 16 - 3k\)
27. \(10z = 12 + 7z\)
28. \(2y + 13 = 57\)
29. \(5M + 24 + 3M = 32\)
30. \(x - 3 = -2\)
31. \(A + 10 = 4\)
32. \(x + 8 = -10\)
# Answer Key for Cumulative Review #1 through Chapter 2a.

<table>
<thead>
<tr>
<th>Question</th>
<th>Reference Chapter</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. equation; it contains an equal sign</td>
<td>1</td>
</tr>
<tr>
<td>2. &gt;</td>
<td>2a</td>
</tr>
<tr>
<td>3. $-(-3) = 3$</td>
<td>2a</td>
</tr>
<tr>
<td>4. a. 8, 0, 5</td>
<td>2a</td>
</tr>
<tr>
<td>b. all are integers</td>
<td>2a</td>
</tr>
<tr>
<td>5. $-7, -4, -3, -1, 0, 2, 5, 6$</td>
<td>2a</td>
</tr>
<tr>
<td>6. $7A + 13B$</td>
<td>1</td>
</tr>
<tr>
<td>7. 8</td>
<td>1</td>
</tr>
<tr>
<td>8. 22</td>
<td>1</td>
</tr>
<tr>
<td>9. 32</td>
<td>1</td>
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<tr>
<td>10. 89</td>
<td>1</td>
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<tr>
<td>11. 3</td>
<td>2a</td>
</tr>
<tr>
<td>12. $-2$</td>
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</tr>
<tr>
<td>13. $-9$</td>
<td>2a</td>
</tr>
<tr>
<td>14. 4</td>
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</tr>
<tr>
<td>15. $-3$</td>
<td>2a</td>
</tr>
<tr>
<td>16. 7</td>
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<tr>
<td>17. $-15$</td>
<td>2a</td>
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<tr>
<td>18.</td>
<td>$2$</td>
</tr>
<tr>
<td>19.</td>
<td>$5x + 1$</td>
</tr>
<tr>
<td>20.</td>
<td>$7$</td>
</tr>
<tr>
<td>21.</td>
<td>$12A - 28$</td>
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<tr>
<td>22.</td>
<td>$4k - 1$</td>
</tr>
<tr>
<td>23.</td>
<td>$14k + 11$</td>
</tr>
<tr>
<td>24.</td>
<td>$3$</td>
</tr>
<tr>
<td>25.</td>
<td>no</td>
</tr>
<tr>
<td>26.</td>
<td>$k = 2$</td>
</tr>
<tr>
<td>27.</td>
<td>$z = 4$</td>
</tr>
<tr>
<td>28.</td>
<td>$y = 22$</td>
</tr>
<tr>
<td>29.</td>
<td>$M = 1$</td>
</tr>
<tr>
<td>30.</td>
<td>$x = 1$</td>
</tr>
<tr>
<td>31.</td>
<td>$A = -6$</td>
</tr>
<tr>
<td>32.</td>
<td>$x = -18$</td>
</tr>
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</table>
Chapter 2b: Integers and Multiplying

You have completed Chapter 2a on combining integers. In that chapter, you learned that addition and subtraction symbols were combining symbols. Then you extended your knowledge by using combining integers with the previously learned ideas of combining like terms and solving equations.

In Chapter 2b, you will continue to learn about integers. You will multiply and divide integers. You will find that the sign of the product of integers depends upon the number of negatives being multiplied. If you have an odd number of negative numbers being multiplied, the product is negative. If you have an even number of negative numbers being multiplied, the product is positive.

Once you multiply (and divide) integers, then you will extend your knowledge by using multiplying integers with the previously learned ideas of order of operations, how to distribute, how to combine like terms, and how to solve equations. You will incorporate both combining integers and multiplying integers with these concepts.

What you should notice by the end of this chapter is that several core ideas are re-addressed: order of operations, combining like terms, how to distribute, how to distribute and combine like terms, and solving equations. In the first chapter, you used whole numbers with these concepts. In this chapter, you are using integers with these concepts.
Section 1: Multiply Integers

To multiply integers, it is helpful to recall a few ideas about whole numbers. Then you will relate these ideas to integer multiplication. Observe.

a. \[ 4 + 4 + 4 + 4 + 4 + 4 \]

\[ 24 \]

b. Another way to write \[ 4 + 4 + 4 + 4 + 4 + 4 \] is to write \[ 6(4) \]

\[ 6(4) \]

\[ 24 \]

c. \[ -4 + (-4) + (-4) + (-4) + (-4) + (-4) \]

\[ -24 \]

d. Another way to write \[ -4 + (-4) + (-4) + (-4) + (-4) + (-4) \] is to write \[ 6(-4) \]

\[ 6(-4) \]

\[ -24 \]

Now you can say that a positive number times a negative number results in a negative number. What would happen if you reversed the order? Observe.

e. \[ 6(4) \text{ or } 4(6) \]

multiplication is commutative (order of numbers can be reversed)

\[ 24 \text{ or } 24 \]

f. Therefore

\[ 6(-4) \text{ or } -4(6) \]

multiplication is commutative

\[ -24 \text{ or } -24 \]

Now you can say the following:

(positive)(negative) results in a negative

(negative)(positive) results in a negative

What would happen if you multiplied two negative numbers? One way to figure out what happens is to complete a pattern and then make an observation.

g. \[ -5(3) = -15 \]

\[ -5(2) = -10 \]

\[ -5(1) = -5 \]

\[ -5(0) = 0 \]

\[ -5(-1) = 5 \]

\[ -5(-2) = 10 \]

\[ -5(-3) = 15 \]

Notice that the product (the result of multiplication) was getting bigger; to keep this pattern going, you had to increase each result by five. Eventually, when you have
a negative times a negative, you can see the result is positive.

To **multiply integers**, use the following:
- positive(positive) results in a positive
- negative(negative) results in a positive
- positive(negative) results in a negative
- negative(positive) results in a negative.

To **multiply integers**, you could say
- the product of two numbers with like signs is positive
- the product of two numbers with different signs is negative.

**CAUTION**: Multiplication is not the same as combining! Observe.

h. \(-5(-3)\) means multiply
i. \(-5 - 3\) means combine

15
-8

j. Notice the operation in each of these examples. Which of these means multiplication?

\[-7 - 5\] \[-7(-5)\] \[-7 + (-5)\]

Answer: \[-7(-5)\] means multiplication.

What would happen if you multiplied a “string” of factors, some negative and some positive? Can you figure out a pattern that will allow you to quickly find the sign of the product? Observe.

k. \(2(3)(-4)\)
l. \(5(-2)(-4)\)

\[6(-4)\] \[-10(-4)\]

\[-24\] \[40\]

m. \(-2(-3)(-4)\)
n. \(-2(-3)(-4)(-5)\)

\[6(-4)\] \[6(-4)(-5)\]

\[-24\] \[-24(-5)\]

\[120\]

What did you notice? You should have noticed that the number of negatives being multiplied affected the sign of the result.

<table>
<thead>
<tr>
<th>When you multiply with:</th>
<th>The result is:</th>
</tr>
</thead>
<tbody>
<tr>
<td>one negative</td>
<td>negative</td>
</tr>
<tr>
<td>two negatives</td>
<td>positive</td>
</tr>
<tr>
<td>three negatives</td>
<td>negative</td>
</tr>
<tr>
<td>four negatives</td>
<td>positive</td>
</tr>
</tbody>
</table>

page 88
To summarize this pattern, when you have an even number of negative factors, the product is positive. When you have an odd number of negative factors, the product is negative.

Using this pattern allows you to find the sign of the result and to then multiply the numbers separately from the sign.

\[
\begin{align*}
o. & \quad 5(-2)(3)(4)(-6) \quad \text{multiply with 2 negatives so result is positive; multiply numbers (not signs) left to right} \\
& \quad 720 \\
p. & \quad -7(-3)(2)(-1)(8)(4) \quad \text{multiply with 3 negatives so result is negative; multiply numbers (not signs) left to right} \\
& \quad -1344
\end{align*}
\]

Finally, you want to extend multiplication to the related idea of exponents.

\[
\begin{align*}
q. & \quad 5^4 \quad \text{(5)(5)(5)(5)} \\
& \quad 625 \\
r. & \quad 6^3 \quad \text{(6)(6)(6)} \\
& \quad 216 \\
s. & \quad (-5)^4 \quad \text{(-5)(-5)(-5)(-5)} \\
& \quad 625 \\
t. & \quad (-6)^3 \quad \text{(-6)(-6)(-6)} \\
& \quad -216 \\
u. & \quad -5^4 \quad \text{-(5)(5)(5)(5)} \\
& \quad -(625) \\
v. & \quad -6^3 \quad \text{-(6)(6)(6)} \\
& \quad -(216)
\end{align*}
\]

Did you notice the number inside the parentheses is the base? Since an exponent indicates a repeated multiplication, you use the same ideas for multiplying. When you multiplied with four negatives on example s, the result was positive. When you multiplied with three negatives on example t, the result was negative. Did you notice on examples u and v that there were no parentheses in the problems? The exponent affects what is directly to the exponent’s left; without parentheses, the exponent does NOT affect the negative symbol. Therefore, the results are negative on examples u and v.

In summary, the sign of the product of integers depends on the number of negatives being multiplied.
NOTE: If you are wondering why a positive times a negative is a negative or why two negatives multiply to a positive, here is an analogy that may demonstrate.

Suppose a water tower is losing water at a rate of 3 gallons per hour. After 5 hours, how much water has been lost?

Think: 3 gallons lost an hour (5 hours) = 15 gallons lost
Symbols: \(-3 \text{ gallons per hour} \times 5 \text{ hours} = -15 \text{ gallons}\)

Suppose a water tower is losing water at a rate of 3 gallons per hour. How much more water was in the tower 5 hours ago?

Think: 3 gallons lost an hour (5 hours ago) = 15 gallons more
Symbols: \(-3 \text{ gallons per hour} \times (-5 \text{ hours}) = 15 \text{ gallons}\)

Suppose a person was dieting and was steadily losing 2 pounds per week. How many pounds were lost after 4 weeks of dieting?

Think: 2 pounds lost per week (4 weeks) = 8 pounds lost
Symbols: \(-2 \text{ pounds per week} \times 4 \text{ weeks} = -8 \text{ pounds}\)

Suppose a person was dieting and was steadily losing 2 pounds per week. How many more pounds did the person weigh 4 weeks ago?

Think: 2 pounds lost per week (4 weeks ago) = 8 pounds more
Symbols: \(-2 \text{ pounds per week} \times (-4 \text{ weeks}) = 8 \text{ pounds}\)
Chapter 2b Section 1
Multiply integers

Homework for Chapter 2b Section 1

Simplify.

1. $5(-6)$
2. $-3 \cdot 7$
3. $8 \cdot 4$
4. $(-9)(-2)$
5. $8(-2)(3)$
6. $-4(2)(-5)$
7. $5(4)(7)(-3)$
8. $-7(5)(-1)(-6)$
9. $3(-6)(-2)(-10)(-5)$
10. $-3(11)(-3)(0)(-2)$
11. $(-7)^2$
12. $(-4)^3$
13. $(-2)^4$
14. $(-3)^5$
15. $-3^4$
16. $-5^2$
17. $-6^2$
18. $-4^2$
19. $-8 - 14$
20. $19 - 30$
21. $-3(-4)$
22. $5(-6)$
23. $-25 + 38$
24. $7 + (-15)$
25. $7(-2)(-3)$
26. $-4(5)(2)$
27. $-35 - (-21)$
28. $6y - 12 - 2y - 8$
29. $8(2m + 1) + 3(m + 2)$
30. $6 + 4(10 - 7) + \left( \frac{12}{4} \right)^2$
31. $36 \div 2 \cdot 6 + 3(8 + 4^2)$
32. $\left( \frac{48}{8} \right)(7) + 3(10 - 4)^2 - 5^2$
Solve.

33. \( m + 30 = 5 \)  
34. \( 2y = 14 \)

35. \( 4w = 27 - 5w \)  
36. \( 20c = 60 + 5c \)

37. \( 5z + 25 = 35 \)  
38. \( 6H + 18 + 3H = 72 \)

39. \( a - 17 = -18 \)  
40. \( A + 28 = 16 \)

41. \( x + 44 = -21 \)  
42. \( 3k - 18 = 33 \)

43. If you multiplied a “string” of fourteen factors where seven factors were negative, would the result be positive or negative?

44. If you raised negative thirteen to the eighth power, would the result be positive or negative?
### Answer Key for Chapter 2b Section 1.

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
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<td>1</td>
<td>$-30$</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>18</td>
<td>5</td>
</tr>
<tr>
<td>7</td>
<td>$-420$</td>
<td>8</td>
</tr>
<tr>
<td>10</td>
<td>0</td>
<td>11</td>
</tr>
<tr>
<td>13</td>
<td>16</td>
<td>14</td>
</tr>
<tr>
<td>16</td>
<td>$-25$</td>
<td>17</td>
</tr>
<tr>
<td>19</td>
<td>$-22$</td>
<td>20</td>
</tr>
<tr>
<td>22</td>
<td>$-30$</td>
<td>23</td>
</tr>
<tr>
<td>25</td>
<td>42</td>
<td>26</td>
</tr>
<tr>
<td>28</td>
<td>$4y - 20$</td>
<td>29</td>
</tr>
<tr>
<td>31</td>
<td>180</td>
<td>32</td>
</tr>
<tr>
<td>34</td>
<td>$y = 7$</td>
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</tr>
<tr>
<td>37</td>
<td>$z = 2$</td>
<td>38</td>
</tr>
<tr>
<td>40</td>
<td>$A = -12$</td>
<td>41</td>
</tr>
<tr>
<td>43</td>
<td>negative</td>
<td>44</td>
</tr>
</tbody>
</table>
Section 2: Divide Integers

You learned in the previous section that the sign of the product of integers depended on the number of negatives being multiplied:
the product of an even number of negatives is positive
the product of an odd number of negatives is negative.
What is the sign of the quotient of integers?

First consider whole numbers; then adapt what you know to integers. Observe.

Multiplication: Two related divisions:

a. \(5(8) = 40\)
   \[\frac{40}{8} = 5\]
   \[\frac{40}{5} = 8\]

b. \(5(-8) = -40\)
   \[\frac{-40}{-8} = 5\]
   \[\frac{-40}{5} = -8\]

c. \((-8)(5) = -40\)
   \[\frac{-40}{5} = -8\]
   \[\frac{-40}{-8} = 5\]

When you have like signs (both positive or both negative), the quotient is a positive number. When you have different signs (one positive and one negative), the quotient is a negative number.

To divide integers, consider the sign:
the quotient of two numbers with like signs is positive
the quotient of two numbers with different signs is negative.

Most times in algebra, a division problem will be shown with a fraction bar line.

d. \(\frac{-8}{-2}\)
   \(\frac{8}{2}\)
   quotient of numbers with like signs

   4  4
   positive result

e. \(\frac{24}{-2}\)
   \(\frac{-24}{2}\)
   quotient of numbers with different signs

   -12 -12
   negative result
Homework for Chapter 2b Section 2

Simplify.

1. \(28 \div (-4)\)  
2. \(-48 \div 6\)

3. \(-50 \div (-5)\)  
4. \(39 \div 3\)

5. \(\frac{100}{-5}\)  
6. \(\frac{-75}{25}\)

7. \(\frac{-33}{-11}\)  
8. \(\frac{16}{-16}\)

9. \(2 - 10\)  
10. \(2(-10)\)

11. \(2 - (-10)\)  
12. \(-10 - (-2)\)

13. \(-10(-2)\)  
14. \(-10 \div (-2)\)

15. \(8(-3)(2)(4)\)  
16. \(-9(2)(-4)(5)(3)\)

17. \((-6)^3\)  
18. \(-7^2\)

19. \(-25z + 8 - 12z - 5\)  
20. \(2(3k + 6)\)

21. \(15 + 6(3m + 5)\)  
22. \(5(2a - 3)\)

23. \(12 + 8(10 - 4)\)  
24. \(30 \div 2 \cdot 5 + 2^3\)

25. \(28 - 5^2\)  
26. \(29 - \left(\frac{8}{2}\right)^2\)
Solve.

27. \(3m = 24\)  
28. \(2y + 9 = 11\)
29. \(5k - 2k = 39\)  
30. \(x + 3x - 16 = 24 + 40\)
31. \(6A = 40 - 2A\)  
32. \(9M = 24 + 3M\)
33. \(z + 17 = 5\)  
34. \(a - 6 = -6\)
35. \(2x - 14 = 22\)  
36. \(k + 13 = -6\)
37. \(y - 29 = -40\)  
38. \(K - 16 = -12\)

In the expression \(4q + 11\),

39. What is the variable?  
40. What is the coefficient?
41. What is the constant?  
42. What are the factors?

Additional Practice with Multiplying and Dividing Integers.

Simplify.

43. If you multiply with four negative numbers, what is the sign of the answer?  
44. If you multiply with eleven negative numbers, what is the sign of the answer?
45. \(8(-3)\)  
46. \(-9(-3)\)
47. \(-7(-5)\)  
48. \(-2(3)(5)(4)\)
49. \(-2(-3)(5)(4)\)  
50. \(-2(-3)(-5)(4)\)
51. \(-2(-3)(-5)(-4)\)  
52. \(-2(3)(-5)(4)\)
53. \(-3(5)(6)\)  
54. \(-5(7)(-2)(10)\)
55. $8(10)(-2)(5)$
56. $9(-5)(2)(3)(-4)$

57. a. $(-5)^1$
    b. $(-5)^2$
    c. $(-5)^3$
    d. $(-5)^4$
58. a. $(-2)^1$
    b. $(-2)^2$
    c. $(-2)^3$
    d. $(-2)^4$

59. a. $-5^1$
    b. $-5^2$
    c. $-5^3$
    d. $-5^4$
60. a. $-2^1$
    b. $-2^2$
    c. $-2^3$
    d. $-2^4$

61. a. $-8(-6)$
    b. $-8 - 6$
    c. $-8 - (-6)$
    d. $-8 \cdot (-6)$
    e. $-8 [-(-6)]$
62. a. $5(-7)$
    b. $5 - 7$
    c. $5 - (-7)$
    d. $5 \cdot (-7)$
    e. $5 [-(-7)]$

Simplify.

63. $18 ÷ (-3)$
64. $-24 ÷ 6$

65. $-40 ÷ (-10)$
66. $\frac{42}{-2}$

67. $\frac{-88}{8}$
68. $\frac{-90}{-10}$
### Answer Key for Chapter 2b Section 2.

1. $-7$
2. $-8$
3. 10
4. 13
5. $-20$
6. $-3$
7. 3
8. $-1$
9. $-8$
10. $-20$
11. 12
12. $-8$
13. 20
14. 5
15. $-192$
16. 1080
17. $-216$
18. $-49$
19. $-37z + 3$
20. $6k + 12$
21. $18m + 45$
22. $10a - 15$
23. 60
24. 83
25. 3
26. 13
27. $m = 8$
28. $y = 1$
29. $k = 13$
30. $x = 20$
31. $A = 5$
32. $M = 4$
33. $z = -12$
34. $a = 0$
35. $x = 18$
36. $k = -19$
37. $y = -11$
38. $K = 4$
39. The variable is $q$.
40. The coefficient is 4.
41. The constant is 11.
42. The factors are 4 and $q$.
43. positive
44. negative
45. $-24$
46. 27
47. 35
48. $-120$
49. 120
50. $-120$
51. 120
52. 120
53. $-90$
54. 700
55. $-800$
56. 1080
### Divide Integers

<p>| | | | |</p>
<table>
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<tr>
<td>57.</td>
<td>a.</td>
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<tr>
<td></td>
<td>b.</td>
<td>$25$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c.</td>
<td>$-125$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>d.</td>
<td>$625$</td>
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<td>58.</td>
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<td></td>
<td>c.</td>
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<td>d.</td>
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<tr>
<td>60.</td>
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<td>c.</td>
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<td></td>
<td>d.</td>
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<tr>
<td>61.</td>
<td>a.</td>
<td>$48$</td>
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<tr>
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<td>b.</td>
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<td></td>
<td>c.</td>
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<tr>
<td></td>
<td>d.</td>
<td>$48$</td>
<td></td>
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<tr>
<td></td>
<td>e.</td>
<td>$-8 [6] \rightarrow -48$</td>
<td></td>
</tr>
<tr>
<td>62.</td>
<td>a.</td>
<td>$-35$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b.</td>
<td>$-2$</td>
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<tr>
<td></td>
<td>c.</td>
<td>$12$</td>
<td></td>
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<td></td>
<td>d.</td>
<td>$-35$</td>
<td></td>
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<td>e.</td>
<td>$5 [7] \rightarrow 35$</td>
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<tr>
<td>65.</td>
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<td>68.</td>
<td>$9$</td>
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</tbody>
</table>
Section 3: Order of Operations with Integers; Evaluate

From Chapter 1, recall the **order of operations:**

1. Simplify inside the **grouping symbols** first.
2. Simplify **exponents** second.
3. From left to right, simplify all **divisions and/or multiplications** whichever comes first.
4. From left to right, simplify all **subtractions and/or additions** whichever comes first.

In this section, you will follow order of operations using integers. The easiest way to become comfortable is to work several examples. Observe the following.

a. \(8 + 6\left(\frac{45}{-9}\right)\)
   - \(8 + 6(-5)\) work inside parentheses
   - \(8 - 30\) multiply positive 6 by negative 5
   - \(-22\) combine

b. \(-16 - 2(5)\)
   - \(-16 - 10\) multiply negative 2 by positive 5
   - \(-26\) combine

c. \(\frac{-36}{2} - 4(5 - 12)\)
   - \(\frac{-36}{2} - 4(-7)\) work inside parentheses
   - \(-18 - 4(-7)\) divide negative 36 by positive 2
   - \(-18 + 28\) multiply negative 4 by negative 7
   - \(10\) combine

d. \(-50 ÷ (-2) + (-4)^3\)
   - \(-50 ÷ (-2) + (-64)\) exponent
   - \(25 + (-64)\) divide negative 50 by negative 2
   - \(-39\) combine

e. \(16 + \frac{35}{-7} - 5^2\)
   - \(16 + \frac{35}{-7} - 25\) exponent: \(5^2\) means 5(5) means 25
   - \(16 - 5 - 25\) divide positive 35 by negative 7
   - \(-14\) combine
Chapter 2 Section 3: Order of operations with integers; evaluate

f. \(27(-2) - 8(-1)
   -54 + 8
   -46\)  multiply on left and on right
   combine

g. \(-7(9) - 2(-6)^2
   -7(9) - 2(36)
   -63 - 72\)  exponent: \((-6)^2\) means \((-6)(-6)\) means 36
   multiply on left and on right
   -135\)  combine

h. \(-9(-5) - (-4)^2
   -9(-5) - 16\)  exponent: \((-4)^2\) means \((-4)(-4)\) means 16
   multiply
   45
   29\)  combine

i. \(8 - 5^2
   8 - 25\)  exponent: \(5^2\) means \((5)(5)\) means 25
   -17\)  combine

j. \(-32 ÷ 8 \cdot 2
   -4 \cdot 2\)  divide since it’s on the left
   -8\)  multiply

k. \(-20 ÷ 4 - 6(-8 + 3)
   -20 ÷ 4 - 6(-5)
   -5 + 30\)  work inside parentheses
   divide on left; multiply on right
   25\)  combine

These examples give you an idea of how to approach an order of operations problem involving integers. You may find that you need to work the above examples several times before you become comfortable with the concept. Also, you will probably encounter problems that are not exactly like the above; remember, order of operations will guide you to the answer.

NOTE: Absolute value symbols are also subject to the order of operations. They are considered a grouping symbol (similar to parentheses). In other words, you need to work inside the absolute value symbols before you would work outside the absolute value symbols. Observe the examples.

l. \(25 + |13| = 25 + 13 = 38\)
m. \(18 + |-32| = 18 + 32 = 50\)

n. \(49 - |5| = 49 - 15 = 34\)
o. \(68 - |-18| = 68 - 18 = 50\)

p. \(29 + |8 + 9 ÷ 2| = 29 + |8 + 18| = 29 + 26 = 55\)

q. \(12 + |4(-3) + 7| = 12 + |-12 + 7| = 12 + |-5| = 12 + 7 = 17\)
In summary, **order of operations** is a method of simplifying so everyone who works an expression will get the same result. As you are simplifying, you need to recall how to combine integers and how to multiply/divide integers:

**To combine integers,**
- if the signs are the same, find the sum and keep the sign
- if the signs are different, find the difference and keep the “bigger” sign;

**To multiply/divide integers,**
- if the signs of two numbers are the same, result is positive
- if the signs of two numbers are different, result is negative.

### Evaluate

In Chapter 1 and Chapter 2a, you evaluated expressions. This section extends your knowledge of evaluation. Observe.

<table>
<thead>
<tr>
<th>r.</th>
<th>6x + 12 for x = -7</th>
<th>s.</th>
<th>3y - 9 when y = -8</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-6(-7) + 12</td>
<td></td>
<td>3(-8) - 9</td>
</tr>
<tr>
<td></td>
<td>42 + 12</td>
<td></td>
<td>-24 - 9</td>
</tr>
<tr>
<td></td>
<td>54</td>
<td></td>
<td>-33</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>t.</th>
<th>4m + 9 when m = -11</th>
<th>u.</th>
<th>-7k - 1 for k = -2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>4(-11) + 9</td>
<td></td>
<td>-7(-2) - 1</td>
</tr>
<tr>
<td></td>
<td>-44 + 9</td>
<td></td>
<td>14 - 1</td>
</tr>
<tr>
<td></td>
<td>-35</td>
<td></td>
<td>13</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>v.</th>
<th>-z + 3 for z = -9</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-(9) + 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>9 + 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>12</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: On example v, you could also think this way.

- z + 3
- 1z + 3
- 1(-9) + 3
9 + 3
12

You get the same result!
Homework for Chapter 2b Section 3

Simplify.

1. \(-24 + 7 - 18\)

2. \(-48 \div 2 \cdot (-3)\)

3. \(\frac{30}{3} + 6(12 - 14)\)

4. \(\left(\frac{36}{4}\right)(-3) + (-20) \div (-2)\)

5. \(-60 \div (-2) - 7(-1 - 5)\)

6. \(\frac{18}{-2} - 4^2\)

7. \(6 - 9(-2 + 7)\)

8. \(-3(5) - 4(2)\)

9. \((6 - 9)(-2 + 7)\)

10. \(2(-3)^2 - 7(-3) + 18\)

11. \(-44 \div 4 - 6(7 - 10)\)

12. \(-7\left(\frac{-30}{-5}\right) + 2(-8 + 5)\)

13. \(3[-7 - 3(8)] + (-5)^3\)

14. \(10 - (13 - 5)^3\)

15. \(5(-4)^2 + 6(-4) - 2\)

16. \(90 - 3^4\)

17. \(|15 + 12 + 2|\)

18. \(|30 - 5^2|\)

19. \(|19 - 25|\)

20. \(|58 - |-35|\)

Evaluate.

21. \(4k - 9\) for \(k = -2\)  

22. \(-8m + 3\) when \(m = -5\)

23. \(-11a - 6\) when \(a = -8\)  

24. \(5y + 2\) when \(y = -1\)

25. \(-w - 7\) for \(w = -12\)
Chapter 2b Section 3

Order of operations with integers; evaluate

Simplify.

26. \(9m + 15 + 12m + 19\)
27. \(-8k + 2 + 9k - 12\)

28. \(7(3y + 8)\)
29. \(\frac{8a - 48}{2}\)

30. \(17 + 2(3a + 4) + 7(5a + 9) + 5a\)

In the expression \(5m + 8\),
31. What are the terms?
32. What are the factors?

33. a. Is 25 a whole number? b. Is 25 an integer?
   c. Is -36 a whole number? d. Is -36 an integer?

34. When you re-write \(8 + 12k\) as \(12k + 8\), what property have you used?
35. Is this an example of commutative or associative:
   \(7 + (3 + 9) = (7 + 3) + 9\)

Solve.

36. \(7m = 45 + 2m\)
37. \(6y = 72 - 3y\)
38. \(4Y + 3Y = 56\)
39. \(2x + 3 + x + 12 = 48\)
40. \(a + 24 = -13\)
41. \(5k - 35 = 40\)

Additional Practice with Order of Operations With Integers; Evaluate

Simplify.

42. \(-13 + 8 - 20\)
43. \(-50 ÷ 2 \cdot (-7)\)

44. \(18 + 3(7 - 12)\)
45. \(-25 + 2(8 - 14)\)

46. \(20 - 6(2 + 3)\)
47. \(-17 - 3(5 + 1)\)

48. \(29 - 4(10 - 16)\)
49. \(-35 - 8(2 - 7)\)

50. \(19 - 7(5 - 2)\)
51. \(-50 - 6(9 - 1)\)
Chapter 2b Section 3

Order of operations with integers; evaluate

52. $5(-9) + 7(-4)$
53. $-3(6) - 5(10)$
54. $-7(-6) + 2(-5)$
55. $-2(-10) - 8(-3)$
56. $-12 \cdot 5 + (-3) + 5^2$
57. $-18 \div (-9) + (-3)^2$
58. $-4(-2) - 7^2$
59. $(-6 + 1)\left(\frac{-54}{-6}\right) - 4^2$
60. $\left(\frac{-32}{4}\right)(-13 + 10) - (-8)^2$
61. $5[9 + 2(-4)]$
62. $(1 - 5)^2 + 3(7 - 18)$
63. $8 - 6(4 - 3 \cdot 5)$
64. $14 - 2(7 - 3)^2$
65. $38 - (2 - 10)^2$
66. $\left|2 - 5 \cdot 7\right|$
67. $25 - \left|10 + 18\right|$
68. $17 - \left|9 - 12\right|$
69. $17 - (9 - 12)$
70. $17(9 - 12)$
71. $17[-(9 - 12)]$

Evaluate.

72. $4m + 6$ when $m = 3$
73. $8y - 2$ for $y = 7$
74. $2k + 9$ for $k = -10$
75. $9a - 10$ when $a = -2$
76. $z + 12$ when $z = -13$
77. $-3x + 12$ for $x = -5$
Chapter 2b Section 3
Order of operations with integers; evaluate

Answer Key for Chapter 2b Section 3.

1. $-35$  
2. $72$  
3. $-2$

4. $-17$  
5. $72$  
6. $-25$

7. $-39$  
8. $-23$  
9. $-15$

10. $57$  
11. $7$  
12. $-48$

13. $-218$  
14. $-54$  
15. $54$

16. $9$  
17. $21$  
18. $5$

19. $48$  
20. $23$  
21. $-17$

22. $43$  
23. $82$  
24. $-3$

25. $5$  
26. $21m + 34$  
27. $k - 10$

28. $21y + 56$  
29. $4a - 24$  
30. $46a + 88$

31. The terms are $5m$ and $8$.  
32. The factors are 5 and m.

33. a. 25 is a whole number  
b. 25 is an integer  
c. $-36$ is not a whole number  
d. $-36$ is an integer

34. commutative property  
35. associative property

36. $m = 9$  
37. $y = 8$  
38. $Y = 8$

39. $x = 11$  
40. $a = -37$  
41. $k = 15$

On this part of the answer key, some problems will have more steps shown.

42. $-25$  
43. $175$

44. $18 + 3(-5)$  
45. $-25 + 2(-6)$  

$18 - 15$  
$-25 - 12$  

$3$  
$-37$
Chapter 2b Section 3  

Order of operations with integers; evaluate

46. \(-10\)

47. \(-35\)

48. \(29 - 4(-6)\)

49. \(-35 - 8(-5)\)

\(29 + 24\)

\(-35 + 40\)

53

5

50. \(-2\)

51. \(-98\)

52. \(-73\)

53. \(-68\)

54. \(42 - 10\)

55. \(20 + 24\)

\(32\)

\(44\)

56. \(45\)

57. \(2 + 9\)

58. \(8 - 49\)

59. \(-5(9) - 16\)

\(-41\)

\(-45 - 16\)

\(-61\)

60. \(-8(-3) - 64\)

61. \(5[9 - 8]\)

\(24 - 64\)

\(5[1]\)

\(-40\)

\(5\)

62. \((-4)^2 + 3(-11)\)

63. \(8 - 6(4 - 15)\)

\(16 - 33\)

\(8 - 6(-11)\)

\(-17\)

\(8 + 66\)

\(-17\)

\(74\)

64. \(14 - 2(4)^2\)

65. \(38 - (-8)^2\)

\(14 - 2(16)\)

\(38 - 64\)

\(14 - 32\)

\(-26\)

\(-18\)

66. \(33\)

67. \(-3\)

68. \(14\)

69. \(20\)

70. \(-51\)

71. \(51\)

72. \(4(3) + 6\)

73. \(8(7) - 2\)

\(18\)

\(54\)

74. \(2(-10) + 9\)

75. \(9(-2) - 10\)

\(-11\)

\(-28\)

76. \(-1\)

77. \(27\)
Section 4: Distribute and Combine with Integers

In Chapter 1, you learned the distributive property of multiplication over addition/subtraction to remove parentheses. In this section, you will distribute with integers.

a. \(5(2m + 8)\)  
   \(10m + 40\)

b. \(6(3y - 7)\)  
   \(18y - 42\)

Recall that you multiplied both terms inside the parentheses by the number on the outside of the parentheses. When you distribute with integers, you need to remember the signs. When you multiply two numbers with the same sign, the result is positive. When you multiply two numbers with different signs, the result is negative.

c. \(-2(3a + 6)\)  
   \(-6a - 12\)

d. \(-4(-5k + 2)\)  
   \(20k - 8\)

e. \(-7(4z - 3)\)  
   \(-28z + 21\)

f. \(5(-2x + 3)\)  
   \(-10x + 15\)

g. \(-(8m + 3)\)  
   \(-8m - 3\)

h. \(-(4y - 12)\)  
   \(-4y + 12\)

In Chapter 1, you also learned to distribute and then to combine like terms. In this section, you will distribute and combine like terms using integers. You need to remember that multiplying has one set of rules while combining has a different set of rules. Observe.

i. \(5(2y + 1) + 3(4y + 8)\)  
   \(10y + 2 + 12y + 24\)

j. \(4(3m - 2) + 5(6m - 1)\)  
   \(12m - 8 + 30m - 5\)

k. \(-6(3k - 7) + 2(5k - 12)\)  
   \(-18k + 42 + 10k - 24\)

l. \(8(3z - 4) - 7(4z - 2)\)  
   \(24z - 32 - 28z + 14\)
You can also extend the above concept to include the following.

\[
\text{o. } \frac{10a + 18}{-2} \quad \text{p. } \frac{-9M + 6}{-3}
\]

\[
\text{means} \quad \text{means}
\]

\[
\frac{10a + 18}{-2} \quad \frac{-9M + 6}{-3}
\]

\[
-5a - 9 \quad 3M - 2
\]

To become comfortable, you may need to work the above examples several times. Each problem you encounter that uses the distributive property also uses the rules for integers. Each problem you encounter that uses combining like terms also uses the rules for integers.
Homework for Chapter 2b Section 4

Simplify.

1. $5(2y - 6)$
2. $-3(4m + 7)$
3. $-2(8z - 5)$
4. $-(-4x + 2)$
5. $8(-3a + 1)$
6. $-(5k - 3)$
7. $4(2y - 8) + 3(y + 2)$
8. $6(11k - 4) - 3(2k + 5)$
9. $3(-2w - 5) - (10w - 7)$
10. $5(9m - 8) - 12(3m - 4)$
11. $2(4m + 9) + 8(3m - 10)$
12. $6(5y + 3) - 4(2y - 7)$
13. $3(x - 6) + 9(10x - 3)$
14. $9(2a - 4) - 5(3a - 11)$
15. $7(x + 6) - (11x + 50)$
16. $11(3x + 4) - (15x - 3)$
17. $8 + 4(6A - 3) + 2(3A + 1)$
18. $-12 - 2(4k - 6) - 5(3k + 4)$
19. $\frac{-14y + 21}{7}$
20. $\frac{-36z + 14}{-2}$
21. $19 - 8^2$
22. $-5(4) - 8(-3)$
23. $\left(\frac{8}{2}\right) - \left(\frac{-26}{13}\right) - 5^2$
24. $3 + 2(8 - 10) + 2\left(\frac{24}{-8}\right)$
25. a. $18 \cdot 2 + 9 + 8^2$
   b. $18 \div 2 + 9 + 8^2$
26. a. $10 + 4(2 + 3)^2$
   b. $10 + 4(2 + 3)^2$

Solve.

27. $5m = 80$
28. $5y - 2y = 18$
29. $48 = c + 32$
30. $2a = 22 - 9a$
31. $7a + 14 = 35 + 21$
32. $7 + 2a + 5 = 18$
33. $k + 16 = -2$
34. $x - 24 = -30$
35. $2z + 18 = 24$
36. $4w - 22 = 22$
Evaluate.

37. \(5k + 18\) for \(k = -6\)  
38. \(-3m - 10\) when \(m = -7\)

Simplify.

39. \(2(5k + 3) + 4(6k - 5) - 9(2k - 6) - 10(4k + 1)\)

40. \((4x + 8) - (10x - 3) + (-9x - 6) - (-21x + 7)\)
Answer Key for Chapter 2b Section 4.

1. \( 10y - 30 \)  
2. \( -12m - 21 \)  
3. \( -16z + 10 \)

4. \( 4x - 2 \)  
5. \( -24a + 8 \)  
6. \( -5k + 3 \)

7. \( 11y - 26 \)  
8. \( 60k - 39 \)  
9. \( -16w - 8 \)

10. \( 9m + 8 \)  
11. \( 32m - 62 \)  
12. \( 22y + 46 \)

13. \( 93x - 45 \)  
14. \( 3a + 19 \)  
15. \( -4x - 8 \)

16. \( 18x + 47 \)  
17. \( 30A - 2 \)  
18. \( -23k - 20 \)

19. \( -2y + 3 \)  
20. \( 18z - 7 \)  
21. \( -45 \)

22. \( 4 \)  
23. \( -33 \)  
24. \( -7 \)

25. a. \( 18 \cdot \frac{2}{9} + 8^2 \)  
26. a. \( 10 + 4(2 + 3)^2 \)

b. \( 18 \cdot \frac{2}{9} + 64 \)  
b. \( 10 + 4(5)^2 \)

36 ÷ 9 + 64  
4 + 64  
4 \( \frac{16}{9} \)  

27. \( m = 16 \)  
28. \( y = 6 \)

29. \( 16 = c \)  
30. \( a = 2 \)

31. \( a = 6 \)  
32. \( a = 3 \)

33. \( k = -18 \)  
34. \( x = -6 \)

35. \( z = 3 \)  
36. \( w = 11 \)

37. \(-12\)  
38. \(11\)

39. \(10k + 6 + 24k - 20 - 18k + 54 - 40k - 10 - 24k + 30\)

40. \(4x + 8 - 10x + 3 - 9x - 6 + 21x - 7 - 6x - 2\)
Section 5: Solve Equations of Type $ax = b$ Using Integers

In Chapter 1, you learned to solve equations by “undoing” the operation in order to isolate the variable. You know to “undo” multiplication, you divide. In this section, you will extend that concept to using integers.

To solve equations of type $ax = b$, you will divide both sides of the equation by the coefficient in order to “undo” the multiplication and in order to completely isolate the variable. Follow the examples.

a. $5m = -30$

\[
\begin{align*}
5m &= -30 \\
\frac{5m}{5} &= \frac{-30}{5} \\
m &= -6
\end{align*}
\]

Divide to “undo” multiplication

Quotient with one negative is negative

Check:

\[
\begin{align*}
5m &= -30 \\
5(-6) &= -30 \\
-30 &= -30
\end{align*}
\]

$m = -6$ is the solution.

b. $-11y = 44$

\[
\begin{align*}
-11y &= 44 \\
\frac{-11y}{-11} &= \frac{44}{-11} \\
y &= -4
\end{align*}
\]

Divide to “undo” multiplication

Quotient with one negative is negative

c. $-7k = -56$

\[
\begin{align*}
-7k &= -56 \\
\frac{-7k}{-7} &= \frac{-56}{-7} \\
k &= 8
\end{align*}
\]

Divide to “undo” multiplication

Quotient with two negatives is positive
Chapter 2b Section 5

Solve equations of type $ax = b$ using integers

d. $24 = -8z$

\[
\begin{align*}
\frac{24}{-8} &= \frac{-8z}{-8} & \text{divide by the coefficient } -8 \text{ to “undo”} \\
-3 &= z & \text{quotient with one negative is negative}
\end{align*}
\]

Remember to simplify before you begin to solve. Observe.

e. $9k - 12k = 33$

\[
\begin{align*}
-3k &= 33 & \text{simplify first} \\
\frac{-3k}{-3} &= \frac{33}{-3} & \text{divide by the coefficient } -3 \text{ to “undo”} \\
k &= -11 & \text{quotient with one negative is negative}
\end{align*}
\]

f. $-2m - 5m = -42$

\[
\begin{align*}
-7m &= -42 & \text{simplify first} \\
\frac{-7m}{-7} &= \frac{-42}{-7} & \text{divide by the coefficient } -7 \text{ to “undo”} \\
m &= 6 & \text{quotient with two negatives is positive}
\end{align*}
\]

Check:

\[
\begin{align*}
-2m - 5m &= -42 \\
-2(6) - 5(6) &= -42 \\
-12 - 30 &= -42 \\
-42 &= -42 \\
m &= 6 \text{ is the solution.}
\end{align*}
\]
Homework for Chapter 2b Section 5

Solve.

1. \(8k = -48\)  
2. \(4z = 52\)

3. \(-10a = -90\)  
4. \(-7b = -7\)

5. \(-6m + 2m = 24\)  
6. \(9Y - 2Y = -35\)

7. \(14W - 19W = -30\)  
8. \(-3d - 5d = 72\)

9. \(-30 = 15x\)  
10. \(-22 = -2m\)

11. \(c - 32 = -38\)  
12. \(6w + 12 = 48\)

Simplify.

13. \(-44 - (-6)\)  
14. \(7(-12)\)

15. \((-4)^3\)  
16. \(13 - (-6)^2\)

17. \(98 - 104\)  
18. \(-12 - 8\)

19. \(-6 + 3 + (-14) - (-8)\)  
20. \(-6(3)(-14)(-8)\)

21. \(5y + 6 + 3y + 11\)  
22. \(8k - 14 + 4k + 3\)

23. \(\frac{-48m + 84}{4}\)  
24. \(5(2z - 9) - 2(3z + 4)\)

25. \(11(a - 5) - (2a - 7)\)  
26. \(16 + 5(3c - 8) - 4c - 12\)

27. \(15 + 2(8 - 12)\)  
28. \((18 - 14)\left(\frac{-9}{3}\right) - 6^2\)

29. In the expression \(6v - 12\),  
a. What is the variable?  
b. What is the coefficient?  
c. What is the constant?

30. What property would rewrite \(6v - 12\) as \(-12 + 6v\)?
Chapter 2b Section 5

Answer Key for Chapter 2b Section 5

1. k = -6
2. z = 13
3. a = 9
4. b = 1
5. m = -6
6. Y = -5
7. W = 6
8. d = -9
9. -2 = x
10. 11 = m
11. c = -6
12. w = 6
13. -38
14. -84
15. -64
16. -23
17. -6
18. -20
19. -9
20. -2016
21. 8y + 17
22. 12k - 11
23. -12m + 21
24. 4z - 53
25. 9a - 48
26. 11c - 36
27. 7
28. -48
29. a. The variable is v.
b. The coefficient is 6.
c. The constant is -12.
30. commutative property
Section 6: Solve Two-Step Equations Using Integers

This section will further extend using integers to solve multi-step equations.

To solve two-step equations, you first need to isolate the variable term on one side of the equal sign. You will add or subtract terms to “undo” the operation that is written. After the variable terms are on one side of the equal sign and the constant terms are on the other side of the equal sign, then you will completely isolate the variable by dividing by the coefficient of the variable. Since you will be using integers, you will need to pay attention to signs. Follow the examples.

a.

Solve vertically:

\[
3k = -4k - 42 \\
+ 4k \quad + 4k \\
7k = -42
\]

\[
\frac{7k}{7} = \frac{-42}{7}
\]

\[k = -6\]

Solve horizontally:

\[
3k = -4k - 42 \\
3k + 4k = -4k + 4k - 42 \\
7k = 0 - 42 \\
7k = -42 \\
7k ÷ 7 = -42 ÷ 7 \\
k = -6
\]

Notice you get the same result!

Check:

\[
3k = -4k - 42 \\
3(-6) = -4(-6) - 42 \\
-18 = 24 - 42 \\
-18 = -18 \\
k = -6 \text{ is the solution.}
\]
b. \[5m + 15 = -30\]
\[5m + 15 - 15 = -30 - 15\]
\[5m = -45\]
\[\frac{5m}{5} = \frac{-45}{5}\]
\[m = -9\]

subtract 15 to “undo” addition

divide to “undo” multiplication

m = -9

c. \[-3v - 6v = -2v - 28\]
\[-9v = -2v - 28\]
simplify first; then begin to solve
\[+2v + 2v\]
\[-7v = -28\]
\[-7v = \frac{-28}{-7}\]
\[v = 4\]

simplify first; then begin to solve

v = 4

d. \[2y + 42 - 8y = 12\]
\[-6y + 42 = 12\]
simplify first; then begin to solve
\[\underline{-42 - 42}\]
\[-6y = -30\]
\[-6y = \frac{-30}{-6}\]
\[y = 5\]

subtract

divide

y = 5

e. \[7k - 49 = -24 - 60\]
\[7k - 49 = -84\]
simplify first; then begin to solve
\[7k - 49 + 49 = -84 + 49\]
\[7k + 0 = -35\]
\[7k = -35\]
\[7k ÷ 7 = -35 ÷ 7\]
\[k = -5\]

simplify first; then begin to solve

Check:
\[7k - 49 = -24 - 60\]
\[7(-5) - 49 = -24 - 60\]
\[-35 - 49 = -24 - 60\]
\[-84 = -84\]
\[k = -5\] is the solution.
Chapter 2b  Section 6  
Solve two-step equations using integers

f. \[-8m - 24 = -72\]
\[
\begin{align*}
+24 & \quad +24 \\
\frac{-8m}{-8} & = \frac{-48}{-8} \\
m & = 6
\end{align*}
\]
g. \[-k + 25 = -38\]
\[
\begin{align*}
-25 & \quad -25 \\
-k & = -63 \\
\frac{-1k}{-1} & = \frac{-63}{-1} \\
k & = 63
\end{align*}
\]
h. \[37 = 2m + 91\]
\[
\begin{align*}
-91 & \quad -91 \\
-54 & = 2m \\
\frac{-54}{2} & = \frac{2m}{2} \\
-27 & = m \text{ or } m = -27
\end{align*}
\]
i. \[2x + 37 = 37\]
\[
\begin{align*}
-37 & \quad -37 \\
2x & = 0 \\
\frac{2x}{2} & = \frac{0}{2} \\
x & = 0
\end{align*}
\]
Check:
\[2x + 37 = 37\]
\[
\begin{align*}
2(0) & + 37 = 37 \\
0 & + 37 = 37 \\
37 & = 37 \\
x & = 0 \text{ is the solution.}
\end{align*}
\]
j. \[-32 = -8m - 24\]

\[
\begin{array}{c}
+ 24 \\
- 8 = -8m
\end{array}
\]

\[-8 = -8m\]

\[
\begin{array}{c}
-8 \\
-8
\end{array}
\]

\[1 = m\]

Check:

\[-32 = -8m - 24\]

\[-32 = -8(1) - 24\]

\[-32 = -8 - 24\]

\[-32 = -32\]

\[1 = m\] is the solution.
Homework for Chapter 2b Section 6

Solve.

1. \(-4A = 18 + 2A\)  
2. \(3B = -20 + 5B\)
3. \(2d = 9d - 56\)  
4. \(-12G = -15G + 21\)
5. \(2x + 18 = 6\)  
6. \(3k - 15 = -51\)
7. \(-4m + 16 = 12\)  
8. \(-5y - 10 = 25\)
9. \(7z + 35 = -14\)  
10. \(11w - 20 = -97\)
11. \(2m + 18 - 8m = -48\)  
12. \(-11h - 28 - 3h = -70\)
13. \(5y = 16 - 11y\)  
14. \(-8R = -6R + 22\)
15. \(-a + 3 = 7\)  
16. \(-c + 18 = 12\)
17. \(-v - 8 = -13\)  
18. \(-q - 12 = -10\)
19. \(28 = 4x + 32\)  
20. \(-34 = -17m - 68\)
21. \(14 = -2m + 22\)  
22. \(-20 = 5y - 5\)
23. \(16 + 3x = 5x\)  
24. \(-10 - 4k = k\)
25. \(9y + 27 = 81\)  
26. \(20z - 28 = 72\)
27. \(4a + 8 = 16 - 40\)  
28. \(-9k - 72 = -18 + 45\)

Simplify.

29. \(25(-2) + 8(3 - 12)\)  
30. \(-17 - 2^4\)
31. \(28 \div (-7) - 5(-9 + 2)\)  
32. \(4 - (3 - 2y) + 2(5 - 4y)\)
33. \(5m + 12 - 6m - 8\)  
34. \(-12y + 6 - 3(2y - 4)\)
35. \(-15 + 7(8 - 3^2)\)  
36. \(5 \cdot (-4) \cdot 3 \cdot 8\)

Evaluate.

37. \(-3b + 8\) when \(b = 6\)  
38. \(5y + 12\) for \(y = -9\)
39. \(z - 11\) for \(z = 2\)  
40. \(-2k - 25\) when \(k = -6\)
Answer Key for Chapter 2b Section 6.

1. A = −3
2. B = 10
3. d = 8
4. G = 7
5. x = −6
6. k = −12
7. m = 1
8. y = −7
9. z = −7
10. w = −7
11. m = 11
12. h = 3
13. y = 1
14. R = −11
15. a = −4
16. c = 6
17. v = 5
18. q = −2
19. −1 = x or x = −1
20. −2 = m or m = −2
21. 4 = m or m = 4
22. −3 = y or y = −3
23. 8 = x or x = 8
24. −2 = k or k = −2
25. y = 6
26. z = 5
27. a = −8
28. k = −11
29. −122
30. −33
31. 31
32. −6y + 11
33. −m + 4
34. −18y + 18
35. −22
36. −480
37. −10
38. −33
39. −9
40. −13
Section 7: Solve Equations Using Combining Like Terms

You have been solving equations and already know the basic steps needed to isolate the variable. This section will more fully expand on simplifying on each side of the equation independently. Observe.

a. \(3m + 2m + 15 = 12 + 18\)
   
   \[5m + 15 = 30\]
   
   \[5m = 15\]
   
   \[m = 3\]

   Check:
   
   \[3m + 2m + 15 = 12 + 18\]
   
   \[3(3) + 2(3) + 15 = 12 + 18\]
   
   \[9 + 6 + 15 = 12 + 18\]
   
   \[30 = 30\]

   \(m = 3\) is the solution.

b. \(4y + 12 + 2y = 80 - 20\)
   
   \[6y + 12 = 60\]
   
   \[6y + 12 - 12 = 60 - 12\]
   
   \[6y = 48\]
   
   \[y = 8\]

c. \(k + k - 49 = 83\)
   
   \[2k - 49 = 83\]
   
   \[2k = 132\]
   
   \[k = 66\]
Chapter 2b Section 7

Solve equations using combining like terms

\[ \begin{align*}
\text{d. } 9a &= 3a + 16 - 10a \\
&= -7a + 16 \\
&\quad + 7a + 7a \\
16a &= 16 \\
\frac{16a}{16} &= \frac{16}{16} \\
a &= 1
\end{align*} \]

\[ \begin{align*}
\text{e. } 2m - 24 + 6m &= 32 + 56 \\
8m - 24 &= 88 \\
\quad + 24 + 24 \\
8m &= 112 \\
\frac{8m}{8} &= \frac{112}{8} \\
m &= 14
\end{align*} \]

\[ \begin{align*}
\text{f. } -k + 25 - 4k &= -35 \\
-5k + 25 &= -35 \\
\quad - 25 - 25 \\
-5k &= -60 \\
\frac{-5k}{-5} &= \frac{-60}{-5} \\
k &= 12
\end{align*} \]

\[ \begin{align*}
\text{g. } 18 + 37 &= m + 91 + m \\
55 &= 2m + 91 \\
\quad - 91 - 91 \\
-36 &= 2m \\
\frac{-36}{2} &= \frac{2m}{2} \\
-18 &= m \text{ or } m = -18
\end{align*} \]
Chapter 2b Section 7  

Solve equations using combining like terms

h. \[2x + 37 - 4x = -12 + 51\]
   
   \[-2x + 37 = 39\]
   
   combine 2x - 4x and -12 + 51
   
   \[-37 - 37\]
   
   \[-2x = 2\]
   
   \[\frac{-2x}{-2} = \frac{2}{-2}\]
   
   \[x = -1\]

i. \[4y + 3y = 2y + 18 + 6y + 20\]
   
   combine 4y + 3y; 2y + 6y; 18 + 20
   
   \[7y = 8y + 38\]
   
   \[7y - 8y = 8y - 8y + 38\]
   
   \[-1y = 0 + 38\]
   
   \[-1y = 38\]
   
   \[-1y ÷ (-1) = 38 ÷ (-1)\]
   
   \[y = -38\]

Check:

\[4y + 3y = 2y + 18 + 6y + 20\]

\[4(-38) + 3(-38) = 2(-38) + 18 + 6(-38) + 20\]

\[-152 - 114 = -76 + 18 - 228 + 20\]

\[-266 = -304 + 38\]

\[-266 = -266\]

\[y = -38\] is the solution.
Homework for Chapter 2b Section 7

Solve.

1. \[2k + 4k + 18 = 72\]
2. \[m + 6 + 3m = -26\]
3. \[4k + 16 + 12k = 48 + 80\]
4. \[-6y + 12 + 2y = 28 - 44\]
5. \[7z = 2z + 55 - 6z\]
6. \[-9w = 3w + 18 - 10w\]
7. \[2y + 42 + 3y = 25 + 52\]
8. \[3a - 24 = 18 - 51\]
9. \[x + x + 19 = 20 + 5\]
10. \[-9F - 2F = -2F - 39 - 6F\]
11. \[6c = 14c - 54 + c\]
12. \[z + 3z - 32 = 100 - 4\]
13. \[4w + 9 + w = 59\]
14. \[3m - 16 + m = -28\]
15. \[44 = z + 18 + z\]
16. \[y - 32 + y + 3 = -14 + 49\]
17. \[-2k + 30 - 4k = -12\]
18. \[5a - 19 - 7a = -3\]
19. \[14k - 9k = -8k + 65 + 6k + 5\]
20. \[-20d + 3d = 16d + 18 - 13d + 22\]
Chapter 2b Section 7

Solve equations using combining like terms

Answer Key for Chapter 2b Section 7.

1. $6k + 18 = 72$
   \[6k = 54\]
   \[k = 9\]

2. $4m + 6 = -26$
   \[4m = -32\]
   \[m = -8\]

3. $16k + 16 = 128$
   \[16k = 112\]
   \[k = 7\]

4. $-4y + 12 = -16$
   \[-4y = -28\]
   \[y = 7\]

5. $7z = -4z + 55$
   \[11z = 55\]
   \[z = 5\]

6. $-9w = -7w + 18$
   \[-2w = 18\]
   \[w = -9\]

7. $5y + 42 = 77$
   \[5y = 35\]
   \[y = 7\]

8. $3a - 24 = -33$
   \[3a = -9\]
   \[a = -3\]

9. $2x + 19 = 25$
   \[2x = 6\]
   \[x = 3\]

10. $-11F = -8F - 39$
    \[-3F = -39\]
    \[F = 13\]

11. $6c = 15c - 54$
    \[-9c = -54\]
    \[c = 6\]

12. $4z - 32 = 96$
    \[4z = 128\]
    \[z = 32\]

13. $5w + 9 = 59$
    \[5w = 50\]
    \[w = 10\]

14. $4m - 16 = -28$
    \[4m = -12\]
    \[m = -3\]

15. $44 = 2z + 18$
    \[26 = 2z\]
    \[13 = z\]

16. $2y - 29 = 35$
    \[2y = 64\]
    \[y = 32\]

17. $-6k + 30 = -12$
    \[-6k = -42\]
    \[k = 7\]

18. $-2a - 19 = -3$
    \[-2a = 16\]
    \[a = 8\]

19. $5k = -2k + 70$
    \[7k = 70\]
    \[k = 10\]

20. $-17d = 3d + 40$
    \[-20d = 40\]
    \[d = -2\]
Chapter 2b Summary

Multiply Integers

two like signs → positive

two unlike signs → negative

\[ 3(2) \rightarrow 6 \]
\[ -4(-5) \rightarrow 20 \]
\[ 6(-3) \rightarrow -18 \]
\[ -4(8) \rightarrow -32 \]

even number of negative factors → positive
odd number of negative factors → negative

\[ 8(-2)(3)(-1)(5) \rightarrow 240 \]
\[ 4(-3)(-6)(-2) \rightarrow -144 \]

Divide Integers

two like signs → positive

two unlike signs → negative

\[ \frac{12}{2} \Rightarrow 6 \]
\[ \frac{-18}{-2} \Rightarrow 9 \]
\[ \frac{8}{-4} \Rightarrow -2 \]
\[ \frac{-24}{6} \Rightarrow -4 \]

Order of Operations

\[ -18 + 3(7 - 12) + \frac{15}{-5} + (-2)^4 \]
\[ -18 + 3(-5) + \frac{15}{-5} + (-2)^4 \]
\[ -18 + 3(-5) + \frac{15}{-5} + 16 \]
\[ -18 - 15 - 3 + 16 \]
\[ -20 \]

Evaluate

2m + 8 when m = -10
2(-10) + 8
-20 + 8
-12
Distribute and Combine

\[7(2y - 9) - 6(3y - 8)\]
\[14y - 63 - 18y + 48\]
\[-4y - 15\]  

Solve

a. \[-9x = -27\]
\[-9x = -27\]
\[-9 \quad \text{and} \quad -9\]
\[x = 3\]

b. \[2y + 20 = 4\]
\[-20 \quad \text{and} \quad -20\]
\[2y = -16\]
\[2\]
\[y = -8\]

c. \[4x + 6 + 3x - 34 = -49\]
\[7x - 28 = -49\]
\[+ 28 \quad + 28\]
\[7x = -21\]
\[7\]
\[x = -3\]

d. \[3y = 6y - 15 + 2y\]
\[3y = 8y - 15\]
\[3y - 8y = 8y - 8y - 15\]
\[-5y = 0 - 15\]
\[-5y = -15\]
\[-5y \div (-5) = -15 \div (-5)\]
\[y = 3\]

e. Recall, you should also check your solutions.
Chapter 2b Review

Simplify.

1. $4(-3)$
2. $-8(-1) \cdot 5$
3. $3(-4)(-3)(-1)$
4. $(-1)^6$
5. $-1^{16}$
6. $-35 \div (-7)$
7. $\frac{39}{-13}$
8. $-32 - (-18) + (-3)$
9. $-28 \div (-4) \cdot 7$
10. $-4 - \left| 4 - 3 \right| \cdot (-2)$
11. $-25 - (-2 - 3^2) + (-2)^4 (1 - 3)^3$
12. $-7 - 5(2 - 8) + \left( \frac{-32}{-16} \right)$
13. $\frac{42a + 54}{-6}$
14. $-4(2a - 3) - 5(4 - a)$
15. $7 + 3(2b - 8) + (-2b + 8)$

Solve.

16. $7x = -42$
17. $-8m + 20m = 36$
18. $-38 = -2y$
19. $4k + 32 = 12$
20. $-5x - 14 = 1$
21. $8T + 18 = -14$
22. $-7m - 3 = -31$
23. $-4 = -4A + 12$
24. $-3R - 8 + 2R = 2$
25. $5w + 14 - 3w = -2$
26. $8 + 5g + 7 = -25 - 30$
27. $-4M = -16 + 44$
28. $x + x - 3 = -3$
29. $4k = 3k - 20 + 6k$
30. $-z = -12 - 8z + 92 - 3z$
Evaluate.

31. $y + 7$ for $y = -8$
32. $2k + 6$ when $k = 4$

33. $3z - 7$ when $z = -1$
34. $-4m - 6$ for $m = -3$

35. To simplify an expression which includes only the operations of addition and subtraction, which would you do first?

36. What is the sign (positive or negative) of the following (do not determine the numerical answer):
   \[
   \frac{8(-2)(14)(-3)(-38)}{(-19)(-28)(-27)}
   \]
### Answer Key for Chapter 2b Review.

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<td>$-7a - 9$</td>
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<td>$x = -6$</td>
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<td>20</td>
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### Reference Section

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<td>$T = -4$</td>
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<td>34.</td>
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<td>35.</td>
<td>Whichever comes first, left to right</td>
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<td>36.</td>
<td>positive</td>
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Chapter 2b Test

Simplify.

1. \((-6)(-3)\)  
2. \(2(-3) \cdot 4\)
3. \((-2)(-2)(-1)\)  
4. \((-2)^3\)
5. \(\frac{-6}{-3}\)  
6. \(-50 + (-6) \div (-3)\)
7. \(-6 + |2 - 3| \cdot 5\)  
8. \(\frac{-60}{-3} - (-1)^3(2 - 1)^3 - \frac{42}{2}\)
9. \(-28 \div 2 + 2(-3x - 5)\)  
10. \(\frac{-9k + 30}{-3}\)
11. \(3(4x - 5) - 2(3x + 4)\)

Solve.

12. \(-5x = 35\)  
13. \(24 = -2y\)
14. \(3x + 18 = -3\)  
15. \(-5k - 4 = 16\)
16. \(-1 + 2x + 13 = -4\)  
17. \(4x + 7 - 8x = 35\)
18. \(-6x - 4 + 16 = 0\)  
19. \(8 = 4x - 2 - 2x\)
20. \(4a + 3a = 6a + 19 + 2a\)  
21. \(-2y = 20 - 3y - y + 12\)

Evaluate.

22. \(3x + 18\) when \(x = -7\)  
23. \(-5t - 4\) when \(t = -4\)
Answer Key for Chapter 2b Test.

1. 18
2. -24
3. -4
4. -8
5. 2
6. -48
7. -1
8. 0
9. -6x - 24
10. 3k - 10
11. 6x - 23
12. x = -7
13. y = -12
14. x = -7
15. k = -4
16. x = -8
17. x = -7
18. x = 2
19. x = 5
20. a = -19
21. y = 16
22. -3
23. 16
Cumulative Review #2 Through Chapter 2b

1. Is subtraction commutative? Give an example.

Evaluate.

2. \( a + 3 \) for \( a = -8 \)

3. \( 4 - b \) when \( b = -5 \)

4. \( -3x - 4 \) for \( x = -2 \)

5. \( -8 + 3a \) when \( a = -3 \)

Simplify.

6. \( 8 \left( \frac{12}{4} \right) - 2 \cdot 1 \)

7. \( 4^2 \cdot 3 \cdot (2^2 - 2^3) \)

8. \( 3(x + 10) - 4^2 \)

9. \( -3x + (-2) + (-1) - x - 5 - (-2x) \)

10. \( -5 + (-3) + (-8) + 4 \)

11. \( -\left| -8 \right| \)

12. \( -3 - (-2) + (1) - 4 \)

13. \( -16 \div (-4) \cdot (-2) \)

14. \( -5(-3x - 2) - 6(2x + 3) \)

15. \( -3 - \left| 1 - 3 \right| \cdot (-5) \)

16. \( (-2^2 - 4)^2 \div (-3 - 1)^3 \)
Simplify.

17. $3(-2)(-4)(-1)$

18. $\frac{27}{-9}$

19. $(4 - 3^2) - (2 - 3)^2 - (-1 + 4)$

20. $-20 ÷ 5 - 7(-3 + 1)$

21. $\frac{-15A + 9B + 21C}{-3}$

Solve.

22. $11x + 8 = 30$

23. $15y = 54 - 3y$

24. $y - 8 = -8$

25. $4m = 6m + 18$

26. $-4x - 3 + 2x = 5$

27. $8k = 2k - 22 - 5k$

28. $-3 = -3a + 6$

29. $5x + (-1) - 2x = -7$

30. $17 = -4x + 8 + x$

31. $6x + 8 - 2x + 12 = 52 - 80$
Answer Key for Cumulative Review #2 through Chapter 2b.

1. no; example: $4 - 2 \times 2 - 4$  
2. $-5$  
3. $9$  
4. $2$  
5. $-17$  
6. $22$  
7. $0$  
8. $3x + 14$  
9. $-2x - 8$  
10. $-12$  
11. $-8$  
12. $-4$  
13. $-8$  
14. $3x - 8$  
15. $7$  
16. $-1$  

Reference Chapter

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<td>17.</td>
<td>$-24$</td>
<td></td>
<td>2b</td>
</tr>
<tr>
<td>18.</td>
<td>$-3$</td>
<td></td>
<td>2b</td>
</tr>
<tr>
<td>19.</td>
<td>$-9$</td>
<td></td>
<td>2b</td>
</tr>
<tr>
<td>20.</td>
<td>$10$</td>
<td></td>
<td>2b</td>
</tr>
<tr>
<td>21.</td>
<td>$5A - 3B - 7C$</td>
<td></td>
<td>2b</td>
</tr>
<tr>
<td>22.</td>
<td>$x = 2$</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>23.</td>
<td>$y = 3$</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>24.</td>
<td>$y = 0$</td>
<td></td>
<td>2a</td>
</tr>
<tr>
<td>25.</td>
<td>$m = -9$</td>
<td></td>
<td>2b</td>
</tr>
<tr>
<td>26.</td>
<td>$x = -4$</td>
<td></td>
<td>2b</td>
</tr>
<tr>
<td>27.</td>
<td>$k = -2$</td>
<td></td>
<td>2b</td>
</tr>
<tr>
<td>28.</td>
<td>$a = 3$</td>
<td></td>
<td>2b</td>
</tr>
<tr>
<td>29.</td>
<td>$x = -2$</td>
<td></td>
<td>2b</td>
</tr>
<tr>
<td>30.</td>
<td>$x = -3$</td>
<td></td>
<td>2b</td>
</tr>
<tr>
<td>31.</td>
<td>$x = -12$</td>
<td></td>
<td>2b</td>
</tr>
</tbody>
</table>
Chapter 3: Algebraic Language and Solving Equations

Preview

Chapter 3 will help you further develop your algebraic skills.

One important concept you will learn deals with translating English words into math symbols. Then you will learn how to apply this translation to solve word problems that have one unknown item. You will also learn how using translation applies to word problems that have two unknown items.

The chapter will end by further developing your skills with solving equations. You will solve equations where variable terms are on both sides of the equal sign. You will also solve equations that contain parentheses.

Within this chapter, you will be applying what you have learned previously from Chapters 1 and 2. You will need to remember the procedures for whole numbers and integers. You will need to remember how to combine like terms, how to distribute, and how to solve equations.
Section 1: Translate Words to Algebra

In algebra, it is important to apply what you have learned to applications (to word problems). A first step is being able to translate from English words into math symbols. In general, what you do not know is written with a variable. Then the other words are translated from English into math symbols. The following is a list of commonly used English words and their math symbols. Notice the phrases with * reverse the order of what you read in English and what you write as math symbols.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Math Symbols</th>
</tr>
</thead>
<tbody>
<tr>
<td>a number</td>
<td>x</td>
</tr>
<tr>
<td>an unknown</td>
<td>x</td>
</tr>
<tr>
<td>what you do not know</td>
<td>x</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Add</th>
<th>English</th>
<th>Math Symbols</th>
</tr>
</thead>
<tbody>
<tr>
<td>sum</td>
<td>The sum of a number and 5</td>
<td>x + 5</td>
</tr>
<tr>
<td>increased by</td>
<td>A number increased by 8</td>
<td>x + 8</td>
</tr>
<tr>
<td>*more than</td>
<td>4 more than a number</td>
<td>x + 4</td>
</tr>
<tr>
<td>*added to</td>
<td>10 added to a number</td>
<td>x + 10</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Subtract</th>
<th>English</th>
<th>Math Symbols</th>
</tr>
</thead>
<tbody>
<tr>
<td>difference</td>
<td>Difference between a number and 3</td>
<td>x - 3</td>
</tr>
<tr>
<td></td>
<td>Difference between 3 and a number</td>
<td>3 - x</td>
</tr>
<tr>
<td>decreased by</td>
<td>A number decreased by 7</td>
<td>x - 7</td>
</tr>
<tr>
<td>*less than</td>
<td>6 less than a number</td>
<td>x - 6</td>
</tr>
<tr>
<td>*subtracted from</td>
<td>14 subtracted from a number</td>
<td>x - 14</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Multiply</th>
<th>English</th>
<th>Math Symbols</th>
</tr>
</thead>
<tbody>
<tr>
<td>product</td>
<td>The product of 5 and a number</td>
<td>5x</td>
</tr>
<tr>
<td>times</td>
<td>4 times a number</td>
<td>4x</td>
</tr>
<tr>
<td>twice</td>
<td>Twice a number</td>
<td>2x</td>
</tr>
<tr>
<td>triple</td>
<td>Triple a number</td>
<td>3x</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Divide</th>
<th>English</th>
<th>Math Symbols</th>
</tr>
</thead>
<tbody>
<tr>
<td>quotient</td>
<td>The quotient of a number and 5</td>
<td>(\frac{x}{5})</td>
</tr>
<tr>
<td>divided by</td>
<td>A number divided by 7</td>
<td>(\frac{x}{7})</td>
</tr>
<tr>
<td>divided by</td>
<td>13 divided by a number</td>
<td>(\frac{13}{x})</td>
</tr>
</tbody>
</table>

NOTE: With subtraction and division, order matters!
Generally, what you read first in English you write first in math symbols. However, you reverse the order when you read the phrases “more than,” “added to,” “less than,” “subtracted from,” or any phrases that have the same meaning, such as “shorter than.” Also, you should identify the unknown by labeling it with a variable. Observe.

a. The height of a tree increased by 8 feet
   height of tree = x
   x + 8

b. A bank balance decreased by $25
   bank balance = x
   x - 25

c. 3 times the cost of a stereo
   cost of stereo = x
   3x

d. 13° more than yesterday’s temperature
   yesterday’s temperature = x
   x + 13 The order reverses because of the words “more than.”

e. 28 feet shorter than the length of a sidewalk
   length of sidewalk = x
   x - 28 The order reverses because of the words “shorter than.”

f. $16 less than 4 times the cost of a ring
   cost of ring = x
   4x - 16 The order reverses because of the words “less than.”

g. An inheritance divided evenly among 4 people
   the inheritance = x
   \[
   \frac{x}{4}
   \]
Chapter 3 Section 1

Translate words to algebra

Homework for Chapter 3 Section 1

For each of the following, first label the unknown. Second, translate the English words into math symbols.

1. Mary’s height increased by 7 inches
2. The sum of Mike’s marbles and 25 marbles
3. 32 feet longer than the length of a street
4. The difference between a number and 12
5. 18 cookies less than the number of cookies for a party
6. The height of a shrub decreased by 5 inches
7. Twice the number of dogs
8. Three times the length of a driveway
9. $12 more than 4 times the cost of a coat
10. 30° less than 3 times the temperature in Omaha
11. 20 feet taller than 5 times the height of a tree
12. 28 shirts fewer than twice the number of Jim’s shirts
13. The quotient of a number and 8
14. The quotient of 8 and a number

Simplify.

15. \(13 + 3k + 7k + 24\)
16. \(\frac{48M + 72}{8}\)
17. \(4(9m - 3) + (m + 7)\)
18. \((-19x + 1) - (2x - 3)\)
19. \(24 - 9^2\)
20. \(\left(\frac{24}{-2}\right)(-5 + 3) - 7(9 - 12)\)
Solve.

21. \( m + 8 = 14 \) 
22. \( y - 17 = 35 \)

23. \( 2x + 18 = 52 \) 
24. \( 3z - 24 = 51 \)

25. \( 4z - 25 = -29 \) 
26. \( 5c + 20 = 20 - 35 \)

27. \( 6a + 14 + a = 56 \) 
28. \( 7m - 33 - 10m = -18 \)

29. \( 5y = -36 - 4y \) 
30. \( -3d = -2d + 24 - 5d \)
Chapter 3 Section 1

Translate words to algebra

Answer Key for Chapter 3 Section 1.

1. Mary’s height = $x$
   \[ x + 7 \]

2. Mike’s marbles = $x$
   \[ x + 25 \]

3. the length of a street = $x$
   \[ x + 32 \]

4. a number = $x$
   \[ x - 12 \]

5. the number of cookies for a party = $x$
   \[ x - 18 \]

6. the height of a shrub = $x$
   \[ x - 5 \]

7. the number of dogs = $2x$

8. the length of a driveway = $x$
   \[ 3x \]

9. the cost of a coat = $x$
   \[ 4x + 12 \]

10. the temperature in Omaha = $x$
    \[ 3x - 30 \]

11. the height of a tree = $x$
    \[ 5x + 20 \]

12. the number of Jim’s shirts = $x$
    \[ 2x - 28 \]

13. the number = $x$
    \[ \frac{x}{8} \]

14. the number = $x$
    \[ \frac{8}{x} \]
<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>15.</td>
<td>$10k + 37$</td>
<td>16.</td>
</tr>
<tr>
<td>17.</td>
<td>$37m - 5$</td>
<td>18.</td>
</tr>
<tr>
<td>19.</td>
<td>$-57$</td>
<td>20.</td>
</tr>
<tr>
<td>21.</td>
<td>$m = 6$</td>
<td>22.</td>
</tr>
<tr>
<td>23.</td>
<td>$x = 17$</td>
<td>24.</td>
</tr>
<tr>
<td>25.</td>
<td>$z = -1$</td>
<td>26.</td>
</tr>
<tr>
<td>27.</td>
<td>$a = 6$</td>
<td>28.</td>
</tr>
<tr>
<td>29.</td>
<td>$y = -4$</td>
<td>30.</td>
</tr>
</tbody>
</table>
Section 2: Word Problems with One Unknown

In the previous section, you learned about translating English words into math symbols. In this section, you will continue translating. Now you will be translating from English words into an equation. You need to know the English words that translate into the equal sign.

Equal Sign
is
equals
is equal to
is the same as
the result is

As before, begin by labeling the unknown with a variable. Then as you read the English words, you write the math symbols. Remember the phrases that reverse the order of the math symbols: “more than,” “less than,” and phrases that have the same meaning. Once you have identified the unknown and translated from English into a math equation, you will then solve the equation and answer the question asked. On these examples, all solving steps will be left up to you. Observe.

a. The sum of a number and 3 is 9. Find the number.

\[
\text{a number} = x \\
x + 3 = 9 \\
x = 6
\]

The number is 6.

b. The height of a tree increased by 7 feet is 25 feet. Find the height of the tree.

\[
\text{height of a tree} = x \\
x + 7 = 25 \\
x = 18
\]

The height of the tree is 18 feet.

c. 15 more than a number equals 31. Find the number.

\[
\text{a number} = x \\
x + 15 = 31 \\
x = 16
\]

The number is 16.
d. The difference between a number and 13 is 21. Find the number.

\[
\begin{align*}
\text{a number} & = x \\
x - 13 & = 21 \\
x & = 34
\end{align*}
\]

The number is 34.

e. The length of a sidewalk decreased by 17 feet is 90 feet. Find the length of the sidewalk.

\[
\begin{align*}
\text{length of sidewalk} & = x \\
x - 17 & = 90 \\
x & = 107
\end{align*}
\]

The length of the sidewalk is 107 feet.

f. $185 less than the cost of a TV is $791. Find the cost of the TV.

\[
\begin{align*}
\text{cost of a TV} & = x \\
x - 185 & = 791 \quad \text{Notice that$185$is written after the subtraction symbol–the words "less than" reverse the order.} \\
x & = 976
\end{align*}
\]

The cost of the TV is $976.

g. $100 subtracted from the cost of a ticket is$35. Find the cost of the ticket.

\[
\begin{align*}
\text{cost of ticket} & = x \\
x - 100 & = 35 \quad \text{Notice$100$is written after the subtraction symbol–the words “subtracted from” reverse the order.} \\
x & = 135
\end{align*}
\]

The cost of the ticket is $135.

h. The difference between$100 and the cost of a ticket is $35. Find the cost of the ticket.

\[
\begin{align*}
\text{cost of ticket} & = x \\
100 - x & = 35 \quad \text{Notice$100$is written first–there are no words to reverse the order.} \\
x & = 65
\end{align*}
\]

The cost of the ticket is $65.
1. 120 feet decreased by the length of a sidewalk is 72 feet. Find the length of the sidewalk.

\[
\text{length of sidewalk} = x \\
120 - x = 72 \\
x = 48
\]

The length of the sidewalk is 48 feet.

2. Twice the cost of a jacket is $122. Find the cost of the jacket.

\[
\text{cost of a jacket} = x \\
2x = 122 \\
x = 61
\]

The cost of the jacket is $61.

3. Three times the temperature in Fairbanks is $-24^\circ$. Find the temperature in Fairbanks.

\[
\text{the temperature in Fairbanks} = x \\
3x = -24 \\
x = -8
\]

The temperature in Fairbanks is $-8^\circ$.

4. $144$ more than four times the price of a stereo system is $5176$. Find the price of the stereo system.

\[
\text{price of stereo system} = x \\
4x + 144 = 5176 \\
x = 1258
\]

The price of the stereo system is $1258$.

5. $858$ feet less than three times the height of a mountain is $15840$ feet. Find the height of a mountain.

\[
\text{height of a mountain} = x \\
3x - 858 = 15840 \\
x = 5566
\]

The height of the mountain is 5566 feet.
Homework for Chapter 3 Section 2

For each of the following, first label the unknown. Second, write an equation. Third, solve. Fourth, answer the question asked.

1. Five feet more than the length of a sidewalk is 22 ft. Find the length of the sidewalk.
2. The length of a ribbon increased by 18 inches is 40 inches. Find the length of the ribbon.
3. The sum of John's age and 6 years equals 23 years. Find John's age.
4. Eight degrees more than the temperature is −6 degrees. Find the temperature.
5. A number increased by 7 is −4. Find the number.
7. Seven miles less than the distance to the store is equal to 14 miles. Find the distance to the store.
8. The height of a tree decreased by 6 feet is 11 feet. Find the height of the tree.
9. The difference between a number and 9 results in 20. Find the number.
10. Three times the cost of dinner is $21. Find the cost of dinner.
11. Four times yesterday's temperature in Omaha is −24 degrees. Find yesterday's temperature in Omaha.
12. Six days more than twice the number of vacation days is 32. Find the number of vacation days.
13. Seven minutes less than 3 times the time to cook oatmeal is 11 minutes. Find the time it takes to cook oatmeal.
14. 25 ft. more than the height of a door is 33 ft. Find the height of the door.
15. 32 in. subtracted from the sidewalk length is 65 in. Find the sidewalk length.
16. Mary's age increased by 17 years is 35 years. Find Mary's age.
17. Glenn's bank account balance decreased by $150 is $920. Find Glenn's bank account balance.
18. 75 decreased by a number is 32. Find the number.

19. The sum of Fred's age and 12 years is 90 years. Find Fred's age.

20. The difference between Julie's wage and $230 is $1,050. Find Julie's wage.

21. The difference between $1258 and Julie's wage is $33. Find Julie's wage.

22. Twice the cost of a TV is $1,250. Find the cost of the TV.

23. Four times the cost of a computer system is $5,236. Find the cost of the computer system.

24. $6 more than three times the price of a house is $312,411. Find the price of the house.

25. $140 less than five times the price of a car is the same as $79,250. Find the price of the car.
Chapter 3 Section 2

Word problems with one unknown

Answer Key for Chapter 3 Section 2.

1. the length of a sidewalk = x
   \[ x + 5 = 22 \]
   \[ x = 17 \]
   The sidewalk is 17 ft long.

2. the length of a ribbon = x
   \[ x + 18 = 40 \]
   \[ x = 22 \]
   The ribbon is 22 inches long.

3. John's age = x
   \[ x + 6 = 23 \]
   \[ x = 17 \]
   John is 17 years old.

4. the temperature = x
   \[ x + 8 = -6 \]
   \[ x = -14 \]
   The temperature is $-14^\circ$.

5. a number = x
   \[ x + 7 = -4 \]
   \[ x = -11 \]
   The number is $-11$.

6. Joe's bank account balance = x
   \[ x + 13 = -7 \]
   \[ x = -20 \]
   Joe's bank account balance is $-20$ dollars.

7. the distance to the store = x
   \[ x - 7 = 14 \]
   \[ x = 21 \]
   It is 21 miles to the store.

8. height of a tree = x
   \[ x - 6 = 11 \]
   \[ x = 17 \]
   The height of a tree is 17 feet.

9. a number = x
   \[ x - 9 = 20 \]
   \[ x = 29 \]
   The number is 29.
Chapter 3 Section 2  

Word problems with one unknown

10. cost of dinner = x
    
    $3x = 21$
    $x = 7$

    The dinner cost $7.

11. yesterday's temperature in Omaha = x
    
    $4x = -24$
    $x = -6$

    Yesterday's temperature in Omaha was -6°.

12. number of vacation days = x
    
    $2x + 6 = 32$
    $x = 13$

    The number of vacation days is 13.

13. time to cook oatmeal = x
    
    $3x - 7 = 11$
    $x = 6$

    It takes 6 minutes to cook oatmeal.

14. height of a door = x
    
    $x + 25 = 33$
    $x = 8$

    The height of a door is 8 ft.

15. the sidewalk length = x
    
    $x - 32 = 65$
    $x = 97$

    The sidewalk is 97 inches long.

16. Mary's age = x
    
    $x + 17 = 35$
    $x = 18$

    Mary is 18 years old.

17. Glenn's bank account balance = x
    
    $x - 150 = 920$
    $x = 1,070$

    Glenn's bank account balance is $1,070.

18. a number = x
    
    $75 - x = 32$
    $x = 43$

    The number is 43.
19. Fred's age = x
   x + 12 = 90
   x = 78
   Fred is 78 years old.

20. Julie's wage = x
   x - 230 = 1,050
   x = 1,280
   Julie's wage is $1280.

21. Julie's wage = x
   1258 - x = 33
   x = 1225
   Julie's wage is $1225.

22. cost of the TV = x
   2x = 1,250
   x = 625
   The TV cost $625.

23. cost of a computer system = x
   4x = 5,236
   x = 1,309
   The computer system cost $1309.

24. price of a house = x
   3x + 6 = 312,411
   x = 104,135
   The house cost $104,135.

25. price of a car = x
   5x - 140 = 79,250
   x = 15,878
   The car cost $15,878.
Section 3: Word Problems with Two Unknowns

Many times in algebra, you are finding more than one unknown. When you are required to find two or more unknowns, you need two or more labels for what you don’t know.

Identify your unknowns. The one you know nothing about will be labeled x. The other will be written as an expression in terms of x. Observe.

a. The dog weighs 8 pounds more than the cat.
   
   weight of cat = x  
   weight of dog = x + 8

b. The shrub is 12 feet shorter than the tree.
   
   height of tree = x  
   height of shrub = x - 12

c. A ring costs $12 less than three times the cost of a bracelet.
   
   cost of bracelet = x  
   cost of ring = 3x - 12

Notice that the bracelet was labeled with x only. All the other words in that comparison statement described the cost of the ring. Also notice that the order reversed because of the words “less than.”

The process for setting up and solving word problems with two unknowns includes the following:

1. List your unknowns.
2. Label your unknowns (one with x; the other with an expression built around x).
3. Write an equation by finding a sentence that gives you the equation in words. You may find it helpful to write the equation, using words, and then to replace those words with the algebraic expressions.
4. Now solve. The answer you get is the answer for the unknown you labeled with x.
5. Find the second unknown; substitute the answer into the expression you built that contains x.
6. Then answer the question(s) asked.
d. In a family, the daughter earns $300 more than the son. If together they earn $1,800, how much does each earn?

son’s earnings = x
daughter’s earnings = x + 300

daughter + son = 1800
(x + 300) + (x) = 1800
2x + 300 = 1800
2x = 1500
x = 750

son’s earnings = x = 750
daughter’s earnings = x + 300 = 750 + 300 = 1050

The son earned $750; the daughter earned $1050.

On all the other examples, the solving will be left up to you.

e. Joe and Gary bicycle a combined total of 80 miles. Gary travels 4 miles further than Joe. How many miles does each bicycle?

miles for Joe = x
miles for Gary = x + 4

Joe + Gary = 80
(x) + (x + 4) = 80
Joe’s miles = x = 38 miles
Gary’s miles = x + 4 = 38 + 4 = 42 miles
Joe biked 38 miles; Gary biked 42 miles.

f. Together, an oven and a microwave cost $620. The microwave costs $240 less than the oven. Find the cost of the microwave.

cost of oven = x
cost of microwave = x - 240

oven + microwave = 620
(x) + (x - 240) = 620
oven = x = $430
microwave = x - 240 = 430 - 240 = 190

The microwave costs $190.

Note: You were asked for the cost of the microwave only.
g. A ring costs 3 times as much as a bracelet. If the total cost was $1600, find the cost of each.

\[
\begin{align*}
\text{cost of bracelet} &= x \\
\text{cost of ring} &= 3x
\end{align*}
\]

\[
\begin{align*}
\text{ring} + \text{bracelet} &= 1600 \\
(3x) + (x) &= 1600
\end{align*}
\]

\[
\begin{align*}
\text{bracelet} &= x = $400 \\
\text{ring} &= 3x = 3(400) = $1200
\end{align*}
\]

The bracelet costs $400; the ring costs $1200.

h. Together, a TV and a stereo system cost $3284. The stereo system costs $36 less than three times the TV system. Find the cost of each.

\[
\begin{align*}
\text{cost of TV} &= x \\
\text{cost of stereo system} &= 3x - 36
\end{align*}
\]

\[
\begin{align*}
\text{stereo} + \text{TV} &= 3284 \\
(3x - 36) + (x) &= 3284
\end{align*}
\]

\[
\begin{align*}
\text{TV} &= x = $830 \\
\text{stereo system} &= 3x - 36 = 3(830) - 36 = $2454
\end{align*}
\]

The cost of the TV is $830; the cost of the stereo system is $2454.
Homework for Chapter 3 Section 3

For each of the following, first label the unknowns. Second, set up an equation. Third, solve. Fourth, answer the question(s) asked.

1. In a school election, Mary received 19 more votes than Judy. Together, they received 41 votes. How many votes did each one receive?

2. In a town election, 2977 votes were cast. If Mr. Jones received 931 votes more than Mr. Moore, how many votes did each one receive?

3. Two friends bicycle a combined total of 60 miles. Jessie travels 8 miles further than Judi. How many miles does each person bicycle?

4. A 100-inch ribbon is cut into two pieces. The long piece is 10 inches longer than the short piece. Find the length of each piece.

5. A cat weighs 12 pounds less than a dog. If together they weigh 32 pounds, find the weight of each one.

6. Gloria is 3 inches shorter than Glenda. Together, they measure 67 inches. How tall is each one?

7. 19,784 votes were cast in a county commission race. Ms. Johnson received 552 votes fewer than Ms. Jackson. Find the number of votes each one received.

8. Jay won $739 playing slots and blackjack. If Jay won $151 fewer at slots than at blackjack, find out how much Jay won at each.

9. Together, an oven and a microwave cost $500. The microwave costs $200 less than the oven. Find the cost of each.

10. A sidewalk is twice the length of a driveway. If together they measure 162 feet, find the length of each.

11. One number is four times the other. If the sum of the numbers is 35, find each number.

12. A ring costs 3 times as much as a bracelet. If the total cost was $1600, find the cost of the ring.

13. Together, a car and a house cost $155,225. If the house costs 6 times as much as the car costs, find the cost of each.
14. Penny was looking at her clothes closet. The number of dresses she owned were 18 more than twice the number of suits. Together, the number of dresses and the number of suits totaled 51. Find the number of each that she owned.

15. Mark and Mike had a total of 700 marbles. If Mike had 45 marbles less than 4 times as many as Mark, find the number each one had.

16. A coat costs $28 less than 3 times the cost of a jacket. Together, they cost $176. Find the cost of each.

17. Together, a TV and stereo cost $3177. The stereo costs $225 more than twice the cost of the TV. Find the cost of each.
Answer Key for Chapter 3 Section 3.

1. Judy’s votes = x
   Mary’s votes = x + 19
   Judy + Mary = 41
   (x) + (x + 19) = 41
   Judy had 11 votes; Mary had 30 votes.

2. Mr. Moore’s votes = x
   Mr. Jones’ votes = x + 931
   Mr. Moore + Mr. Jones = 2977
   (x) + (x + 931) = 2977
   Mr. Moore had 1023 votes; Mr. Jones had 1954 votes.

3. miles for Judi = x
   miles for Jessie = x + 8
   Judi + Jessie = 60
   (x) + (x + 8) = 60
   Judi bicycled 26 miles; Jessie bicycled 34 miles.

4. inches for shorter piece = x
   inches for longer piece = x + 10
   shorter + longer = 100
   (x) + (x + 10) = 100
   The shorter piece is 45 inches; the longer piece is 55 inches.

5. dog’s weight = x
   cat’s weight = x - 12
   dog + cat = 32
   (x) + (x - 12) = 32
   The dog weighs 22 pounds; the cat weighs 10 pounds.

6. Glenda’s height = x
   Gloria’s height = x - 3
   Glenda + Gloria = 67
   (x) + (x - 3) = 67
   Glenda is 35 inches tall; Gloria is 32 inches tall.
7. Ms. Jackson’s votes = x  
   Ms. Johnson’s votes = x - 552  
   Ms. Jackson + Ms. Johnson = 19784  
   (x) + (x - 552) = 19784  

   Ms. Jackson had 10168 votes; Ms. Johnson had 9616 votes.

8. winnings at blackjack = x  
   winnings at slots = x - 151  
   blackjack + slots = 739  
   (x) + (x - 151) = 739  

   Jay won $445 at blackjack and $294 at slots.

9. cost of oven = x  
   cost of microwave = x - 200  
   oven + microwave = 500  
   (x) + (x - 200) = 500  

   The oven cost $350; the microwave cost $150.

10. length of driveway = x  
    length of sidewalk = 2x  
    driveway + sidewalk = 162  
    (x) + (2x) = 162  

    The driveway is 54 ft; the sidewalk is 108 ft.

11. other number = x  
    one number = 4x  
    other number + one number = 35  
    (x) + (4x) = 35  

    The other number is 7; one number is 28.

12. cost of bracelet = x  
    cost of ring = 3x  
    bracelet + ring = 1600  
    (x) + (3x) = 1600  

    The ring costs $1200.
13. cost of car = x
   \[6x = \text{cost of house} = 6x\]
   \[\text{car} + \text{house} = 155225\]
   \[(x) + (6x) = 155225\]

   The car costs $22,175; the house costs $133,050.

14. number of suits = x
   number of dresses = 2x + 18
   \[\text{suits} + \text{dresses} = 51\]
   \[(x) + (2x + 18) = 51\]

   Penny has 11 suits and 40 dresses.

15. Mark's marbles = x
    Mike's marbles = 4x - 45
    Mark + Mike = 700
    \[(x) + (4x - 45) = 700\]

    Mark has 149 marbles; Mike has 551 marbles.

16. cost of jacket = x
    cost of coat = 3x - 28
    \[\text{jacket} + \text{coat} = 176\]
    \[(x) + (3x - 28) = 176\]

    The jacket costs $51; the coat costs $125.

17. cost of TV = x
    cost of stereo = 2x + 225
    \[\text{TV} + \text{stereo} = 3177\]
    \[(x) + (2x + 225) = 3177\]

    The TV costs $984; the stereo costs $2193.
Section 4: Solve Equations of Type $ax + b = cx + d$ Using Integers

You have been solving equations and already know the basic steps needed in order to isolate the variable. This section will extend your knowledge of multi-step equations. You add or subtract to get all variable terms on only one side of the equal sign. Then you add or subtract to get all constant terms on the OTHER side of the equal sign. You will finish solving by completely isolating the variable. Observe.

a. Solve vertically:

\[
\begin{align*}
5m + 6 &= 2m + 15 \\
-2m &\quad -2m \quad \text{Subtract variable term 2m from both sides;} \\
3m + 6 &= 15 \quad \text{2m - 2m = 0} \\
-6 &\quad -6 \quad \text{Subtract constant term 6 from both sides;} \\
3m &= 9 \\
\frac{3m}{3} &= \frac{9}{3} \quad \text{Divide by coefficient 3;} \\
m &= 3
\end{align*}
\]

Solve horizontally:

\[
\begin{align*}
5m + 6 &= 2m + 15 \\
5m - 2m + 6 &= 2m - 2m + 15 \\
3m + 6 &= 0 + 15 \\
3m + 6 &= 15 \\
3m + 6 - 6 &= 15 - 6 \\
3m &= 9 \\
\frac{3m}{3} &= \frac{9}{3} \\
m &= 3
\end{align*}
\]

You get the same result!

Check:

\[
\begin{align*}
5m + 6 &= 2m + 15 \\
5(3) + 6 &= 2(3) + 15 \\
15 + 6 &= 6 + 15 \\
21 &= 21 \\
m &= 3 \text{ is the solution.}
\end{align*}
\]
Chapter 3 Section 4  
Solve equations of type $ax + b = cx + d$ using integers

b. \[ 4y - 12 = -2y - 60 \]
\[
\begin{align*}
+ 2y & + 2y \\
6y - 12 &= -60 \\
+ 12 &+ 12 \\
6y &= -48 \\
\frac{6y}{6} &= \frac{-48}{6} \\
y &= -8
\end{align*}
\]
Add variable term $2y$ to both sides; 
$-2y + 2y = 0$ 
Add constant term $12$ to both sides; 
$-12 + 12 = 0$ 
Divide by coefficient $6$; 
$6y \div 6 = 1y = y$

c. \[ -4k - 48 = -k + 81 \]
\[
\begin{align*}
+ 1k &+ k \\
-3k - 48 &= 81 \\
+ 48 &+ 48 \\
-3k &= 129 \\
\frac{-3k}{-3} &= \frac{129}{-3} \\
k &= -43
\end{align*}
\]
Recall: $k$ means $1k$

d. \[ 2m + 24 = 6m - 56 \]
\[
\begin{align*}
-6m &- 6m \\
-4m + 24 &= -56 \\
-24 &- 24 \\
-4m &= -80 \\
\frac{-4m}{-4} &= \frac{-80}{-4} \\
m &= 20
\end{align*}
\]
e. \[ 7m - 16 = 6m - 3 \]
\[
\begin{align*}
-6m &- 6m \\
m - 16 &= -3 \\
+ 16 &+ 16 \\
m &= 13
\end{align*}
\]
Solve equations of type $ax + b = cx + d$ using integers

Chapter 3 Section 4

Simplify each side of the equation independently before you begin solving equations. Observe.

f. \(-k + 25 - 4k = -4k - 35\)

\[
\begin{align*}
-5k + 25 &= -4k - 35 \\
+4k &= +4k \\
-1k + 25 &= -35 \\
-25 &= -25 \\
-1k &= -60
\end{align*}
\]

\[
\frac{-1k}{-1} = \frac{-60}{-1}
\]

\[k = 60\]

Check:
\[-k + 25 - 4k = -4k - 35\]
\[-(60) + 25 - 4(60) = -4(60) - 35\]
\[-60 + 25 - 240 = -240 - 35\]
\[-275 = -275\]

\[k = 60\text{ is the solution.}\]

g. \[2m + 25 + 4m + 81 = 3m + 91 + 7m + 11\]
\[6m + 106 = 10m + 102\]
\[-10m \quad -10m\]
\[-4m + 106 = 102 \\
-106 = -106 \\
-4m = -4 \\
\frac{-4m}{-4} = \frac{-4}{-4}
\]

\[m = 1\]

h. \[16 + 3z + 4 - 5z = -19 - 9z + 3 - 2z\]
\[-2z + 20 = -11z - 16\]
\[-2z + 11z + 20 = -11z + 11z - 16\]
\[9z + 20 = -16\]
\[9z + 20 - 20 = -16 - 20\]
\[9z = -36\]
\[9z + 9 = -36 + 9\]
\[z = -4\]
**Chapter 3 Section 4**

_Solve equations of type ax + b = cx + d using integers_

**Homework for Chapter 3 Section 4**

_Solve._

1. \(5y + 27 = 3y + 89\)
2. \(8m + 16 = 3m - 84\)
3. \(10k - 14 = 4k + 52\)
4. \(9z - 15 = 6z - 87\)
5. \(-11a + 7 = 3a + 35\)
6. \(-12w + 16 = 4w - 96\)
7. \(-2x - 5 = 7x + 67\)
8. \(-5c - 28 = 2c - 21\)
9. \(4m + 13 = -2m + 61\)
10. \(3d - 31 = -4d - 87\)
11. \(-5z - 19 = -7z - 43\)
12. \(-14a + 17 = -4a + 87\)
13. \(m + 15 = 5m + 3\)
14. \(13y - 18 = y + 30\)
15. \(2x + 47 = x + 90\)
16. \(-3m - 80 = -m + 22\)
17. \(2z + 16 + 5z + 28 = 3z - 52\)
18. \(-2A - 3 = 5A + 7 + 3A\)
19. \(6 + 4d - 8 = 18 - 6d - 20 + 14d\)
20. \(8z + 3 + 2z + 12 = 5z + 18 + 2z + 24\)
21. \(3w + 15 + 4w - 10 = -8w + 29 + 13w + 8\)

_Set up and solve the following._

22. $24 more than 3 times the cost of a jacket is $123. Find the cost of the jacket.
23. A coat costs $24 more than 3 times the cost of a jacket. Together, they cost $196. Find the cost of each.
24. Joe is 15 years younger than Mary. Together, the sum of their ages is 21 years. How old is each person?

_Simplify._

25. \(11 + 3 \left( \frac{-35}{-7} \right) - 8^2\)
26. \(\frac{100D + 80}{-4}\)
Chapter 3 Section 4 Solve equations of type $ax + b = cx + d$ using integers

Answer Key for Chapter 3 Section 4.

1. $5y + 27 = 3y + 89$
   $2y + 27 = 89$
   $2y = 62$
   $y = 31$

2. $8m + 16 = 3m - 84$
   $5m + 16 = -84$
   $5m = -100$
   $m = -20$

3. $10k - 14 = 4k + 52$
   $6k - 14 = 52$
   $6k = 66$
   $k = 11$

4. $9z - 15 = 6z - 87$
   $3z - 15 = -87$
   $3z = -72$
   $z = -24$

5. $-11a + 7 = 3a + 35$
   $-14a + 7 = 35$
   $-14a = 28$
   $a = -2$

6. $-12w + 16 = 4w - 96$
   $-16w + 16 = -96$
   $-16w = -112$
   $w = 7$

7. $-2x - 5 = 7x + 67$
   $-9x - 5 = 67$
   $-9x = 72$
   $x = -8$

8. $-5c - 28 = 2c - 21$
   $-7c - 28 = -21$
   $-7c = 7$
   $c = -1$

9. $4m + 13 = -2m + 61$
   $6m + 13 = 61$
   $6m = 48$
   $m = 8$

10. $3d - 31 = -4d - 87$
    $7d - 31 = -87$
    $7d = -56$
    $d = -8$

11. $-5z - 19 = -7z - 43$
    $2z - 19 = -43$
    $2z = -24$
    $z = -12$

12. $-14a + 17 = -4a + 87$
    $-10a + 17 = 87$
    $-10a = 70$
    $a = -7$

13. $m = 3$

14. $y = 4$

15. $x = 43$

16. $m = -51$

17. $2z + 16 + 5z + 28 = 3z - 52$
    $7z + 44 = 3z - 52$
    $4z + 44 = -52$
    $4z = -96$
    $z = -24$
18. \( -2A - 3 = 5A + 7 + 3A \)
\[-2A - 3 = 8A + 7 \]
\[-10A - 3 = 7 \]
\[-10A = 10 \]
\[A = -1 \]

19. \( 6 + 4d - 8 = 18 - 6d - 20 + 14d \)
\[4d - 2 = 8d - 2 \]
\[-4d - 2 = -2 \]
\[-4d = 0 \]
\[d = 0 \]

20. \( 8z + 3 + 2z + 12 = 5z + 18 + 2z + 24 \)
\[10z + 15 = 7z + 42 \]
\[3z = 27 \]
\[z = 9 \]

21. \( 3w + 15 + 4w - 10 = -8w + 29 + 13w + 8 \)
\[7w + 5 = 5w + 37 \]
\[2w + 5 = 37 \]
\[2w = 32 \]
\[w = 16 \]

22. $24 more than 3 times the cost of a jacket is $123. Find the cost of the jacket.

\[
\text{cost of jacket} = x \\
3x + 24 = 123 \\
3x = 99 \\
x = 33
\]

The cost of the jacket is $33.
23. A coat costs $24 more than 3 times the cost of a jacket. Together, they cost $196. Find the cost of each.

\[
\begin{align*}
cost \, of \, jacket &= x \\
cost \, of \, coat &= 3x + 24 \\
\text{jacket} + \text{coat} &= 196 \\
(x) + (3x + 24) &= 196 \\
4x + 24 &= 196 \\
4x &= 172 \\
x &= 43
\end{align*}
\]

cost of jacket = \( x = 43 \)  

\[
\text{cost of coat} = 3x + 24 = 3(43) + 24 = 153
\]

The jacket costs $43; the coat costs $153.

24. Joe is 15 years younger than Mary. Together, the sum of their ages is 21 years. How old is each person?

\[
\begin{align*}
\text{Mary's age} &= x \\
\text{Joe's age} &= x - 15 \\
\text{Mary} + \text{Joe} &= 21 \\
(x) + (x - 15) &= 21 \\
2x - 15 &= 21 \\
2x &= 36 \\
x &= 18
\end{align*}
\]

Mary’s age = \( x = 18 \) years  

Joe’s age = \( x - 15 = 18 - 15 = 3 \) years

Mary is 18 years old; Joe is 3 years old.

25. \( 11 + 3(5) - 64 \)  

26. \( -25D - 20 \)
Section 5: Solve Equations Using Distributive Property and Integers

You are now ready to solve equations in which you have to use the distributive property to remove parentheses. You will use all the techniques for solving that you have previously learned. Observe.

a. \(2(m + 3) = 14\)
   \[2m + 6 = 14\]
   \[-6 \quad -6\]
   \[2m = 8\]
   \[
   \frac{2m}{2} = \frac{8}{2}
   \]
   \[m = 4\]

b. \(3(k - 4) = -51\)
   \[3k - 12 = -51\]
   \[+12 + 12\]
   \[3k = -39\]
   \[
   \frac{3k}{3} = \frac{-39}{3}
   \]
   \[k = -13\]

c. \(4(2y + 6) = -72\)
   \[8y + 24 = -72\]
   \[-24 - 24\]
   \[8y = -96\]
   \[
   \frac{8y}{8} = \frac{-96}{8}
   \]
   \[y = -12\]

d. \(-k - 46 = -3(5k + 6)\)
   \[-1k - 46 = -15k - 18\]
   \[+15k + 15k\]
   \[14k - 46 = -18\]
   \[+46 + 46\]
   \[14k = 28\]
   \[
   \frac{14k}{14} = \frac{28}{14}
   \]
   \[k = 2\]

e. \(-8(4x - 9) = -7(5x + 6)\)
   \[-32x + 72 = -35x - 42\]
   \[+35x + 35x\]
   \[3x + 72 = -42\]
   \[-72 - 72\]
   \[3x = -114\]
   \[
   \frac{3x}{3} = \frac{-114}{3}
   \]
   \[x = -38\]

f. \(3(2y + 9) - 4(5y - 2) = 63\)
   \[6y + 27 - 20y + 8 = 63\]
   \[-14y + 35 = 63\]
   \[+35 + 35\]
   \[14y = 28\]
   \[
   \frac{14y}{14} = \frac{28}{14}
   \]
   \[y = -2\]
Chapter 3 Section 5  
Solve equations using distributive property and integers

\[ g. \quad 6 + 2(4d + 5) = 2(7 + 3d) - 20 \]
\[ \begin{align*}
6 + 8d + 10 &= 14 + 6d - 20 \\
8d + 16 &= 6d - 6 \\
8d - 6d + 16 &= 6d - 6d - 6 \\
2d + 16 &= -6 \\
2d + 16 - 16 &= -6 - 16 \\
2d &= -22 & \text{Divide by 2 to isolate } d \\
d &= -11 \\
\end{align*} \]

Check:
\[ \begin{align*}
6 + 2(4d + 5) &= 2(7 + 3d) - 20 \\
6 + 2[4(-11) + 5] &= 2[7 + 3(-11)] - 20 \\
6 + 2[-44 + 5] &= 2[7 - 33] - 20 \\
6 + 2[-39] &= 2[-26] - 20 \\
6 - 78 &= -52 - 20 \\
-72 &= -72 \\
d &= -11 \text{ is the solution.} \\
\end{align*} \]

\[ h. \quad \text{Twice the sum of a number and 7 is 44. Find the number.} \]
\[ x = \text{the number} \]
\[ 2(x + 7) = 44 \quad \text{Notice the use of parentheses.} \]
\[ 2x + 14 = 44 \]
\[ 2x = 30 \]
\[ x = 15 \]

The number is 15.

\[ i. \quad \text{Three times the difference between a number and 4 is 66. Find the number.} \]
\[ x = \text{the number} \]
\[ 3(x - 4) = 66 \]
\[ 3x - 12 = 66 \]
\[ 3x = 78 \]
\[ x = 26 \]

The number is 26.

Notice the word problems used parentheses and required distribution to solve.
Homework for Chapter 3 Section 5

Solve.

1. \(5(x + 3) = 40\)
2. \(6(y - 7) = 24\)
3. \(-2(z + 7) = -56\)
4. \(-3(a - 8) = 33\)
5. \(-4(d - 9) = -96\)
6. \(4(2f + 6) = 8\)
7. \(y + 26 + y = 3(5y + 13)\)
8. \(6(9y + 5) = 3(17y + 4)\)
9. \(2(4m - 8) = -2(-7 + m)\)
10. \(-3(5B - 3) = 2(3B + 15)\)
11. \(-5(6y + 3) = -7(4y + 5)\)
12. \(3m + 2(7m + 5) = 8(2m - 3)\)
13. \(5 - 2(3c + 6) = 2c + 17 + 4c\)
14. \(9(4z - 3) - 5(7z + 8) = -68\)
15. \(2(6T - 3) + 4(5T + 6) = 146\)
16. \(8(3m + 4) - 2(4m - 8) = 4(3m + 2)\)
Set up and solve.

17. The price of a ticket increased by $21 is $72. Find the price of the ticket.

18. 18 less than 3 times Mike’s age is 99. How old is Mike?

19. Fido weighs 8 pounds more than Fluffy. Together, they weigh 18 pounds. Find the weight of each.

20. Together, Glenn and David bikes 48 miles. Glenn biked 12 miles less than twice what David biked. How far did each one bike?

21. Four times the sum of a number and 5 is 96. Find the number.

22. Five times the difference between a number and 11 is 50. Find the number.

Evaluate.

23. $7k - 35$ for $k = 2$

24. $-3z - 20$ when $z = -4$

25. $x + y$ when $x = 3$ and $y = -2$

26. $2x - y$ for $x = 3$ and $y = 12$

Simplify.

27. $-12 ÷ 3 + 7(-8 - 2)$

28. $\left(\frac{6}{2}\right)\left(-\frac{18}{9}\right) - 2^4$

29. $-3(4x - 8)$

30. $(x + 5) - (-2x + 6)$

31. $-13 - 3(2m + 4) + 9m$

32. $\frac{-44F + 28}{-4}$
Answer Key for Chapter 3 Section 5.

1. \(5(x + 3) = 40\)
   \[5x + 15 = 40\]
   \[5x = 25\]
   \[x = 5\]

2. \(6(y - 7) = 24\)
   \[6y - 42 = 24\]
   \[6y = 66\]
   \[y = 11\]

3. \(-2(z + 7) = -56\)
   \[-2z - 14 = -56\]
   \[-2z = -42\]
   \[z = 21\]

4. \(-3(a - 8) = 33\)
   \[-3a + 24 = 33\]
   \[-3a = 9\]
   \[a = -3\]

5. \(-4(d - 9) = -96\)
   \[-4d + 36 = -96\]
   \[-4d = -132\]
   \[d = 33\]

6. \(4(2f + 6) = 8\)
   \[8f + 24 = 8\]
   \[8f = -16\]
   \[f = -2\]

7. \(y + 26 + y = 3(5y + 13)\)
   \[2y + 26 = 15y + 39\]
   \[-13y + 26 = 39\]
   \[-13y = 13\]
   \[y = -1\]

8. \(6(9y + 5) = 3(17y + 4)\)
   \[54y + 30 = 51y + 12\]
   \[3y + 30 = 12\]
   \[3y = -18\]
   \[y = -6\]
Chapter 3 Section 5  Solve equations using distributive property and integers

9. $2(4m - 8) = -2(-7 + m)$
   $8m - 16 = 14 - 2m$
   $10m - 16 = 14$
   $10m = 30$
   $m = 3$

10. $-3(5B - 3) = 2(3B + 15)$
    $-15B + 9 = 6B + 30$
    $-21B + 9 = 30$
    $-21B = 21$
    $B = -1$

11. $-5(6y + 3) = -7(4y + 5)$
    $-30y - 15 = -28y - 35$
    $-2y - 15 = -35$
    $-2y = -20$
    $y = 10$

12. $3m + 2(7m + 5) = 8(2m - 3)$
    $3m + 14m + 10 = 16m - 24$
    $17m + 10 = 16m - 24$
    $m + 10 = -24$
    $m = -34$

13. $5 - 2(3c + 6) = 2c + 17 + 4c$
    $5 - 6c - 12 = 6c + 17$
    $-6c - 7 = 6c + 17$
    $-12c - 7 = 17$
    $-12c = 24$
    $c = -2$

14. $9(4z - 3) - 5(7z + 8) = -68$
    $36z - 27 - 35z - 40 = -68$
    $z - 67 = -68$
    $z = -1$

15. $2(6T - 3) + 4(5T + 6) = 146$
    $12T - 6 + 20T + 24 = 146$
    $32T + 18 = 146$
    $32T = 128$
    $T = 4$
16. \[8(3m + 4) - 2(4m - 8) = 4(3m + 2)\]
\[24m + 32 - 8m + 16 = 12m + 8\]
\[16m + 48 = 12m + 8\]
\[4m + 48 = 8\]
\[4m = -40\]
\[m = -10\]

17. The price of a ticket increased by $21 is $72. Find the price of the ticket.

price of ticket = \(x\)
\[x + 21 = 72\]
\[x = 51\]

The price of the ticket is $51.

18. 18 less than 3 times Mike’s age is 99. How old is Mike?

Mike’s age = \(x\)
\[3x - 18 = 99\]
\[3x = 117\]
\[x = 39\]

Mike is 39 years old.

19. Fido weighs 8 pounds more than Fluffy. Together, they weigh 18 pounds. Find the weight of each.

Fluffy’s weight = \(x\)
Fido’s weight = \(x + 8\)
Fluffy + Fido = 18
\[(x) + (x + 8) = 18\]
\[2x + 8 = 18\]
\[2x = 10\]
\[x = 5\]

Fluffy’s weight = \(x = 5\) pounds
Fido’s weight = \(x + 8 = 5 + 8 = 13\) pounds

Fluffy weighs 5 pounds; Fido weighs 13 pounds.
20. Together, Glenn and David bikes 48 miles. Glenn biked 12 miles less than twice what David biked. How far did each one bike?

David’s miles = \( x \)
Glenn’s miles = \( 2x - 12 \)
David + Glenn = 48

\[
(x) + (2x - 12) = 48 \\
3x - 12 = 48 \\
3x = 60 \\
x = 20
\]

David’s miles = \( x = 20 \) miles
Glenn’s mile = \( 2x - 12 = 2(20) - 12 = 28 \) miles

David biked 20 miles; Glenn biked 28 miles.

21. Four times the sum of a number and 5 is 96. Find the number.

the number = \( x \)

\[
4(x + 5) = 96 \\
4x + 20 = 96 \\
4x = 76 \\
x = 19
\]

The number is 19.

22. Five times the difference between a number and 11 is 50. Find the number.

the number = \( x \)

\[
5(x - 11) = 50 \\
5x - 55 = 50 \\
5x = 105 \\
x = 21
\]

The number is 21.

23. \(-21\)  
24. \(-8\)
25. \(1\)  
26. \(-6\)
27. \(-74\)  
28. \(-22\)
29. \(-12x + 24\)  
30. \(3x - 1\)
31. \(3m - 25\)  
32. \(11F - 7\)
Chapter 3 Summary

Translate words to algebra

Label the unknown; what you read in English you write in algebra symbols; remember “more than” and “less than” reverse order

The weight of a dog increased by 3 pounds
weight of dog = x
x + 3

7 feet more than the height of a tree
height of tree = x
x + 7

$8 less than twice the cost of a radio
cost of radio = x
2x - 8

One and Two Unknowns

Label the unknown with a variable (and for two unknowns, a variable and an expression containing a variable)

Write an equation
Solve
Answer the question(s) asked

a. The sum of Joe’s bank balance and $307 is $940. Find Joe’s bank balance.

Joe’s bank balance = x

x + 307 = 940
x = 633

Joe’s bank balance is $633
One and Two Unknowns  

b. A dog weighs 3 pounds more than a cat. Together they weigh 13 pounds. Find the weight of each.

weight of cat = x  
weight of dog = x + 3

(x) + (x + 3) = 13  
x = 5

The cat weighs 5 pounds; the dog weighs 8 pounds.

Solve equations  
a.  

\[ 7x + 12 = 10x - 33 \]

\[ -10x \hspace{1cm} -10x \]
\[ -3x + 12 = -33 \]
\[ -12 \hspace{1cm} -12 \]

\[ -3x = -45 \]
\[ -3 \hspace{1cm} -3 \]

\[ x = 15 \]

b.  

\[ 4(2x - 5) = -28 \]

\[ 8x - 20 = -28 \]
\[ + 20 \hspace{1cm} + 20 \]

\[ 8x = -8 \]
\[ 8 \hspace{1cm} 8 \]

\[ x = -1 \]

c. Recall, you should also check your solutions.
Chapter 3 Review

For each of the following, first label the unknown. Second, translate the English words into math symbols.

1. Triple the ounces of ice cream
2. The number of tickets divided by four
3. A number subtracted from 28
4. 8 more than the quotient of a number and 5

For each of the following, first label the unknowns. Second, set up an equation. Third, solve. Fourth, answer the question(s) asked.

5. 32° more than the low temperature for the day in International Falls, Minnesota, was the high temperature of \(-3°\). Find the low temperature for the day.
6. A number multiplied by \(-3\) equals 9. Find the number.
7. 18 dollars more than the cost of 3 tickets is $105. Find the cost of one ticket.
8. A gambler won $7 on slot machines. Her total win on slots and blackjack was \(-$29\). What was the win on blackjack? (Hint: a loss is a negative win.)
9. Nancy’s first two test scores totaled 163. If the second test score was 11 more than the first, find the score of the second test.
10. A car traveled 782 miles on two tanks of gas. If the car traveled 38 miles less on the second tank than the first, find the miles traveled on each tank.
11. A farmer uses 4 times as much fertilizer A as fertilizer B on a field. If he used a total of 1790 pounds of the two fertilizers, how many pounds of fertilizer A did he use?
12. A student finds that one certain book costs $18 more than three times another. If the two books cost $126 together, find the cost of the cheaper book.
13. A play production requires the length of a red streamer to be 32 feet shorter than 3 times the length of a gold streamer. Find the length of the red streamer if the total length of the two streamers is 200 feet.
Solve.

14. \[3A + 9 = 2A - 8\]
15. \[4x + 13 = -3x - 15\]
16. \[-5z + 14 = -3z + 2\]
17. \[3b + 8 - 2b - 6 = -3b + 4 + b - 2\]
18. \[18y - 3 - 5y + 10 = -2y - 13 + 6y + 2\]
19. \[-3(X - 8) = -27\]
20. \[k + 33 + k = 3(k - 11)\]
21. \[6r + 3(-2r + 5) = 3(r - 2)\]
22. \[-2(y + 3) - y = 2(2y + 4)\]
23. \[2(3x - 5) - 5(x - 3) = -18\]

Set up and solve.

24. Five times the sum of twice a number and 8 is the same as \(-30\). Find the number.

25. Three times the difference between the price of a ticket and $14 equals $45. Find the price of the ticket.
Answer Key for Chapter 3 Review.

1. ounces of ice cream = \( x \)  
   \[ 3x \]

2. number of tickets = \( x \)  
   \[ \frac{x}{4} \]

3. the number = \( x \)  
   \[ 28 - x \]

4. the number = \( x \)  
   \[ \frac{x}{5} + 8 \]

5. low temperature for the day = \( x \)  
   \[ x + 32 = -3 \]  
   \[ x = -35 \]  
   The low temperature for the day was \(-35^\circ\).

6. the number = \( x \)  
   \[ x (-3) = 9 \]  
   \[ x = -3 \]  
   The number equals \(-3\).

7. the cost of one ticket = \( x \)  
   \[ 3x + 18 = 105 \]  
   \[ x = 29 \]  
   The cost of one ticket is $29.

8. the win on blackjack = \( x \)  
   \[ 7 + x = -29 \]  
   \[ x = -36 \]  
   The gambler’s win on blackjack was \(-36\) (i.e., a $36 loss)
9. first test score $= x$
   $x + 11 = \text{second test score} = x + 11$
   $(x) + (x + 11) = 163$
   $x = 76$
   The second test score $(x + 11)$ was 87.

10. number of miles traveled on first tank of gas $= x$
    number of miles traveled on second tank of gas $= x - 38$
    $(x) + (x - 38) = 782$
    $x = 410$
    The car traveled 410 miles on the first tank of gas; the car traveled 372 miles on the second tank of gas.

11. pound of fertilizer B used on the field $= x$
    pounds of fertilizer A used on the field $= 4x$
    $(4x) + (x) = 1790$
    $x = 358$
    The farmer used 1432 pounds of fertilizer A on the field.

12. cost of the cheaper book $= x$
    cost of more expensive book $= 3x + 18$
    $x + (3x + 18) = 126$
    $x = 27$
    The cost of the cheaper book is $27.

13. length of gold streamer $= x$
    length of red streamer $= 3x - 32$
    $(x) + (3x - 32) = 200$
    $x = 58$
    The length of the red streamer $(3x - 32)$ is 142 feet.

14. $A = -17$

15. $x = -4$

16. $z = 6$

17. $b = 0$

18. $y = -2$
19. $X = 17$

20. $k = 66$

21. $r = 7$

22. $y = -2$

23. $x = -23$

24. the number = $x$
   
   $5(2x + 8) = -30$
   
   $x = -7$
   
   The number is $-7$.

25. price of a ticket = $x$
    
    $3(x - 14) = 45$
    
    $x = 29$
    
    The price of a ticket is $29$. 
Chapter 3 Test

For each of the following, first label the unknown. Second, translate the English words into math symbols.

1. 5 less than a number
2. Twice the number of people
3. The quotient of a number and 6
4. 6 more than the product of 2 and a number

For each of the following, first label the unknowns. Second, set up an equation. Third, solve. Fourth, answer the question(s) asked.

5. The product of 3 and a number equals –12. Find the number.
6. Water boils at a temperature that is 180°F more than the temperature at which it freezes. If water boils at 212°F, find the temperature at which it freezes.
7. Joe’s golf score was –8 (8 below par). Joe and Fred’s scores combined totaled 3. What was Fred’s score?
8. My new car costs $400 more than 3 times the amount I got for my old car. My new car cost $19,300. How much did I get for my old car?
9. Joan made 2 deposits totaling $113 this month. Her deposit on the 15th was $25 more than her deposit on the 30th. Find the amount of the deposits for each date.
10. My cousin and I bought the same leather jacket at different stores. The jacket I bought cost $150 less than twice the cost of my cousin’s. Together, we spent $330. How much did we each pay for our jackets?
11. One piece of wood for my project must be 11 inches longer than three times the length of the other piece. Together, they measure 27 inches. How long is each piece?
Solve.

12. \(4x - 3 = 3x + 7\)
13. \(-7x - 3 = -5x - 3\)
14. \(-2y + 5 = -5y - 4\)
15. \(6x - 5 + 2x - 7 = 5x + 3 - 2x + 10\)
16. \(2(x + 3) = 2\)
17. \(y + 13 - 4y = 2(5y - 13)\)
18. \(8x + 5(-2x + 3) = 3x + 25\)
19. \(2(11a + 6) = 5(5a - 9)\)
20. \(4(z - 6) - 3(z + 7) = -10\)
21. \(7y - 2(2y + 5) = 5(y - 4)\)

Set up and solve.

22. Twice the sum of a number and 12 is 96. Find the number.
23. Three times the difference between a number and 9 is 45. Find the number.
Answer Key for Chapter 3 Test.

1. the number = \( x \)
   \( x - 5 \)

2. the number of people = \( x \)
   \( 2x \)

3. the number = \( x \)
   \( \frac{x}{6} \)

4. the number = \( x \)
   \( 2x + 6 \)

5. the number = \( x \)
   \( 3x = -12 \)
   \( x = -4 \)
   The number is -4.

6. temperature at which water freezes = \( x \)
   \( 212 = x + 180 \)
   \( 32 = x \) or \( x = 32 \)
   Water freezes at 32° F.

7. Fred’s score = \( x \)
   \(-8 + x = 3 \)
   \( x = 11 \)
   Fred’s score was 11.

8. old car amount = \( x \)
   \( 3x + 400 = 19300 \)
   \( x = 6300 \)
   The old car amount was $6,300.

9. deposit on 30th = \( x \)
deposit on 15th = \( x + 25 \)
   \((x) + (x + 25) = 113 \)
   \( x = 44 \)
   $69 was deposited on the 15th; $44 was deposited on the 30th.

10. cost of cousin’s jacket = \( x \)
cost of my jacket = \( 2x - 150 \)
    \((x) + (2x - 150) = 330 \)
    \( x = 160 \)
    My cousin’s jacket was $160; my jacket was $170.
11. first piece = x  
   second piece = 3x + 11  
   (x) + (3x + 11) = 27  
   x = 4  
   The first piece is 4 inches long; the second piece is 23 inches long.

12. x = 10  
13. x = 0  
14. y = -3  
15. x = 5  
16. x = -2  
17. y = 3  
18. x = -2  
19. a = 19  
20. z = 35  
21. y = 5  
22. number = x  
   2(x + 12) = 96  
   x = 36  
   The number is 36.

23. number = x  
   3(x - 9) = 45  
   x = 24  
   The number is 24.
Cumulative Review #3 Through Chapter 3

1. Compare, using the appropriate symbol (< or >): \(-14 \) \(3\)

Evaluate.

2. \(5x - 3\) when \(x = 4\)
3. \(-4b + 8\) for \(b = -2\)

Simplify.

4. \(14 \div 2 - 2\)
5. \(-14 + 2 - (-11) + (-3) - 7\)
6. \(-4x - 3x + (-1)\)
7. \(3(x + 8) - 5x\)
8. a. \((-1)^{10}\) b. \(-1^{10}\) c. \((-9)^2\) d. \(-9^2\)
9. \(-3 + |22 - 24|\)
10. \(\frac{12m + 3}{-3} + 6m + 8 - (3m - 2)\)
11. \(-56 \div (-7) + 3 \cdot (-4)\)
12. \(-32 \div 4 - 6(-4 - 5) - \left(\frac{-16}{4}\right)\)

Solve.

13. \(4x - 8 = 32\)
14. \(6x = 21 + 3x\)
15. \(3x - 18 = -12\)
Solve.

16. \(-3x - 8 + x = -2\)
17. \(4k + 19 - 6k = -18 + 31\)
18. \(5y = 6y + 48 - 9y\)
19. \(18w - 13 = 5w\)
20. \(4x - 3 = 3x - 8\)
21. \(3a - 4 + 2a - 3 = -4a + 11\)
22. \(-2(r - 6) = -14\)
23. \(5(x + 10) = 3(7 + 2x)\)
24. \(-2(x + 3) - 3(x - 2) = -35\)
25. \(4K + 2(-K - 7) = -8\)

26. First, label the unknown. Second, translate the English words into math symbols.

The difference between a number and \(-7\)

For each of the following, first label the unknowns. Second, set up an equation. Third, solve. Fourth, answer the question(s) asked.

27. An number multiplied by \(-8\) equals 72. Find the number.

28. An adult ticket costs $13 more than a child’s. If the cost for the sum of a child’s and an adult’s ticket is $41, find the cost of each ticket.

29. The total number of hours Betsy worked in two weeks was 83. If she worked 11 more hours the first week than the second, how many hours did she work in the second week?

30. The cost of a dress was $13 more than twice the cost of a skirt. If Josephine paid $139 for both the dress and the skirt, how much did she pay for the dress?
### Answer Key for Cumulative Review #3 through Chapter 3.

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
<th>Reference Chapter</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>&gt;</td>
<td>2a</td>
</tr>
<tr>
<td>2.</td>
<td>17</td>
<td>1</td>
</tr>
<tr>
<td>3.</td>
<td>16</td>
<td>2b</td>
</tr>
<tr>
<td>4.</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>5.</td>
<td>−11</td>
<td>2a</td>
</tr>
<tr>
<td>6.</td>
<td>−7x − 1</td>
<td>2a</td>
</tr>
<tr>
<td>7.</td>
<td>−2x + 24</td>
<td>2a</td>
</tr>
<tr>
<td>8. a.</td>
<td>1</td>
<td>2b</td>
</tr>
<tr>
<td></td>
<td>b. −1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>c. 81</td>
<td></td>
</tr>
<tr>
<td></td>
<td>d. −81</td>
<td></td>
</tr>
<tr>
<td>9.</td>
<td>−1</td>
<td>2b</td>
</tr>
<tr>
<td>10.</td>
<td>−m+ 9</td>
<td>2b</td>
</tr>
<tr>
<td>11.</td>
<td>−4</td>
<td>2b</td>
</tr>
<tr>
<td>12.</td>
<td>50</td>
<td>2b</td>
</tr>
<tr>
<td>13.</td>
<td>x = 10</td>
<td>1</td>
</tr>
<tr>
<td>14.</td>
<td>x = 7</td>
<td>1</td>
</tr>
<tr>
<td>15.</td>
<td>x = 2</td>
<td>2a</td>
</tr>
<tr>
<td>16.</td>
<td>x = −3</td>
<td>2b</td>
</tr>
<tr>
<td>17.</td>
<td>k = 3</td>
<td>2b</td>
</tr>
<tr>
<td>18.</td>
<td>y = 6</td>
<td>2b</td>
</tr>
<tr>
<td>19.</td>
<td>w = 1</td>
<td>2b</td>
</tr>
</tbody>
</table>
20. \( x = -5 \)  

21. \( a = 2 \)  

22. \( r = 13 \)  

23. \( x = 29 \)  

24. \( x = 7 \)  

25. \( K = 3 \)  

26. the number = \( x \)  
   \[ x - (-7) \]  

27. the number = \( x \)  
   \[ x(-8) = 72 \]  
   \[ x = -9 \]  
   The number is -9.  

28. cost of child’s ticket = \( x \)  
   cost of adult’s ticket = \( x + 13 \)  
   \[ (x) + (x + 13) = 41 \]  
   \[ x = 14 \]  
   \[ x + 13 = 14 + 13 = 27 \]  
   The cost of the child’s ticket is $14; the cost of the adult’s ticket is $27.  

29. hours worked during 2\(^{nd}\) week = \( x \)  
   hours worked during 1\(^{st}\) week = \( x + 11 \)  
   \[ (x) + (x + 11) = 83 \]  
   \[ x = 36 \]  
   Betsy worked 36 hours during the 2\(^{nd}\) week.  

30. cost of skirt = \( x \)  
   cost of dress = \( 2x + 13 \)  
   \[ (x) + (2x + 13) = 139 \]  
   \[ x = 42 \]  
   dress = \( 2x + 13 = 2(42) + 13 = 97 \)  
   The cost of the dress was $97.
Chapter 4 presents ideas about fractions.

The first several sections will present concepts about simplifying: how to reduce a fraction to lowest terms; how to multiply, divide, add, and subtract fractions; and how to multiply, divide, add, and subtract mixed numbers. When you learn about simplifying, you will briefly review arithmetic of fractions; the main focus of this chapter will be on the algebra of fractions so you will continue to integrate the ideas about positive and negative numbers and about variables.

The last section of this chapter will talk about solving equations with fractions. Some of the equations will have results which are fractions while some of the equations will have coefficients and/or constants which are fractions.

Your goal is to become very comfortable with fractions, along with perfecting your skills with algebra.
Section 1: Reduce Algebraic Fractions

Before you begin to reduce algebraic fractions to lowest terms, review the following ideas. First, remember \[ \frac{2}{3} \] means numerator divided by denominator: \( 2 \div 3 \).

Second, to reduce fractions to lowest terms, you need to review factors. Factors are numbers that multiply to give a product. Usually, you have the product and are asked to list the factors of the number. Observe.

a. List the factors of 18

\[ 18 = 1 \cdot 18 \]
\[ 2 \cdot 9 \]
\[ 3 \cdot 6 \]

b. List the factors of 48

\[ 48 = 1 \cdot 48 \]
\[ 2 \cdot 24 \]
\[ 3 \cdot 16 \]
\[ 4 \cdot 12 \]
\[ 6 \cdot 8 \]

How do you find the complete list of factors of a number? In general, you divide the number you are given by 1, 2, 3, 4, 5, 6, and so on. The numbers that divide into the given number evenly are the factors of that number. You continue dividing until you reach a pair of numbers that you already have listed. For example, when you factor 18, you think

\[ 18 \div 1 = 18 \text{ so } 18 = 1 \cdot 18 \]
\[ 18 \div 2 = 9 \text{ so } 18 = 2 \cdot 9 \]
\[ 18 \div 3 = 6 \text{ so } 18 = 3 \cdot 6 \]

18 doesn’t divide by 4
18 doesn’t divide by 5
18 \div 6 = 3 \text{ so } 18 = 6 \cdot 3

However, you already have 3 \cdot 6 on the list which is equivalent to 6 \cdot 3 so you can quit trying to find factors.

Why might you want to know what the factors of a number are? One of the reasons is to use the factors to help you reduce fractions. For example, suppose you want to reduce \( \frac{32}{48} \). If you know the factors of both the numerator and denominator, that can help you decide how to reduce to lowest terms.

Third, remember that to reduce a fraction to lowest terms, you divide both numerator and denominator by the same factor (usually the greatest factor that divides into both the numerator and the denominator). Using your multiplication table and remembering factors can help with this. If you can’t find the greatest factor, find ANY factor that divides into both the numerator and denominator and keep dividing by a common factor until you cannot divide the numerator and denominator by the same factor (other than 1) any longer. Observe.
c. \( \frac{32}{48} \)  

One way to reduce is to factor each number. Then look for the largest factor that occurs on both lists.

\[
32 = \frac{1 \cdot 32}{2 \cdot 16} = \frac{2 \cdot 16}{4 \cdot 8} = \frac{4 \cdot 8}{6 \cdot 8} \\
48 = \frac{1 \cdot 48}{2 \cdot 24} = \frac{3 \cdot 16}{4 \cdot 12} 
\]

What is the largest number that occurs on both lists? Sixteen is the largest common factor (also called the greatest common factor). How can you use this fact to reduce to lowest terms? There are a couple of ways. Observe.

\[
\frac{32}{48} = \frac{2 \cdot 16}{3 \cdot 16} = \frac{2}{3} \quad \text{or} \quad \frac{32}{48} = \frac{32 \div 16}{48 \div 16} = \frac{2}{3}
\]

Notice you get the same answer either way you show this. Now look at a few more examples of ways to reduce to lowest terms.

As you are beginning to think about reducing fractions to lowest terms, it might be helpful for you to review the rules of divisibility.

**A number is divisible by** if the number is an even number (it ends in 0, 2, 4, 6, 8)

- \( 2 \)

\[
\text{ex: } \ 48 \text{ divides by 2 because 48 ends in 8} \quad \text{ex: } \ 94 \text{ divides by 2 because 94 ends in 4}
\]

**3** if the digits add up to a number that’s divisible by 3

\[
\text{ex: } \ 36 \text{ divides by 3 because the digits 3 and 6 add to 9, which divides by 3} \quad \text{ex: } \ 156 \text{ divides by 3 because the digits 1, 5, and 6 add to 12, which divides by 3} \quad \text{ex: } \ 87 \text{ divides by 3 because the digits 8 and 7 add to 15, which divides by 3}
\]
A number is divisible by if the number

<table>
<thead>
<tr>
<th>Divisible by</th>
<th>Condition</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>4</strong></td>
<td>if the last two digits of the number divide by 4</td>
<td>124 divides by 4 because 24 divides by 4&lt;br&gt;316 divides by 4 because 16 divides by 4&lt;br&gt;1348 divides by 4 because 48 divides by 4</td>
</tr>
<tr>
<td><strong>5</strong></td>
<td>if the number ends in 0 or 5</td>
<td>65 divides by 5 because 65 ends in 5&lt;br&gt;170 divides by 5 because 170 ends in 0</td>
</tr>
<tr>
<td><strong>6</strong></td>
<td>if number is even AND the digits add up to a number that divides by 3</td>
<td>126 divides by 6 because 126 is even AND 1+2+6 = 9, which divides by 3&lt;br&gt;234 divides by 6 because 123 is even AND 2+3+4 = 9, which divides by 3</td>
</tr>
<tr>
<td><strong>7</strong></td>
<td>no easy test; divide the number by 7 to see if 7 works</td>
<td>98 divides by 7 because 98 ÷ 7 = 14</td>
</tr>
<tr>
<td><strong>8</strong></td>
<td>if the last three digits of the number divide by 8</td>
<td>3112 divides by 8 because 112 divides by 8</td>
</tr>
<tr>
<td><strong>9</strong></td>
<td>if the digits add up to a number that divides by 9</td>
<td>126 divides by 9 because 1+2+6 = 9, which divides by 9&lt;br&gt;783 divides by 9 because 7+8+3 = 18, which divides by 9</td>
</tr>
<tr>
<td><strong>10</strong></td>
<td>if the number ends in 0</td>
<td>140 ends in 0 so 140 divides by 10</td>
</tr>
</tbody>
</table>

How do you use this information? Many times, you use it to help you reduce fractions to lowest terms. Observe.
d. \[ \frac{248}{372} = \frac{248 \div 4}{372 \div 4} = \frac{62}{93} \]

these are both even numbers so both divide by 2
2+ 4+ 8= 14, which doesn’t divide by 3 so 248 doesn’t divide by 3 so you
don’t need to check whether 372 divides by 3
248 ends in 48, which divides by 4 so 248 divides by 4
372 ends in 72, which divides by 4 so 372 divides by 4
Begin by dividing both numerator and denominator by 4.

How do you check to see if \( \frac{62}{93} \) reduces further? You could factor 62 and
factor 93 to see if there are any common factors: \( \frac{62}{93} = \frac{2 \cdot 31}{3 \cdot 31} \). Since 31 is a
common factor to 62 and 93, the fraction will reduce further! \( \frac{62}{93} = \frac{2}{3} \)

Notice that you may use several techniques in one problem.

e. \[ \frac{36}{57} = \frac{36 + 3}{57 + 3} = \frac{12}{19} \]

3+ 6 = 9, which divides by 3 so 36 divides by 3
5+ 7 = 12, which divides by 3 so 57 divides by 3
Begin by dividing both numerator and denominator by 3.

Observe the following to see the various ways you might reduce a fraction to
lowest terms.

f. \[ \frac{42}{56} = \frac{42 \div 2}{56 \div 2} = \frac{21}{28} = \frac{3}{4} \]

OR

\[ \frac{42}{56} = \frac{3 \cdot 14}{4 \cdot 14} = \frac{3}{4} \]

OR

\[ \frac{42}{56} = \frac{42 \div 14}{56 \div 14} = \frac{3}{4} \]
Chapter 4 Section 1

Reduce algebraic fractions

Another method of reducing a fraction to lowest terms is to rewrite the numerator and rewrite the denominator by factoring each into its primes. Prime numbers include 2, 3, 5, 7, 11, 13, etc. Each prime number is divisible only by 1 and itself. Once all the factoring has occurred, you then reduce identical factors in the numerator and in the denominator. Observe.

\[
g. \quad \frac{27}{72} = \frac{27 \div 9}{72 \div 9} = \frac{3}{8}
\]

\[
h. \quad \frac{48}{60} = \frac{48 \div 12}{60 \div 12} = \frac{4}{5}
\]

How would you reduce the following to lowest terms? Observe.

\[
i. \quad \frac{24}{28} = \frac{2 \cdot 2 \cdot 2 \cdot 3}{2 \cdot 2 \cdot 7} = \frac{2 \cdot 3}{7} = \frac{6}{7}
\]

\[
j. \quad \frac{48}{112} = \frac{2 \cdot 2 \cdot 2 \cdot 2 \cdot 3}{2 \cdot 2 \cdot 2 \cdot 7} = \frac{3}{7}
\]

\[
k. \quad \frac{30}{165} = \frac{2 \cdot 3 \cdot 5}{3 \cdot 5 \cdot 11} = \frac{2}{11}
\]

l. \[
\frac{-14}{21} \quad \text{divide with one negative; result is negative}
\]

\[
\frac{-14}{21} = \frac{-14 \div 7}{21 \div 7} = \frac{-2}{3}
\]

OR

\[
\frac{-14}{21} = \frac{-2 \cdot 7}{3 \cdot 7} = \frac{-2}{3}
\]

m. \[
\frac{-27}{63} \quad \text{divide with two negatives; result is positive}
\]

\[
\frac{27}{63} = \frac{27 \div 9}{63 \div 9} = \frac{3}{7} \quad \text{OR} \quad \frac{3 \cdot 3 \cdot 3}{3 \cdot 3 \cdot 7} = \frac{3}{7}
\]
Recall \( \frac{2}{2} = 1 \). For the same reason \( \frac{x}{x} = 1 \). Using this fact, you can also reduce fractions to lowest terms when the fractions contain variables. Observe the following. You will notice the use of the “understood 1” coefficient on some of these.

n. \( \frac{x^5}{x^2} = \frac{1 \cdot x \cdot x \cdot x \cdot x}{1 \cdot x \cdot x} = \frac{1}{1} = x^3 \)

o. \( \frac{m^7}{m^5} = \frac{1 \cdot m \cdot m \cdot m \cdot m}{1 \cdot m \cdot m \cdot m \cdot m} = \frac{1}{1} = 1 \)

p. \( \frac{y^4z^2}{y^2z^5} = \frac{y \cdot y \cdot y \cdot y \cdot z \cdot z}{y \cdot y \cdot z \cdot z \cdot z \cdot z} = \frac{y}{z \cdot z \cdot z} = \frac{y}{z^3} \)

Now you can reduce the following to lowest terms by integrating all ideas into one fraction. Observe.

q. \( \frac{-25a^7b^3}{-35ab^5} \)

\[ + \frac{5 \cdot 5 \cdot a \cdot a \cdot a \cdot a \cdot b \cdot b \cdot b}{5 \cdot 7 \cdot a \cdot b \cdot b \cdot b \cdot b} = \frac{5 \cdot a \cdot a \cdot a \cdot a}{7 \cdot b \cdot b} \]

\[ = \frac{5a^6}{7b^2} \]

One final observation. There are times when the simplifying involves only the signs of the numbers.

r. \( \frac{12}{-25} \)

\[ = -\frac{12}{25} \]
Chapter 4 Section 1

Reduce algebraic fractions

Homework for Chapter 4 Section 1

Reduce the following to lowest terms.

1. \( \frac{12}{21} \)  
2. \( \frac{56}{72} \)  
3. \( \frac{22}{39} \)

4. \( \frac{25}{70} \)  
5. \( \frac{36}{84} \)  
6. \( \frac{-24}{33} \)

7. \( \frac{-51}{-57} \)  
8. \( \frac{27}{-39} \)  
9. \( \frac{-10}{17} \)

10. \( \frac{-48}{96} \)  
11. \( \frac{-15}{-44} \)  
12. \( \frac{k^7}{k^5} \)

13. \( \frac{m^4}{m} \)  
14. \( \frac{z^3}{z^8} \)  
15. \( \frac{y^3}{y^4} \)

16. \( \frac{x^3 y^7}{x^2 y^3} \)  
17. \( \frac{a^2 b}{a^4 b^5} \)

18. \( \frac{e^7 d^3}{c^3 d^5} \)  
19. \( \frac{e^3 f^9}{e^7 f^3} \)

20. \( \frac{24a^6}{32a^4} \)  
21. \( \frac{14x^2 y}{-7x^3 y^4} \)

22. \( \frac{4a^3 b^5}{4a^3 b^5} \)  
23. \( \frac{2x^3}{7y^3} \)

24. \( \frac{0}{3abc} \)
Chapter 4 Section 1  
Reduce algebraic fractions

Solve.

25. \(2y + 18 = 6\)  
26. \(6a + 28 - 10a = 32\)

27. \(7m + 12 = 3m + 4\)  
28. \(3(4x - 8) = 7(2x + 6)\)

29. A bank balance decreased by $17 is $975. Find the bank balance.

30. Today’s temperature was 5 degrees more than yesterday’s temperature. The sum of the temperatures was 173 degrees. Find the temperature for each day.

Additional Practice with Reducing Fractions to Lowest Terms

31. \(\frac{6}{15}\)  
32. \(\frac{14}{35}\)  
33. \(\frac{12}{32}\)

34. \(\frac{5}{30}\)  
35. \(\frac{30}{75}\)  
36. \(\frac{48}{64}\)

37. \(\frac{44}{99}\)  
38. \(\frac{96}{132}\)  
39. \(\frac{-9}{30}\)

40. \(\frac{12}{-24}\)  
41. \(\frac{-20}{-36}\)  
42. \(\frac{27}{-45}\)

43. \(\frac{-39}{-87}\)  
44. \(\frac{-15}{48}\)  
45. \(\frac{35}{-14}\)

46. \(\frac{-10}{-21}\)  
47. \(\frac{x^5}{x^2}\)  
48. \(\frac{y^4}{y^9}\)

49. \(\frac{a^4}{a^3}\)  
50. \(\frac{k^3}{k^{10}}\)  
51. \(\frac{a^6b^4}{a^2b^2}\)

52. \(\frac{x^3y^2}{x^7y^5}\)  
53. \(\frac{a^8b^3}{a^2b^4}\)  
54. \(\frac{x^4b^5}{x^6b}\)

55. \(\frac{15a^{10}}{35a^7}\)  
56. \(\frac{-16x^3}{40x^7}\)  
57. \(\frac{12a^2b^6}{-9a^2b^2}\)

58. \(\frac{-18x^9y^2}{-42x^3y^2}\)  
59. \(\frac{3m^4n}{3m^2n}\)  
60. \(\frac{6y^4}{25y^4}\)

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Answer Key for Chapter 4 Section 1.

1. \( \frac{4}{7} \)  
2. \( \frac{7}{9} \)  
3. \( \frac{22}{39} \)

4. \( \frac{5}{14} \)  
5. \( \frac{3}{7} \)  
6. \( -\frac{8}{11} \)

7. \( \frac{17}{19} \)  
8. \( -\frac{9}{13} \)  
9. \( -\frac{10}{17} \)

10. \( -\frac{1}{2} \)  
11. \( \frac{15}{44} \)  
12. \( k^2 \)

13. \( m^3 \)  
14. \( \frac{1}{z^5} \)  
15. \( \frac{1}{y} \)

16. \( xy^4 \)  
17. \( \frac{1}{a^2b^4} \)

18. \( \frac{c^4}{d^2} \)  
19. \( \frac{f^6}{e^4} \)

20. \( \frac{3a^2}{4} \)  
21. \( -\frac{2}{xy^3} \)

22. 1  
23. \( \frac{2x^3}{7y^3} \)

24. 0  
25. \( y = -6 \)

26. \( a = -1 \)  
27. \( m = -2 \)

28. \( x = -33 \)

29. The bank balance is $992.

30. Yesterday’s temperature was 84°. Today’s temperature was 89°.

31. \( \frac{2}{5} \)  
32. \( \frac{2}{5} \)  
33. \( \frac{3}{8} \)
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<tr>
<td>49.</td>
<td>$a$</td>
<td>50.</td>
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<tr>
<td>52.</td>
<td>$\frac{1}{x^4 y^3}$</td>
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<tr>
<td>55.</td>
<td>$\frac{3a^3}{7}$</td>
<td>56.</td>
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<tr>
<td>58.</td>
<td>$\frac{3x^4}{7}$</td>
<td>59.</td>
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Section 2: Multiply/Divide Algebraic Fractions

Multiply Fractions

As you have probably noticed, the procedures for algebraic fractions are very similar to the procedures for “normal” fractions. To multiply fractions, multiply the numerators and multiply the denominators. Then check for reducing. Observe the following.

a. \[
\frac{2}{3} \cdot \frac{5}{7} \Rightarrow \frac{2 \cdot 5}{3 \cdot 7} \Rightarrow \frac{10}{21}
\]

b. \[
\frac{3}{4} \cdot \frac{8}{15} \Rightarrow \frac{3 \cdot 8}{4 \cdot 15} \Rightarrow \frac{24}{60}
\]

then reduce so \[
\frac{24}{60} \Rightarrow \frac{24 \div 12}{60 \div 12} \Rightarrow \frac{2}{5}
\]

On example b, you may also remember that you can reduce before you start multiplying. One way to do this is to factor all numbers into their primes. Another way to do this is to divide a numerator and a denominator by the same factor simultaneously. Observe.

c. \[
\frac{3}{4} \cdot \frac{8}{15} \Rightarrow \frac{3 \cdot 8}{2 \cdot 2 \cdot 3 \cdot 5} \Rightarrow \frac{2}{5}
\]

d. \[
\frac{3}{4} \cdot \frac{8}{15} \Rightarrow \frac{3 \div 3}{4 \div 4} \cdot \frac{8 \div 4}{15 \div 3} \Rightarrow \frac{1}{1} \cdot \frac{2}{5} \Rightarrow \frac{2}{5}
\]

Multiplying algebraic fractions is very similar. Observe.

e. \[
\frac{-12}{35} \cdot \frac{49}{54}
\]

result will be negative (why?)

\[
\frac{-12 \div 6}{35 \div 7} \cdot \frac{49 \div 7}{54 \div 6} \Rightarrow \frac{-2 \cdot 7}{5 \cdot 9} \Rightarrow \frac{-14}{45}
\]
There are several different approaches to the next example. This example will use factoring into primes.

f. \[ \left( \frac{-8}{21} \right) \left( \frac{33}{25} \right) \left( \frac{-35}{52} \right) \] result is positive (why?)

\[ + \frac{2 \cdot 2 \cdot 2}{3 \cdot 7} \cdot \frac{3 \cdot 11}{5 \cdot 5} \cdot \frac{5 \cdot 7}{2 \cdot 2 \cdot 13} \]

\[ \frac{2}{1} \cdot \frac{11}{5} \cdot \frac{1}{13} \]

\[ \frac{22}{65} \]

g. \[ \left( \frac{1}{3} \right) \left( \frac{4}{9} \right) \] result is negative (why?)

\[ \left( \frac{1}{3} \right) \left( \frac{4}{9} \right) \]

\[ - \frac{28}{27} \]

h. \[ (-6) \left( \frac{1}{8} \right) \left( -9 \right) \Rightarrow \left( -\frac{6}{1} \right) \left( \frac{1}{8} \right) \left( -\frac{9}{1} \right) \]

\[ \left( -\frac{6 \cdot 2}{1} \right) \left( \frac{1}{8 + 2} \right) \left( -\frac{9}{1} \right) \Rightarrow \left( -\frac{3}{1} \right) \left( \frac{1}{4} \right) \left( -\frac{9}{1} \right) \]

\[ \frac{27}{4} \]

i. \[ \left( \frac{9x^3}{15} \right) \left( \frac{-20}{21x^4} \right) \] result is negative (why?)

\[ - \frac{3 \cdot 3 \cdot x \cdot x \cdot x}{3 \cdot 5} \cdot \frac{2 \cdot 2 \cdot 5}{3 \cdot 7 \cdot x \cdot x \cdot x} \Rightarrow - \frac{4}{7x} \]
Chapter 4 Section 2
Multiply/divide algebraic fractions

j. \[ \left( -\frac{3}{5} \right)^2 \]

k. \[ \left( -\frac{2}{3} \right)^2 \]

\[ \left( -\frac{3}{5} \right) \left( -\frac{3}{5} \right) \]

\[ \left( -\frac{2}{3} \right) \left( -\frac{2}{3} \right) \left( -\frac{2}{3} \right) \]

\[ \frac{9}{25} \]

\[ -\frac{8}{27} \]

Divide Fractions

Observe the following.

l. \[ 8 \div 2 \Rightarrow 4 \]

m. \[ 8 \cdot \frac{1}{2} \Rightarrow \frac{8}{1} \cdot \frac{1}{2} \Rightarrow \frac{8}{2} \Rightarrow 4 \]

n. \[ 6 \div \frac{1}{3} \Rightarrow 6 \cdot 3 \Rightarrow 18 \]

Notice that on examples l and m both results were the same. One way to approach division, especially with fractions, is to rewrite division problems as multiplication by the reciprocal. A reciprocal is a number “flipped upside down.” For example, the number \( \frac{3}{4} \) has the reciprocal \( \frac{4}{3} \).

The number \( -\frac{9}{7} \) has the reciprocal \( -\frac{7}{9} \).

o. \[ \frac{2}{3} \div \frac{10}{21} \]

\[ \frac{2 \cdot 21}{3 \cdot 10} \Rightarrow \frac{2 \div 2 \cdot 21 \div 3}{3 \div 3 \cdot 10 \div 2} \Rightarrow \frac{1 \cdot 7}{5} \Rightarrow \frac{7}{5} \]

Remember \( \frac{2}{3} \) is the dividend;

\( \frac{10}{21} \) is the divisor. Notice the divisor inverts.

The result \( \frac{7}{5} \) is the quotient.
The instructions would say “Simplify.” You would be expected to know how to proceed.

NOTE: You may wonder about using multiplication by the reciprocal when you divide fractions. Observe the following.

\[ \frac{2}{3} \div \frac{10}{21} = \frac{2}{3} \cdot \frac{21}{10}, \] and you would reduce from there. Another way to approach this complex fraction is to figure out a way to make the denominator of the complex fraction become 1. To change the look of the denominator to 1, you could multiply \( \frac{10}{21} \) by its reciprocal \( \frac{21}{10} \). You would also simultaneously multiply the numerator by \( \frac{21}{10} \) in order to keep the value of the fraction the same (multiplying numerator and denominator by the same number changes its looks and not its value). Therefore, you could think like this:

\[ \frac{2}{3} \div \frac{10}{21} \Rightarrow \frac{2}{3} \cdot \frac{21}{10} \Rightarrow \frac{2}{3} \cdot \frac{21}{10} \Rightarrow \frac{2}{3} \cdot \frac{21}{10} \]

Notice you eventually have changed the look of this problem from a division to a multiplication by the reciprocal. This may help you remember to multiply by the reciprocal when you divide fractions.
Homework for Chapter 4 Section 2

Simplify.

1. \( \frac{3}{8} \cdot \frac{5}{11} \)
2. \( \frac{14}{27} \left( -\frac{9}{28} \right) \)
3. \( \frac{-10}{21} \left( -\frac{36}{15} \right) \)
4. \( \frac{1}{4} \left( -9 \right) \left( \frac{1}{5} \right) \)
5. \( (-7) \left( \frac{1}{14} \right) \left( -11 \right) \)
6. \( \left( \frac{2}{15} \right) \left( \frac{25}{49} \right) \left( -\frac{63}{72} \right) \)
7. \( \frac{3x^6}{5} \cdot \frac{15}{21x^{10}} \)
8. \( \frac{16}{5y^2} \cdot \frac{35y^7}{12} \)
9. \( \left( \frac{-3}{8} \right)^2 \)
10. \( \left( \frac{-2}{5} \right)^3 \)
11. \( -10 \div \frac{1}{4} \)
12. \( -\frac{1}{9} \div (-3) \)
13. \( \frac{2}{3} \div \left( -\frac{4}{15} \right) \)
14. \( -\frac{8}{33} \div \left( -\frac{20}{39} \right) \)
15. \( \frac{6}{w^3} \div \frac{6}{w^2} \)
16. \( \frac{5}{z^4} \div \frac{7}{z} \)
17. \( \frac{a^2}{2} \div \frac{a^2}{8} \)
18. \( \frac{9}{x^3} \div \frac{9}{x^2} \)
19. \( \frac{x^3}{y^3} \div \frac{x^6}{y^4} \)
20. \( -\frac{10x^6}{48} \div \frac{15x^5}{56} \)
21. \( -\frac{12}{17} \div \left( -\frac{12}{17} \right) \)
22. \( \frac{3}{8} \cdot \frac{3}{8} \)
23. \( \frac{4a^3}{3} \div \frac{3c^2}{4} \)
24. \( \frac{48x^3}{yz^2} \cdot \frac{yz^2}{48x^3} \)
25. \( \frac{6}{x^3} \cdot \frac{x^2}{18} \)

26. \( \frac{y^4}{8} \div \frac{y^6}{2} \)

Solve.

27. \( 2k + 15 = -17 \)

28. \( 3a - 24 = 10 - 37 \)

29. \( -x + 6 = 10 \)

30. \( 12 - 4x + 6 = 2x - 42 \)

31. 22 inches more than twice Jan’s height is 104 inches. Find Jan’s height.

32. Mike spent $28 more than three times what Jack spent. Together, they spent $296. What did Mike spend?

Additional Practice with Multiplying or Dividing Fractions

33. \( \frac{2}{3} \cdot \frac{5}{7} \)

34. \( \frac{3}{8} \cdot \frac{16}{21} \)

35. \( \frac{8}{9} \cdot \frac{21}{20} \)

36. \( \frac{12}{35} \cdot \frac{21}{24} \)

37. \( \frac{5}{9} \cdot \frac{12}{15} \)

38. \( \frac{24}{30} \cdot \frac{-45}{18} \)

39. \( -\frac{6}{14} \cdot \frac{49}{27} \)

40. \( \frac{3}{8} (-10) \)

41. \( \frac{1}{2} \left( \frac{3}{7} \right) \left( \frac{8}{9} \right) \)

42. \( \frac{2}{21} \left( \frac{-14}{33} \right) \left( \frac{-55}{6} \right) \)

43. \( -15 \left( \frac{4}{25} \right) \left( \frac{7}{24} \right) \)

44. \( \frac{5x^4}{12} \cdot \frac{18}{35x^7} \)

45. \( \frac{14}{3z^2} \cdot \frac{9z^5}{21} \)

46. \( \frac{a^4}{b^2} \cdot \frac{b^6}{a} \)

47. \( -12 \div \frac{1}{5} \)

48. \( -\frac{1}{10} \div (-7) \)

49. \( \frac{10}{21} \div \left( -\frac{15}{24} \right) \)

50. \( -\frac{12}{49} \div \left( -\frac{42}{35} \right) \)

51. \( \frac{7}{a^5} \div \frac{7}{a^7} \)

52. \( \frac{3}{z^3} \div \frac{8}{z^2} \)

53. \( \frac{18}{25} \div \left( -\frac{18}{25} \right) \)

54. \( \frac{6a^2}{7} \div \frac{7c^2}{6} \)

55. \( \frac{50a^3}{b^2c^4} \cdot \frac{b^2c^4}{50a^5} \)

56. \( \frac{5}{m^6} \cdot \frac{m^4}{20} \)

57. \( \frac{k}{12} \div \frac{k^5}{4} \)
### Answer Key for Chapter 4 Section 2.

1. \(\frac{15}{88}\)  
2. \(-\frac{1}{6}\)  
3. \(\frac{8}{7}\)  

4. \(-\frac{9}{20}\)  
5. \(\frac{11}{2}\)  
6. \(-\frac{5}{84}\)  

7. \(\frac{3}{7x^4}\)  
8. \(\frac{28y^5}{3}\)  
9. \(\frac{9}{64}\)  

10. \(-\frac{8}{125}\)  
11. \(-40\)  
12. \(\frac{1}{27}\)  

13. \(-\frac{5}{2}\)  
14. \(\frac{26}{55}\)  
15. \(w\)  

16. \(\frac{5}{7z^3}\)  
17. 4  
18. 1  

19. \(\frac{y^2}{x^3}\)  
20. \(-\frac{7x}{9}\)  
21. 1  

22. \(\frac{9}{64}\)  
23. \(\frac{16a^3}{9e^2}\)  
24. 1  

25. \(\frac{1}{3x}\)  
26. \(\frac{1}{4y^2}\)  
27. \(k = -16\)  

28. \(a = -1\)  
29. \(x = -4\)  
30. \(x = 10\)  

31. Jan’s height is 41 inches.  
32. Mike spent $229.  

33. \(\frac{10}{21}\)  
34. \(\frac{2}{7}\)  
35. \(\frac{14}{15}\)  

36. \(\frac{3}{10}\)  
37. \(-\frac{4}{9}\)  
38. 2  

39. \(-\frac{7}{9}\)  
40. \(-\frac{15}{4}\)  
41. \(\frac{4}{21}\)
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<td>( \frac{10}{27} )</td>
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<td>( -\frac{7}{10} )</td>
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<td>44.</td>
<td>( \frac{3}{14x^3} )</td>
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<td>45.</td>
<td>( 2z^3 )</td>
<td>46.</td>
<td>( a^3 b^4 )</td>
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<td>( -60 )</td>
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<td>48.</td>
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<td>( -\frac{16}{21} )</td>
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<td>50.</td>
<td>( \frac{10}{49} )</td>
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<tr>
<td>51.</td>
<td>( a^2 )</td>
<td>52.</td>
<td>( \frac{3}{8z^3} )</td>
</tr>
<tr>
<td>53.</td>
<td>( -1 )</td>
<td></td>
<td></td>
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<td>54.</td>
<td>( \frac{36a^2}{49c^2} )</td>
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<td>1</td>
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<td>57.</td>
<td>( \frac{1}{3k^4} )</td>
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Section 3: Add/Subtract Algebraic Fractions

Like Denominators

In Chapter 1, you learned about combining like terms. Combining fractions with like denominators is similar. When the denominators are the same, keep the denominator and combine the numerators. Remember the procedures for positive and negative numbers! After combining, reduce the result to lowest terms if possible. Observe.

a. \[ \frac{1}{10} + \frac{3}{10} \]
   \[-\frac{4}{10} \]
   \[-\frac{2}{5} \]
large{like denominators}

b. \[ \frac{3}{8} + \left( -\frac{7}{8} \right) \]
   \[-\frac{4}{8} \]
   \[-\frac{1}{2} \]

find difference; keep “larger” sign

c. \[ -\frac{11}{12} - \left( -\frac{2}{12} \right) \]
   \[-\frac{11}{12} + \frac{2}{12} \]
   \[-\frac{9}{12} \]
   \[-\frac{3}{4} \]
reduce to lowest terms
The concept remains the same even when you have “algebraic” fractions. Observe.

\[\frac{4a}{15} - \frac{11a}{15} \quad \text{same denominator}\]

\[- \frac{7a}{15} \quad \text{combine numerators; keep denominator}\]

Notice this result was already reduced to lowest terms. Also notice that you combined \(4a\) with \(-11a\) because those were like terms (both had the variable \(a\)).

\[\frac{4}{15} - \frac{11}{15} \quad \text{same denominator}\]

\[- \frac{7}{15} \quad \text{algebraic fractions}\]

Notice you are still combining like terms (keep the variable, combine the coefficients). Notice example e is the same as example d.

\[\frac{-3m}{25} - \frac{12m}{25} \quad \text{same denominator}\]

\[- \frac{15m}{25} \quad \text{combine numerators; keep denominator}\]

\[- \frac{3m}{5} \quad \text{algebraic fractions}\]

Notice on example g that the denominator was the same so you could combine numerators and keep the denominator. However, when you looked at the numerators, you realized that \(2x\) had a variable and \(5\) did not. So you rewrote that as the expression \(2x + 5\) and placed it in the numerator.

\[\frac{2x}{7} + \frac{5}{7} \quad \text{algebraic fractions}\]

\[\frac{2x + 5}{7} \quad \text{algebraic fractions}\]

Notice on example h that you had a common denominator so you need to combine numerators. However, you were adding expressions so needed to remember to combine like terms only.

\[\frac{3k + 17}{8} + \frac{2k + 4}{8} \quad \text{same denominator}\]

\[\frac{(3k + 17) + (2k + 4)}{8} \quad \text{combine numerators; keep denominator}\]

\[\frac{3k + 17 + 2k + 4}{8} \quad \text{algebraic fractions}\]

\[\frac{5k + 21}{8} \quad \text{algebraic fractions}\]
Chapter 4 Section 3  
Add/subtract algebraic fractions

i. \[ \frac{13x + 9}{7} - \frac{10x - 4}{7} \]

\[ \frac{(13x + 9) - (10x - 4)}{7} \Rightarrow \frac{13x + 9 - 10x + 4}{7} \Rightarrow \frac{3x + 13}{7} \]

Notice on example h that you had a common denominator so you need to combine numerators. However, you had to think first numerator (in parentheses) subtract second numerator (in parentheses). To actually finish, you removed parentheses and combined like terms, using all the rules you learned so far.

j. \[ \frac{5}{x} + \frac{3}{x} \]

\[ \frac{8}{x} \]

Recall, you cannot divide by zero; therefore, in this example, \( x \) cannot equal zero.
Unlike Denominators

When the denominators are not the same, you will need to find a lowest common denominator (LCD) before you can combine. There are many different ways to do this.

You can find the LCD by listing multiples of each denominator until each list has the same number. Observe.

k. Find the LCD for 12 and 9.

12, 24, 36, 48, ...
9, 18, 27, 36, 45, ...

LCD is 36 (that’s the first number that is identical in both lists).

l. Find the LCD for 24 and 30.

24, 48, 72, 96, 120, 144, ...
30, 60, 90, 120, 150, ...

LCD is 120.

There is also another way to find the LCD. Factor each denominator into its primes. Then multiply together each factor, the most number of times it is used in any one denominator. Observe.

m. Find the LCD for 18 and 24.

\[
\begin{align*}
18 & = 2 \cdot 3 \cdot 3 \\
24 & = 2 \cdot 2 \cdot 2 \cdot 3
\end{align*}
\]

LCD \(= 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3 = 72\)

Both methods give the same result!
Another approach to finding the LCD is to organize denominators into three categories.

<table>
<thead>
<tr>
<th>Category</th>
<th>Method of finding LCD:</th>
</tr>
</thead>
<tbody>
<tr>
<td>n. The denominators have no common factors.</td>
<td>Multiply denominators together for LCD.</td>
</tr>
<tr>
<td>Examples: 5 and 7 5(7) = 35 is the LCD 3 and 11 3(11) = 33 is the LCD</td>
<td></td>
</tr>
<tr>
<td>o. One denominator is a factor of the other denominator.</td>
<td>The larger of the two is the LCD.</td>
</tr>
<tr>
<td>Examples: 4 and 12 12 is LCD 10 and 5 10 is LCD</td>
<td></td>
</tr>
<tr>
<td>p. The two denominators have a common factor.</td>
<td>Multiply the denominators; divide the result by the largest common factor</td>
</tr>
<tr>
<td>Examples: 6 and 8 (2 is largest common factor) 6(8) = 48; 48 ÷ 2 = 24 is LCD 12 and 18 (6 is largest common factor) 12(18) = 216; 216 ÷ 6 = 36 is LCD 24 and 16 (8 is largest common factor) 24(16) = 384; 384 ÷ 8 = 48 is LCD</td>
<td></td>
</tr>
</tbody>
</table>
Once you have decided upon the LCD, you then need to rewrite each fraction so each fraction will have the same LCD. You do this by multiplying each fraction by the “missing” factor that will change the current denominator to the LCD. Remember to multiply both numerator and denominator by this “missing” factor (this process changes the “look” of the fraction but not its numerical value). Observe.

q. \[ \frac{7}{12} - \frac{17}{20} \]

First, find the LCD by one of the above methods.

By listing: 12, 24, 36, 48, 60, 72,…
20, 40, 60, 80,…
so LCD = 60.

By primes: \[ 12 = 2 \cdot 2 \cdot 3 \]
\[ 20 = 2 \cdot 2 \cdot 5 \]
so LCD = \(2 \cdot 2 \cdot 3 \cdot 5 = 60.\)

By category: 12 and 20 have a common factor of 4
12(20) = 240; 240 ÷ 4 = 60
so LCD = 60.

Now that you’ve chosen 60 as the LCD, multiply the first fraction by \(\frac{5}{5}\) to change the denominator 12 into 60 and multiply the second fraction by \(\frac{3}{3}\) to change the denominator 20 into 60. Observe the steps.

\[ \frac{7}{12} - \frac{17}{20} \]

\[ \frac{7}{12} \left( \frac{5}{5} \right) - \frac{17}{20} \left( \frac{3}{3} \right) \]

\[ \frac{35}{60} - \frac{51}{60} \]

\[ \frac{35 - 51}{60} \]

\[ \frac{-16}{60} \]

\[ \frac{-4}{15} \]
Chapter 4 Section 3  
Add/subtract algebraic fractions

r. \[ \frac{3}{8} - \frac{5}{6} + \frac{2}{3} \]

First, find the LCD.
- By listing: 8, 16, 24, 32,
6, 12, 18, 24, 30,
3, 6, 9, 12, 15, 18, 21, 24, 27,
so LCD = 24.
- By primes:
  \[ 8 = 2 \cdot 2 \cdot 2 \]
  \[ 6 = 2 \cdot 3 \]
  \[ 3 = 3 \]
so LCD = 2\cdot2\cdot3 = 24.
- By category: 6 is a multiple of 3 so 6 is the LCD for 6 and 3.
8 and 6 have a common factor of 2
8(6) = 48; 48 ÷ 2 = 24
so LCD = 24.

\[ \frac{3}{8} - \frac{5}{6} + \frac{2}{3} \]
\[ \frac{3}{8} \left( \frac{3}{3} \right) - \frac{5}{6} \left( \frac{4}{4} \right) + \frac{2}{3} \left( \frac{8}{8} \right) \]
\[ \frac{9}{24} - \frac{20}{24} + \frac{16}{24} \Rightarrow \frac{9 - 20 + 16}{24} \Rightarrow \frac{5}{24} \]

s. \[ -\frac{3a}{7} - \frac{11a}{15} \]

LCM is 105. Why?
\[ -\frac{3a}{7} \left( \frac{15}{15} \right) - \frac{11a}{15} \left( \frac{7}{7} \right) \]
\[ -\frac{45a}{105} - \frac{77a}{105} \]
\[ -\frac{122a}{105} \]
First find LCD. In this case, $x(9) = 9x$ is the LCD.

\[
\frac{2}{x} \left( \frac{9}{9} \right) + \frac{5}{9} \left( \frac{x}{9x} \right) = \frac{18}{9x} + \frac{5x}{9x} = \frac{5x + 18}{9x}
\]

The most challenging part of combining fractions with unlike denominators is finding the LCD. You have to choose a method and practice that method until you are comfortable. If you have never conquered fractions before, this is the time to really persevere and understand. Get extra help if you need to. Be patient, but keep working!

How would you approach this problem? Order of operations still applies.

Note: 2 and 10 divided by 2; 9 and 27 divided by 9

\[
-\frac{7}{10} + \left( \frac{2}{27} \right) \cdot \left( \frac{9}{10} \right) = -\frac{7}{10} + \left( \frac{1}{3} \cdot \frac{1}{5} \right)
\]

\[
-\frac{7}{10} + \frac{1}{15}
\]

LCD is 30.

\[
-\frac{7}{10} \left( \frac{3}{3} \right) + \frac{1}{15} \left( \frac{2}{2} \right) \Rightarrow -\frac{21}{30} + \frac{2}{30} = -\frac{19}{30}
\]
NOTE: How would you approach the following problem?

v. \[ \frac{10 + 2 \cdot 8}{30 - 17} \]

When you have operations (addition, subtraction, multiplication, or division) within the numerator or denominator of a fraction, the fraction bar acts as a grouping symbol. Find the value of the numerator, using order of operations; separately, find the value of the denominator, using order of operations. Once you have the values of the numerator and denominator, simplify the resulting fraction, if possible. Observe.

\[
\begin{align*}
\frac{10 + 2 \cdot 8}{30 - 17} & \Rightarrow \frac{10 + 16}{30 - 17} \Rightarrow \frac{26}{13} \Rightarrow 2 \\
\end{align*}
\]

w. \[ \frac{12 \div 2 - 4^{2}}{7 \cdot 3 + 20 \div 5} \]

\[
\begin{align*}
\frac{12 \div 2 - 4^{2}}{7 \cdot 3 + 20 \div 5} & \Rightarrow \frac{12 \div 2 - 16}{7 \cdot 3 + 20 \div 5} \Rightarrow \frac{6 - 16}{7 \cdot 3 + 20 \div 5} \Rightarrow \frac{-10}{7 \cdot 3 + 20 \div 5} \\
\frac{-10}{7 \cdot 3 + 20 \div 5} & \Rightarrow \frac{-10}{21 + 4} \Rightarrow \frac{-10}{25} \Rightarrow -\frac{2}{5} \\
\end{align*}
\]
Chapter 4 Section 3  
Add/subtract algebraic fractions

Homework for Chapter 4 Section 3

Simplify.

1. \( \frac{3}{8} - \frac{5}{8} \)
2. \( -\frac{1}{12} - \frac{3}{12} + \frac{7}{12} \)
3. \( \frac{3m}{20} + \frac{7m}{20} \)
4. \( \frac{5y}{14} - \frac{7y}{14} \)
5. \( -\frac{8z}{25} + \frac{3z}{25} \)
6. \( -\frac{18x}{30} - \frac{2x}{30} \)
7. \( \frac{2a+11}{5} + \frac{4a+8}{5} \)
8. \( \frac{10z-3}{7} + \frac{5z+5}{7} \)
9. \( \frac{5x+14}{11} - \frac{3x+13}{11} \)
10. \( \frac{12k-3}{9} - \frac{2k-7}{9} \)
11. \( -\frac{9m-13}{10} - \frac{8m+14}{10} \)
12. \( \frac{6a-16}{3} - \frac{11a-20}{3} \)
13. \( \frac{2z}{5} + \frac{3}{5} \)
14. \( \frac{2}{m} + \frac{8}{m} \)
15. \( -\frac{1}{4} + \frac{5}{14} \)
16. \( \frac{5}{9} + \frac{7}{12} - \frac{5}{6} \)
17. \( \frac{5}{12} - \frac{13}{30} \)
18. \( -\frac{10}{15} + \frac{16}{24} \)
19. \( \frac{3a}{8} + \frac{5a}{6} \)
20. \( -\frac{2m}{5} + \frac{5m}{6} \)
21. \( \frac{7y}{10} - \frac{2y}{15} \)
22. \( -\frac{8x}{12} - \frac{6x}{18} \)
23. \( -\frac{4m}{9} - \left(-\frac{2m}{15}\right) \)
24. \( \frac{5x}{8} - \left(-\frac{7x}{12}\right) \)
25. \( \frac{8}{9} + \frac{2}{y} \)
26. \( \frac{3}{m} + \frac{5}{6} \)
Chapter 4 Section 3  
Add/subtract algebraic fractions

27. \( \left( \frac{12}{35} \right) \left( \frac{-56}{42} \right) \)

28. \( -\frac{3}{8} \div \left( -\frac{27}{20} \right) \)

29. \( -\frac{3}{8} - \frac{5}{12} \left( \frac{18}{25} \right) \)

30. \( \frac{1}{2} \left( \frac{-5}{7} \right) + \frac{3}{28} \div \frac{3}{4} \)

31. \( 5y + 18 + 7y + 2 \)

32. \( 7 + 3(m + 8) - (5m - 6) \)

33. \( \left( \frac{18}{2} \right) \left( 8 + 2^3 \right) - 8^2 \)

34. \( 4(2x)(3) \)

35. \( \frac{-6z + 12}{-3} \)

36. \( -\frac{45k + 20}{5} \)

Solve.

37. \( 2y + 18 = 11 + 27 \)

38. \( 4z + 14 + 3z = 56 \)

39. \( 15 = 24 - 3k \)

40. \( 3(2m + 7) + 3 = 12 \)

41. \( 4z = 18 - 2z \)

42. \( 5(2a + 6) = 3(4a + 12) \)

43. Mary received 148 votes less than Joan. Together they received 1972 votes. How many votes did each one receive?

Evaluate.

44. \( 5y + 6 \) for \( y = -4 \)

45. \( -z - 3 \) when \( z = -12 \)

46. \( x + \frac{1}{6} \) for \( x = \frac{5}{6} \)

47. \( y + \frac{1}{12} \) when \( y = \frac{5}{12} \)

Simplify.

48. \( \frac{6 + 3(7 - 2)}{20 - 2 - 10} \)

49. \( \frac{5 - 24 \div 4}{2^3 + 6} \)

50. \( \frac{2(8) - 80 \div 5}{7^2} \)

51. \( \frac{7 + 5^2}{3(-4) + 6(2)} \)
Additional Practice with Adding or Subtracting Fractions

Simplify.

52. \( \frac{1}{12} + \frac{7}{12} \)

53. \( \frac{3}{25} - \frac{18}{25} \)

54. \( -\frac{11}{30} - \frac{7}{30} \)

55. \( \frac{8a}{15} + \frac{2a}{15} \)

56. \( -\frac{4m}{11} + \frac{9m}{11} \)

57. \( \frac{29y}{35} - \frac{24y}{35} \)

58. \( \frac{5}{x} + \frac{8}{x} \)

59. \( \frac{2y+1}{17} - \frac{3y-14}{17} \)

60. \( \frac{3}{4} + \frac{2}{3} \)

61. \( \frac{5}{6} - \frac{2}{3} \)

62. \( -\frac{1}{6} - \frac{5}{9} \)

63. \( -\frac{7}{10} + \frac{2}{15} \)

64. \( \frac{7}{8} - \left( -\frac{5}{12} \right) \)

65. \( \frac{7}{24} + \frac{1}{30} \)

66. \( \frac{5a}{7} + \frac{2a}{3} \)

67. \( \frac{2m}{15} - \frac{3m}{12} \)

68. \( -\frac{4y}{9} - \frac{5y}{8} \)

69. \( -\frac{9k}{20} + \frac{3k}{4} \)

70. \( -\frac{3x}{16} - \left( -\frac{7x}{24} \right) \)

71. \( \frac{2}{\checkmark} + \frac{3}{\checkmark} \)

72. \( \frac{2}{x} + \frac{3}{5} \)

73. \( \frac{5}{6} - \frac{7}{m} \)

74. \( \frac{1}{4} + \frac{5}{6} + \frac{7}{9} \)

75. \( \frac{5a}{12} - \frac{3a}{8} - \frac{a}{6} \)
Chapter 4 Section 3

Add/subtract algebraic fractions

Answer Key for Chapter 4 Section 3.

1. $\frac{-2}{8} = \frac{-1}{4}$
2. $\frac{3}{12} = \frac{1}{4}$
3. $\frac{10m}{20} = \frac{m}{2}$
4. $\frac{-2y}{14} = \frac{-y}{7}$
5. $\frac{-5z}{25} = \frac{-z}{5}$
6. $\frac{-20x}{30} = \frac{-2x}{3}$
7. $\frac{6a + 19}{5}$
8. $\frac{15z + 2}{7}$
9. $\frac{2x + 1}{11}$
10. $\frac{10k + 4}{9}$
11. $\frac{-17m - 27}{10}$
12. $\frac{-5a + 4}{3}$
13. $\frac{2z + 3}{5}$
14. $\frac{10}{m}$
15. $\frac{3}{28}$
16. $\frac{11}{36}$
17. $\frac{-1}{60}$
18. $0$
19. $\frac{29a}{24}$
20. $\frac{13m}{30}$
21. $\frac{17y}{30}$
22. $\frac{-36x}{36} = -x$
23. $\frac{-14m}{45}$
24. $\frac{29x}{24}$
25. $\frac{8y + 18}{9y}$
26. $\frac{18 + 5m}{6m}$ or $\frac{5m + 18}{6m}$
Chapter 4 Section 3  
Add/subtract algebraic fractions

27. \(-\frac{16}{35}\)  
28. \(\frac{5}{18}\)

29. \(-\frac{27}{40}\)  
30. \(-\frac{3}{14}\)

31. \(12y + 20\)  
32. \(-2m + 37\)

33. \(80\)  
34. \(24x\)

35. \(2Z - 4\)  
36. \(-9k + 4\)

37. \(y = 10\)  
38. \(z = 6\)

39. \(3 = k \text{ or } k = 3\)  
40. \(m = -2\)

41. \(z = 3\)  
42. \(a = -3\)

43. Joan received 1060 votes; Mary received 912 votes.

44. \(-14\)  
45. \(9\)

46. \(\frac{6}{6} = 1\)  
47. \(\frac{6}{12} = \frac{1}{2}\)

48. \(\frac{21}{30} = \frac{7}{10}\)  
49. \(\frac{-1}{14} = -\frac{1}{14}\)

50. \(\frac{0}{49} = 0\)  
51. \(\frac{32}{0}\) is undefined

52. \(\frac{8}{12} = \frac{2}{3}\)  
53. \(-\frac{15}{25} = -\frac{3}{5}\)

54. \(-\frac{18}{30} = -\frac{3}{5}\)  
55. \(\frac{10a}{15} = \frac{2a}{3}\)

56. \(\frac{5m}{11}\)  
57. \(\frac{5y}{35} = \frac{y}{7}\)

58. \(\frac{13}{x}\)  
59. \(-\frac{y + 15}{17}\)

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<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>60.</td>
<td>17/12</td>
<td>61.</td>
</tr>
<tr>
<td>62.</td>
<td>-13/18</td>
<td>63.</td>
</tr>
<tr>
<td>64.</td>
<td>31/24</td>
<td>65.</td>
</tr>
<tr>
<td>66.</td>
<td>29a/21</td>
<td>67.</td>
</tr>
<tr>
<td>68.</td>
<td>-77y/72</td>
<td>69.</td>
</tr>
<tr>
<td>70.</td>
<td>5x/48</td>
<td>71.</td>
</tr>
<tr>
<td>72.</td>
<td>10 + 3x/5x</td>
<td>73.</td>
</tr>
<tr>
<td>74.</td>
<td>67/36</td>
<td>75.</td>
</tr>
</tbody>
</table>
Section 4: Mixed Numbers, Using Signs

When you add, subtract, multiply, or divide with signed (positive or negative) mixed numbers, one approach is to convert all mixed numbers to improper fractions.

What is a mixed number? Mixed numbers are whole numbers plus fractions: \(1\frac{3}{4}\) or \(-3\frac{5}{6}\) are examples of mixed numbers. What does it mean when you see a mixed number?

\[
1\frac{3}{4} \quad \text{means} \quad 1 + \frac{3}{4}; \\
-3\frac{5}{6} \quad \text{means} \quad -3 + \frac{5}{6}.
\]

This gives us a way to convert from mixed number form to improper form. What is an improper fraction? An improper fraction has a value greater than or equal to 1 (the numerator is greater than or equal to the denominator). Examples of improper fractions include \(\frac{3}{2}, -\frac{7}{4}, \frac{8}{8}\), or \(\frac{7}{4}\).

Observe the following approach to convert a mixed number to an improper fraction.

a. Convert \(1\frac{3}{4}\) to improper fraction form.

\[
1\frac{3}{4} \Rightarrow 1 + \frac{3}{4} \Rightarrow 1 + \frac{3}{4} \quad \text{rewrite with + and 1 as } \frac{1}{1} \\
1\left(\frac{4}{4}\right) + \frac{3}{4} \Rightarrow \frac{4}{4} + \frac{3}{4} \quad \text{find LCD and continue} \\
\frac{4+3}{4} \Rightarrow \frac{7}{4}
\]

An alternative to this is to proceed as follows: multiply the whole number by the denominator, add this result to the numerator, place this new result in the numerator, and keep the original denominator.

For \(1\frac{3}{4}\), think “1 times 4 is 4; 4 plus 3 is 7; the new numerator is 7 and keep the original denominator of 4.” Therefore, \(1\frac{3}{4} \Rightarrow \frac{1(4)+3}{4} \Rightarrow \frac{4+3}{4} \Rightarrow \frac{7}{4}\).
How would you convert a negative mixed number to an improper fraction?
Again, observe.

b. \(-3\frac{5}{6} \Rightarrow -3 + \frac{5}{6} \Rightarrow -3 + \frac{5}{6}
\]
\[-\frac{3}{1} \cdot \frac{5}{6} = \frac{-18}{6} + \frac{5}{6} \Rightarrow \frac{-23}{6} \lor \frac{-23}{6}
\]

An alternative would be to think like this: you know the result will be negative so carry the negative over; multiply the whole number (without the sign) by the denominator, add the numerator (without the sign), place that result in the numerator, and keep the original denominator. On \(-3\frac{5}{6}\) think “Result is negative; 3 times 6 is 18; 18 plus 5 is 23; the numerator will be 23 and the original denominator is 5.”

Therefore, \[-3\frac{5}{6} \Rightarrow (3(6) + 5) \Rightarrow \frac{18 + 5}{6} \Rightarrow \frac{23}{6} \Rightarrow \frac{-23}{6}
\]

The following examples will show how to utilize the above thinking in addition, subtraction, multiplication, and division of signed mixed numbers.

c. \(-3\frac{1}{4} + 1\frac{11}{12}
\]
\[-\frac{13}{4} + \frac{23}{12}
\]
\[-\frac{13(3)}{4} + \frac{23}{12}
\]
\[-\frac{39}{12} + \frac{23}{12}
\]
\[-\frac{62}{12} = -\frac{4}{3} \lor -\frac{1}{3}
\]

d. \(-9\frac{7}{8} - 1\frac{1}{3}
\]
\[-\frac{79}{8} - \frac{34}{3}
\]
\[-\frac{79(3)}{8} - \frac{34(8)}{3}
\]
\[-\frac{237}{24} - \frac{272}{24}
\]
\[-\frac{509}{24} \lor -\frac{21 \frac{5}{24}}{24}
\]

On addition and subtraction, you need to remember to find the LCD! You also need to remember when to find the sum, when to find the difference, and what sign the result would have.
On division, you need to remember to invert the divisor and multiply. Then you need to remember to reduce to lowest terms (these examples used factoring into primes). You also need to remember to find the sign of the result!
Homework for Chapter 4 Section 4

Simplify.

1. \(-2\frac{7}{12} - 4\frac{2}{12}\)  
2. \(3\frac{2}{11} - 8\frac{1}{11}\)

3. \(-6\frac{1}{8} + 2\frac{11}{12}\)  
4. \(-2\frac{7}{8} + 5\frac{1}{4}\)

5. \(1\frac{6}{7} - 8\frac{1}{4}\)  
6. \(3\frac{14}{15} - 9\frac{2}{3}\)

7. \(-4\frac{3}{5} - 8\frac{2}{7}\)  
8. \(5\frac{1}{8} - 2\frac{2}{3}\)

9. \((-2\frac{2}{3})(5\frac{1}{4})\)  
10. \((-1\frac{1}{2})(-3\frac{1}{3})\)

11. \(4\frac{1}{4} ÷ (-1\frac{1}{8})\)  
12. \(-2\frac{3}{5} ÷ (-1\frac{1}{25})\)

13. \(-6\frac{1}{2} + 2\frac{2}{3} - 3\frac{1}{4}\)  
14. \(7\frac{1}{6} - 10\frac{1}{3} - 2\frac{1}{2}\)

15. \(5 + 18(1 - 3)\)  
16. \(\left(\frac{10}{-2}\right)(-3) + 2(7 - 3)^2\)

17. \(4k + 3(9 + 2k)\)  
18. \(3 + 4(7m - 5) - 10m\)

19. \(\frac{6m + 18}{2}\)  
20. \(\frac{4\alpha + 19}{23} + \frac{3\alpha + 1}{23}\)

21. \(\frac{7x + 13}{17} - \frac{2x + 10}{17}\)  
22. \(\frac{14y - 30}{29} - \frac{10y - 27}{29}\)
Chapter 4 Section 4

Mixed numbers, using signs

Solve.

23. \(-3x - 17 = -x - 35\)  
24. \(2m + 18 + m + 19 = 1\)

25. \(4z + 30 - 3z + 11 = -25\)  
26. \(-5y = -45\)

27. Three times the height of a tree is the same as 48 feet. What is the height of the tree?

28. Jackie drove 18 miles less than twice what Mary drove. Together they drove 483 miles. How far did each person drive?

Additional Practice with Mixed Numbers

Simplify.

29. \(-1 \frac{4}{9} - 3 \frac{7}{9}\)  
30. \(2 \frac{3}{13} - 5 \frac{1}{13}\)

31. \(2 \frac{7}{8} - 5 \frac{1}{2}\)  
32. \(-6 \frac{10}{21} - 2 \frac{1}{7}\)

33. \(-5 \frac{2}{15} + 3 \frac{5}{6}\)  
34. \(-2 \frac{1}{6} - 4 \frac{3}{4}\)

35. \(9 - 4 \frac{2}{5}\)  
36. \(1 \frac{2}{7} - 4\)

37. \(2 \frac{1}{4} + \frac{5}{6} - 6 \frac{3}{8}\)  
38. \(8 - 4 \frac{11}{12} - 5 \frac{2}{3}\)

39. \((-3 \frac{1}{5})(1 \frac{1}{4})\)  
40. \(-2 \frac{1}{4} \div (-1 \frac{1}{2})\)
Chapter 4 Section 4

Mixed numbers, using signs

Answer Key for Chapter 4 Section 4.

1. \(-\frac{81}{12} = -\frac{27}{4} \quad \text{or} \quad -6\frac{3}{4}\)

2. \(-\frac{54}{11} \quad \text{or} \quad -4\frac{10}{11}\)

3. \(-\frac{77}{24} \quad \text{or} \quad -3\frac{5}{24}\)

4. \(\frac{19}{8} \quad 2\frac{3}{8}\)

5. \(-\frac{179}{28} \quad -6\frac{11}{28}\)

6. \(-\frac{86}{15} \quad -5\frac{11}{15}\)

7. \(-\frac{451}{35} \quad -12\frac{31}{35}\)

8. \(\frac{59}{24} \quad 2\frac{11}{24}\)

9. \(-14\)

10. \(5\)

11. \(-\frac{34}{9} \quad \text{or} \quad -3\frac{7}{9}\)

12. \(\frac{5}{2} \quad 2\frac{1}{2}\)

13. \(-\frac{85}{12} \quad \text{or} \quad -7\frac{1}{12}\)

14. \(-\frac{34}{6} = -\frac{17}{3} = -5\frac{2}{3} \quad \text{or}\)

15. \(-31\)

16. \(47\)

17. \(10k + 27\)

18. \(18m - 17\)

19. \(3m + 9\)

20. \(\frac{7a + 20}{23}\)

21. \(\frac{5x + 3}{17}\)

22. \(\frac{4y - 3}{29}\)

23. \(x = 9\)

24. \(m = -12\)

25. \(z = -66\)

26. \(y = 9\)

27. The tree is 16 feet tall.

28. Mary drove 167 miles; Jackie drove 316 miles.

29. \(-\frac{47}{9} \quad \text{or} \quad -5\frac{2}{9}\)

30. \(-\frac{37}{15} \quad \text{or} \quad -2\frac{11}{13}\)
31. \(-\frac{21}{8} \text{ or } -2\frac{5}{8}\) 
32. \(-\frac{181}{21} \text{ or } -8\frac{13}{21}\)

33. \(-\frac{13}{10} \text{ or } -1\frac{3}{10}\) 
34. \(-\frac{83}{12} \text{ or } -6\frac{11}{12}\)

35. \(\frac{23}{5} \text{ or } 4\frac{3}{5}\) 
36. \(-\frac{19}{7} \text{ or } -2\frac{5}{7}\)

37. \(-\frac{79}{24} \text{ or } -3\frac{7}{24}\) 
38. \(-\frac{31}{12} \text{ or } -2\frac{7}{12}\)

39. \(-4\) 
40. \(\frac{3}{2} \text{ or } 1\frac{1}{2}\)
Section 5: Solve Equations Involving Fractions

You are now ready to solve equations involving fractions. You will use all the techniques for solving that you have previously learned. First, review the following (equations from chapters 1 through 3).

a. \[ 2x + 9 = 15 \]
   \[ 2x = 6 \]
   \[ x = 3 \]

b. \[ 4(2x + 6) = -16 \]
   \[ 8x + 24 = -16 \]
   \[ 8x = -40 \]
   \[ x = -5 \]

c. \[ \frac{7x}{12} = \frac{4x}{48} \]
   \[ \frac{3x}{12} = \frac{-48}{4x} \]
   \[ 3x = -36 \]
   \[ x = -12 \]

Notice: all the results were integers.

Now, you will apply the same techniques to similar equations. In this first set of examples, the results are fractions. Observe.

d. \[ 3m + 1 = 8 \]
   \[ 3m = 7 \]
   \[ m = \frac{7}{3} \]

e. \[ -2y - 17 = 4 \]
   \[ -2y = 21 \]
   \[ y = \frac{-21}{2} \]
Chapter 4 Section 5

Solve equations involving fractions

f. \[ 14 - 24x = -2x + 3 \]
\[ +2x \quad +2x \]
\[ 14 - 22x = \frac{3}{14} \]
\[ -14 \quad -14 \]
\[ -22x = -11 \]
\[ -22 \quad -22 \]
\[ x = \frac{1}{2} \]

g. \[ -3(x + 1) + 5 = 18 - 9x \]
\[ -3x - 3 + 5 = 18 - 9x \]
\[ -3x + 2 = 18 - 9x \]
\[ +9x \quad +9x \]
\[ 6x + 2 = 18 \]
\[ -2 \quad -2 \]
\[ 6x = 16 \]
\[ 6 \quad 6 \]
\[ x = \frac{16}{6} = \frac{8}{3} \]

h. \[ 2z + 8 = 10 - 5z \]
\[ 2z + 5z + 8 = 10 - 5z + 5z \]
\[ 7z + 8 = 10 \]
\[ 7z + 8 - 8 = 10 - 8 \]
\[ 7z = 2 \]
\[ 7z + 7 = 2 + 7 \]
\[ z = \frac{2}{7} \]

How do you solve equations when the equation itself contains fractions? One approach to solving equations involving fractions is to clear the equation of fractions. This means you multiply ALL terms by the same LCD, eliminating all denominators. The numerators will change accordingly. The new equation will look different but will have the same numerical value. Then solve, using all the methods you know. Observe.

i. \[ \frac{3}{8} x + \frac{5}{6} = \frac{2}{3} \]

multiply all terms by LCD 24

\[ 24 \left( \frac{3}{8} x \right) + 24 \left( \frac{5}{6} \right) = 24 \left( \frac{2}{3} \right) \]

reduce 24 with each denominator

\[ 3(3x) + 4(5) = 8(2) \]

now multiply to remove parentheses

\[ 9x + 20 = 16 \]
\[ 9x + 20 - 20 = 16 - 20 \]
\[ 9x = -4 \]
\[ 9x + 9 = -4 + 9 \]

\[ x = -\frac{4}{9} \]
Chapter 4 Section 5  
Solve equations involving fractions

j. \[ \frac{5}{6}x + \frac{7}{12} = \frac{3}{8}x + \frac{4}{3} \]

\[
24 \left( \frac{5}{6}x \right) + 24 \left( \frac{7}{12} \right) = 24 \left( \frac{3}{8}x \right) + 24 \left( \frac{4}{3} \right) 
\]

reduce 24 with each denominator

\[ 4(5x) + 2(7) = 3(3x) + 8(4) \]

now multiply to remove parentheses

\[ 20x + 14 = 9x + 32 \]

\[ -9x \quad -9x \]

\[ 11x + 14 = 32 \]

\[ -14 \quad -14 \]

\[ 11x = 18 \]

\[ \frac{11x}{11} = \frac{18}{11} \]

\[ x = \frac{18}{11} \]

Suppose you multiply the equation in example j by the common denominator 48 (48 is a common denominator, but 24 is the lowest common denominator). You should still eventually get the same result. Observe.

\[ \frac{5}{6}x + \frac{7}{12} = \frac{3}{8}x + \frac{4}{3} \]

\[
48 \left( \frac{5}{6}x \right) + 48 \left( \frac{7}{12} \right) = 48 \left( \frac{3}{8}x \right) + 48 \left( \frac{4}{3} \right) 
\]

\[ 8(5x) + 4(7) = 6(3x) + 16(4) \]

\[ 40x + 28 = 18x + 64 \]

\[ -18x \quad -18x \]

\[ 22x + 28 = 64 \]

\[ -28 \quad -28 \]

\[ 22x = 36 \]

\[ \frac{22x}{22} = \frac{36}{22} \]

\[ x = \frac{36}{22} = \frac{18}{11} \]

Notice the result is the same!
\[ \frac{3x}{4} - \frac{1}{10} = \frac{-7}{10} \]

multiply all terms by LCD 20

\[ 20(3x) - 20\left(\frac{1}{4}\right) = 20\left(\frac{-7}{10}\right) \]

reduce 20 with each denominator

\[ 20(3x) - 5(1) = 2(-7) \]

now multiply to remove parentheses

\[ 60x - 5 = -14 \]

\[ 60x + 5 + 5 \]

\[ 60x = -9 \]

\[ \frac{60x}{60} \]

\[ x = -\frac{9}{60} = -\frac{3}{20} \]

\[ \frac{3}{8}x = 11 \]

multiply all terms by LCD 8

\[ \frac{8}{8}\left(\frac{3}{8}x\right) = 8(11) \]

\[ 1(3x) = 8(11) \]

\[ 3x = 88 \]

\[ x = \frac{88}{3} \]

\[ \frac{2}{3}x + 5 = 8 \]

multiply all terms by LCD 3

\[ \frac{3}{3}\left(\frac{2}{3}x\right) + 3(5) = 3(8) \]

\[ 1(2x) + 3(5) = 3(8) \]

\[ 2x + 15 = 24 \]

\[ -15 -15 \]

\[ 2x = 9 \]

\[ \frac{2x}{2} = \frac{9}{2} \]

\[ x = \frac{9}{2} \]
Suppose on example m, you saw the following:

\[
\left(\frac{2}{3}\right)x + 5 = 8 \quad \text{remove parentheses}
\]

\[
\frac{2}{3}x + 5 = 8
\]

Now you have the same equation as above; continue to solve by clearing the fraction.

n. multiply all terms by LCD 24

\[
24(k) + 24\left(\frac{2}{3}\right) = 24\left(\frac{5}{8}\right)
\]

\[
24(k) + 8(2) = 3(5)
\]

\[
24k + 16 = 15
\]

\[
24k = -1
\]

\[
k = -\frac{1}{24}
\]

Suppose you want to check a result. Observe the check for example m.

\[
\frac{3}{8}x + \frac{5}{6} = \frac{2}{3}
\]

\[
\text{check } x = -\frac{4}{9}
\]

\[
\frac{3}{8}\left(-\frac{4}{9}\right) + \frac{5}{6} = \frac{2}{3}
\]

\[
-\frac{12}{72} + \frac{5}{6} = \frac{2}{3}
\]

\[
-\frac{1}{6} + \frac{5}{6} = \frac{2}{3}
\]

\[
\frac{4}{6} = \frac{2}{3}
\]

Since this checks (the two numbers are equal to each other), the solution is

\[
x = -\frac{4}{9}
\]

Notice, when you are checking, you replace the variable with the solution,
placing the solution inside parentheses. Then you simplify, using order of operations. If the two simplified expressions (numbers) are equal to each other, the solution checks.

Are there other ways of solving equations that contain fractions? Yes, but those ways have many intricate details. If you are interested in seeing some other methods, look at the following.

o. \[ \frac{2}{3} + \frac{5}{8} = \frac{5}{8} \]

- \[ \frac{2}{3} \]
- \[ \frac{2}{3} \]
\[ k = \frac{5}{8} - \frac{2}{3} \]
\[ k = \frac{15}{24} - \frac{16}{24} \]
\[ k = -\frac{1}{24} \]

p. \[ 3x - \frac{1}{4} = -\frac{7}{10} \]
+ \[ \frac{1}{4} + \frac{1}{4} \]
\[ 3x = -\frac{7}{10} + \frac{1}{4} \]
\[ 3x = -\frac{14}{20} + \frac{5}{20} \]
\[ 3x = -\frac{9}{20} \]

\[ \frac{3x}{3} = -\frac{9}{20} \quad \text{OR} \quad \frac{1}{3}(3x) = \frac{1}{3}\left(-\frac{9}{20}\right) \]
\[ x = -\frac{3}{20} \]

Notice you had already worked examples o and p previously. You got the same result, no matter which way you solved the equations. You need to decide which method is more understandable for you. Then practice until you are comfortable with the method you have chosen.
q. Could you work example k by first subtracting 5 from both sides and then by isolating \( x \) (either by dividing by \( \frac{2}{3} \) or by multiplying \( \frac{3}{2} \))? Observe.

\[
\frac{2}{3}x + 5 = 8
\]

\[
\begin{align*}
\frac{2}{3}x + 5 & = 8 \\
-5 & -5 \\
\frac{2}{3}x & = 3
\end{align*}
\]

\[
\frac{2}{3}x = \frac{3}{2} \quad \text{OR} \quad \frac{3}{2} \left( \frac{2}{3}x \right) = \frac{3}{2} \left( \frac{3}{1} \right)
\]

\[
\begin{align*}
x & = \frac{3}{\frac{2}{3}} = \frac{3}{\frac{2}{3}} \cdot \frac{3}{1} \cdot \frac{3}{1} \\
x & = \frac{9}{2}
\end{align*}
\]

Compare that to clearing the fractions:

\[
\begin{align*}
3 \left( \frac{2}{3}x \right) + 3(5) & = 3(8) \\
1(2x) + 3(5) & = 3(8) \\
2x + 15 & = 24 \\
-15 & -15 \\
2x & = 9
\end{align*}
\]

\[
\begin{align*}
\frac{2x}{2} & = \frac{9}{2} \\
x & = \frac{9}{2}
\end{align*}
\]

You get the same result! Which method do you prefer?
Homework for Chapter 4 Section 5

Solve.

1. $5x + 7 = 10$
2. $4m - 3 = 2$
3. $-9k + 3 = 4$
4. $-10v - 6 = 3$
5. $6w + 11 = 6$
6. $11x - 5 = -14$
7. $12y + 8 = 10$
8. $18A - 10 = -16$
9. $-8m + 15 = 9$
10. $-15B - 14 = -4$
11. $5(x + 3) = 7$
12. $6(2x - 4) = -20$
13. $4k - 27 = 15k - 18$
14. $-8m + 12 = -2m + 27$
15. $15z = 25$
16. $7 + 19m = 3m + 2$
17. $x + 9 + 12x - 15 = -7$
18. $-6 + 10(5 - 2x) = 1$
19. $3(4x + 8) = 9(3x + 2)$
20. $-4(2M - 3) = 2(M + 3)$
21. $m + \frac{3}{8} = \frac{7}{12}$
22. $y + \frac{1}{5} = -\frac{8}{15}$
23. $k - \frac{5}{12} = -\frac{3}{10}$
24. $\frac{3}{5}z = 7$
25. $-\frac{2}{3}a = \frac{8}{15}$
26. $\frac{5}{12}g + \frac{2}{3} = \frac{1}{8}$
27. $\frac{2}{9}w - \frac{1}{4} = \frac{5}{6}$
28. $\frac{7}{15}x - 4 = 2$
29. $\left(\frac{3}{7}\right)x + 2 = 4$
30. $\left(\frac{7}{9}\right)x - \frac{1}{2} = 3$
31. $\frac{2}{3}x + \frac{5}{6} = \frac{1}{2}x - \frac{7}{9}$
32. $\frac{3}{4}x + \frac{7}{12} = \frac{1}{9}x + \frac{5}{6}$
33. $\frac{1}{8}x - \frac{3}{4} = \frac{7}{16}x + \frac{1}{2}$
34. $\frac{3}{5}x - \frac{7}{10} = \frac{1}{2}x - \frac{5}{4}$
35. \( \frac{5}{8}x - \frac{1}{3} = 2x - \frac{5}{6} \)

36. \( \frac{2}{3}x + 1 = \frac{5}{6}x + \frac{3}{4} \)

37. \( 2x - \frac{4}{7} = \left(\frac{1}{2}\right)x + \frac{9}{14} \)

38. \( \frac{3}{11}x - 2 = 5x - \frac{1}{2} \)

39. \( -\frac{7}{8}x - \frac{5}{6} = \frac{11}{24}x - \frac{5}{16} \)

40. \( \frac{5}{7}x - \frac{3}{14} = -\frac{3}{8}x + \frac{3}{4} \)

41. Nine feet more than twice the height of a shrub is 12 feet. Find the height of the shrub.

42. The difference between Joe’s bank balance and $159 is $2162. Find Joe’s bank balance.

43. There are 82 more green marbles than blue marbles. The sum of the green and blue marbles is 958. How many green marbles are there?

**Evaluate.**

44. \( 5y + 2 \) when \( y = \frac{1}{5} \)

45. \( 4k - 3 \) for \( k = \frac{1}{4} \)

46. \( 8m + 1 \) when \( m = \frac{3}{8} \)
Additional Practice with Solving Equations Involving Fractions

Solve.

47. $5x + 2 = 6$
48. $6x - 10 = -14$
49. $8x + 9 = 2x + 12$
50. $4x - 6 = -2x - 13$
51. $-8(2x + 1) = 20$
52. $-3(4x - 2) = -21$
53. $\frac{1}{3}x + \frac{2}{9} = \frac{5}{6}x + \frac{1}{2}$
54. $\frac{3}{4}x - \frac{4}{9} = \frac{7}{18}x - \frac{2}{3}$
55. $\frac{1}{5}x + 2 = \frac{3}{8}$
56. $\frac{3}{10}x - \frac{1}{4} = 3$
57. $x + \frac{7}{12} = 2$
58. $x - \frac{2}{3} = \frac{7}{6}$
59. $\left(\frac{2}{7}\right)x - 3 = -8$
60. $\frac{5}{12}x + \frac{1}{6} = -\frac{4}{3}$
61. $\frac{3}{5}x = \frac{7}{10}$
62. $\frac{3}{8}x = 6
Answer Key for Chapter 4 Section 5.

1. \( x = \frac{3}{5} \)
2. \( m = \frac{5}{4} \)
3. \( k = -\frac{1}{9} \)
4. \( v = -\frac{9}{10} \)
5. \( w = -\frac{5}{6} \)
6. \( x = -\frac{9}{11} \)
7. \( y = \frac{2}{12} = \frac{1}{6} \)
8. \( A = \frac{-6}{18} = -\frac{1}{3} \)
9. \( m = \frac{-6}{-8} = \frac{3}{4} \)
10. \( B = \frac{10}{-15} = -\frac{2}{3} \)
11. \( x = -\frac{8}{5} \)
12. \( x = \frac{4}{12} = \frac{1}{3} \)
13. \( k = -\frac{9}{11} \)
14. \( m = \frac{15}{-6} = -\frac{5}{2} \)
15. \( z = \frac{25}{15} = \frac{5}{3} \)
16. \( m = -\frac{5}{16} \)
17. \( x = -\frac{1}{13} \)
18. \( x = \frac{-43}{-20} = \frac{43}{20} \)
19. \( x = \frac{-6}{-15} = \frac{2}{5} \)
20. \( M = \frac{-6}{-10} = \frac{3}{5} \)
21. \( m = \frac{5}{24} \)
22. \( y = -\frac{11}{15} \)
23. \( k = \frac{7}{60} \)
24. \( z = \frac{35}{3} \)
25. \( a = \frac{-4}{5} \)
26. \( g = -\frac{13}{10} \)
27. \( w = \frac{39}{8} \)
28. \( x = \frac{90}{7} \)
Chapter 4 Section 5
Solve equations involving fractions

29. \( x = \frac{14}{3} \)

30. \( x = \frac{9}{2} \)

31. \( x = -\frac{29}{3} \)

32. \( x = \frac{9}{23} \)

33. \( x = -4 \)

34. \( x = -\frac{11}{2} \)

35. \( x = \frac{4}{11} \)

36. \( x = \frac{3}{2} \)

37. \( x = \frac{17}{21} \)

38. \( x = -\frac{33}{104} \)

39. \( x = -\frac{25}{64} \)

40. \( x = \frac{54}{61} \)

41. The shrub is \(1\frac{1}{2}\) feet tall.

42. Joe’s bank balance is $2321.

43. There are 520 green marbles.

44. 3

45. -2

46. 4

47. \( x = \frac{4}{5} \)

48. \( x = -\frac{4}{6} = -\frac{2}{3} \)

49. \( x = \frac{3}{6} = \frac{1}{2} \)

50. \( x = -\frac{7}{6} = -\frac{7}{6} \)

51. \( x = \frac{28}{-16} = -\frac{7}{4} \)

52. \( x = -\frac{27}{-12} = \frac{9}{4} \)

53. \( x = \frac{5}{-9} = -\frac{5}{9} \)

54. \( x = -\frac{8}{13} = -\frac{8}{13} \)

55. \( x = -\frac{65}{8} = -\frac{65}{8} \)

56. \( x = \frac{65}{6} \)

57. \( x = \frac{17}{12} \)

58. \( x = \frac{11}{6} \)
Chapter 4 Section 5

Solve equations involving fractions

59. \( x = \frac{-35}{2} = -\frac{35}{2} \)

60. \( x = \frac{-18}{5} = -\frac{18}{5} \)

61. \( x = \frac{7}{6} \)

62. \( x = 16 \)
Chapter 4 Summary

Reduce Algebraic Fractions
\[
\frac{12x^5}{18x^2} = \frac{2 \cdot 2 \cdot 3 \cdot x \cdot x \cdot x \cdot x}{2 \cdot 3 \cdot 3 \cdot x \cdot x} = \frac{2x^3}{3} \rightarrow
\]

Multiply Algebraic Fractions
\[
\left( -\frac{2x}{5} \right) \left( \frac{15}{18x^3} \right) = \frac{-2 \cdot x \cdot 3 \cdot 5}{5 \cdot 2 \cdot 3 \cdot 3 \cdot x \cdot x} = \frac{-1}{3x^2} \rightarrow
\]

Divide Algebraic Fractions
\[
\left( -\frac{8}{15a^2} \right) \div \left( -\frac{6}{5a} \right) = \left( -\frac{8}{15a^2} \right) \cdot \left( \frac{5a}{6} \right) = \frac{-2 \cdot 2 \cdot 2}{3 \cdot 5 \cdot a \cdot a} \cdot \left( \frac{-5 \cdot a}{2 \cdot 3} \right) = \frac{4}{9a} \rightarrow
\]

Add/Subtract Algebraic Fractions
a. \[
\frac{2x}{15} - \frac{12x}{15} = \frac{-10x}{15} = -\frac{2x}{3} \rightarrow
\]
b. \[
\frac{2x}{15} - \frac{12}{15} = \frac{2x - 12}{15} \rightarrow
\]
c. \[
\frac{12x - 3}{19} - \frac{2x - 10}{19} = \frac{10x + 7}{19} \rightarrow
\]
d. \[
\frac{5}{x} + \frac{2}{x} = \frac{7}{x} \rightarrow
\]
e. \[
-\frac{5x}{6} - \frac{3x}{4} = -\frac{5x}{6} \left( \frac{2}{2} \right) - \frac{3x}{4} \left( \frac{3}{3} \right) = -\frac{10x}{12} - \frac{9x}{12} = -\frac{19x}{12} \rightarrow
\]
Chapter 4

Mixed Numbers

Change mixed numbers to improper fractions; then add, subtract, multiply, or divide as indicated

a. \(-2\frac{5}{6} + 3\frac{1}{5} - \frac{17}{6} + \frac{16}{5}\)
   
   \[-\frac{17}{6}\left(\frac{5}{5}\right) + \frac{16}{5}\left(\frac{6}{6}\right)\]
   
   \[-\frac{85}{30} + \frac{96}{30} \quad \frac{11}{30} \quad \Rightarrow\]

b. \(-3\frac{3}{4} \cdot -2\frac{2}{5} \cdot -\frac{15}{4} \cdot \frac{12}{5}\)
   
   \[-\frac{3\cdot5}{2\cdot2} \cdot \left(-\frac{2\cdot2\cdot3}{5}\right) \quad \Rightarrow 9\]

Solve Equations

a. \(2x + 7 = 2\)
   
   \[-7 - 7\]
   
   \[-2x = -5\]
   
   \[\frac{2x}{2} = \frac{-5}{2}\]
   
   \[x = \frac{-5}{2}\]
Chapter 4 Summary

Solve Equations

When the equation contains fractions, one technique for solving is to clear the fractions by multiplying all terms by the same common denominator.

b. \[ \frac{1}{6}x + 5 = \frac{4}{9} \]

\[ 18 \left( \frac{1}{6}x \right) + 18(5) = 18 \left( \frac{4}{9} \right) \]

\[ 3(1x) + 18(5) = 2(4) \]

\[ 3x + 90 = 8 \]

\[ -90 \quad -90 \]

\[ 3x = -82 \]

\[ \frac{3x}{3} = \frac{-82}{3} \]

\[ x = -\frac{82}{3} \]

c. Recall, you should also check your solutions.
Chapter 4 Review

After the answer key to Chapter 4 Review, there are additional problems involving fractions; these are to give you more practice.

Reduce the following to lowest terms.

1. \( \frac{48}{-64} \)
2. \( \frac{17}{29} \)
3. \( \frac{x^3 y^5}{xy^2} \)
4. \( \frac{-a^3 b^4}{-a^3 b^5} \)
5. \( \frac{-45RT^4}{75R^2T} \)

Simplify.

6. \( \left( -\frac{4}{9} \right) \left( \frac{35}{8} \right) \left( -\frac{3}{25} \right) \)
7. \( \frac{3a^3}{14} \cdot \left( -\frac{42}{7a} \right) \)
8. \( \left( -\frac{1}{3} \right)^2 \)
9. \( -\frac{z^4}{9} \div \frac{z^3}{27} \)
10. \( \frac{a^4}{b^4} \div \frac{a^5}{b^3} \)
11. \( -\frac{5x}{14} + \frac{3x}{14} \)
12. \( \frac{6a + 18}{5} - \frac{2a + 4}{5} \)
13. \( \frac{7x - 19}{6} - \frac{6x - 12}{6} \)
14. \( \frac{3}{10} - \frac{2}{15} + \frac{1}{6} \)
15. \( -\frac{3x}{5} \left( -\frac{2x}{3} \right) \)
16. \( \frac{-11x}{14} + \frac{2}{21} \)
17. \( \frac{4}{k} - \frac{1}{4} \)
18. \( -2 \frac{1}{4} \left( -3 \frac{1}{3} \right) \)
19. \( -\frac{3}{2} \div 5 \frac{1}{10} \)
Chapter 4

Review

20. \[-2 \frac{1}{4} + 3 \frac{2}{3} - 1 \frac{7}{10}\]

21. \[-1 \frac{1}{3} - 1 \frac{1}{6} + 2 \frac{3}{4}\]

22. \[3 \frac{5}{9} - 8 \frac{1}{8}\]

Solve.

23. \[3m + 2 = 4\]

24. \[5y - 6 = -10\]

25. \[4(k + 2) = 3\]

26. \[-4x - 9 = 2x - 11\]

27. \[2y - (5y + 1) = 1\]

28. \[a + \frac{2}{3} = -\frac{3}{4}\]

29. \[x - \frac{1}{5} = -\frac{1}{2}\]

30. \[\frac{3}{5}x - 2 = \frac{1}{3}\]

31. \[\frac{1}{4}t - \frac{1}{6} = -\frac{4}{9}\]

32. \[-3y - \frac{1}{8} = -2 \frac{1}{2}\]

33. \[\left(\frac{3}{11}\right)x + \frac{1}{2} = \frac{3}{4}\]

34. \[\left(-\frac{7}{9}\right)x - 2 = 3\]

35. \[\frac{2}{3}x + \frac{5}{6} = \frac{7}{8}x - \frac{1}{2}\]

36. \[\frac{1}{6}x - \frac{7}{9} = \frac{2}{3}x + \frac{3}{2}\]
## Answer Key for Chapter 4 Review.

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>$-\frac{3}{4}$</td>
</tr>
<tr>
<td>2.</td>
<td>$\frac{17}{29}$</td>
</tr>
<tr>
<td>3.</td>
<td>$x^2 y^3$</td>
</tr>
<tr>
<td>4.</td>
<td>$b$</td>
</tr>
<tr>
<td>5.</td>
<td>$-\frac{37^3}{5R^2}$</td>
</tr>
<tr>
<td>6.</td>
<td>$\frac{7}{30}$</td>
</tr>
<tr>
<td>7.</td>
<td>$-\frac{9a^2}{7}$</td>
</tr>
<tr>
<td>8.</td>
<td>$\frac{1}{9}$</td>
</tr>
<tr>
<td>9.</td>
<td>$-3z$</td>
</tr>
<tr>
<td>10.</td>
<td>$\frac{1}{ab}$</td>
</tr>
<tr>
<td>11.</td>
<td>$-\frac{x}{7}$</td>
</tr>
<tr>
<td>12.</td>
<td>$\frac{4a + 14}{5}$</td>
</tr>
<tr>
<td>13.</td>
<td>$\frac{x - 7}{6}$</td>
</tr>
<tr>
<td>14.</td>
<td>$\frac{1}{3}$</td>
</tr>
</tbody>
</table>
15. \( \frac{x}{15} \)  

16. \( \frac{-33x + 4}{42} \)  

17. \( \frac{16 - k}{4k} \)  

18. \( 7 \frac{1}{2} \)  

19. \( - \frac{2}{3} \)  

20. \( - \frac{17}{60} \)  

21. \( 1 \frac{1}{4} \)  

22. \( -4 \frac{41}{72} \text{ or } -\frac{329}{72} \)  

23. \( m = \frac{2}{3} \)  

24. \( y = -\frac{4}{5} \)  

25. \( k = -\frac{5}{4} \)
26. \[ x = \frac{-2}{-6} = \frac{1}{3} \] 5

27. \[ y = \frac{-2}{3} \] 5

28. \[ a = -\frac{5}{12} \] 5

29. \[ x = \frac{-3}{10} \] 5

30. \[ x = \frac{35}{9} \] 5

31. \[ t = -\frac{10}{9} \] 5

32. \[ y = \frac{19}{24} \] 5

33. \[ x = \frac{11}{12} \] 5

34. \[ x = -\frac{45}{7} \] 5

35. \[ x = \frac{32}{5} \] 5

36. \[ x = -\frac{41}{9} \] 5
Additional Practice with Fractions

Simplify appropriately.

1. \(\frac{24}{30}\)  

2. \(\frac{-48}{75}\)

3. \(\frac{-54}{-72}\)  

4. \(\frac{x^6}{x^4}\)

5. \(\frac{18m^2}{30m^3}\)  

6. \(\frac{15a^3b^7}{-33a^7b^6}\)

7. \(\frac{12}{49} \div \left(\frac{-35}{32}\right)\)  

8. \(-\frac{10}{21} \div \left(-\frac{28}{15}\right)\)

9. \(\frac{k^6}{20} \cdot \frac{36}{k^2}\)  

10. \(-\frac{16}{27} \div \frac{40}{63}\)

11. \(-\frac{18}{56} \div \left(\frac{-30}{77}\right)\)  

12. \(\frac{m^2}{20} \div \frac{m}{30}\)

13. \(\frac{3}{8} - \frac{11}{12}\)  

14. \(-\frac{3a}{7} - \frac{2a}{5}\)

15. \(-\frac{5a}{6} + \frac{4a}{9}\)  

16. \(\frac{7a}{12} + \frac{9a}{20}\)

17. \(1\frac{1}{2} \left(-2\frac{2}{5}\right)\)  

18. \(-9\frac{1}{3} \div \left(-1\frac{1}{6}\right)\)

19. \(-5\frac{1}{8} + 2\frac{4}{5}\)  

20. \(-4\frac{1}{6} - 1\frac{2}{9}\)

21. \(-\frac{5m}{18} + \frac{-7m}{24}\)  

22. \(\frac{3a}{10} - \frac{7a}{15}\)

23. \(-\frac{18}{-40} \div \frac{27}{-56}\)  

24. \(\frac{24a^3}{36a}\)
Solve. Where appropriate, clear the fractions (by multiplying all terms by the same LCD so the original denominators cancel). However, use any method that makes sense to you!

25. \( \frac{3}{8}x + \frac{5}{6} = \frac{5}{4}x + \frac{2}{3} \)  
26. \( \frac{2}{9}x - \frac{1}{3} = \frac{5}{6}x - \frac{1}{2} \)

27. \( \frac{2}{5}x + \frac{1}{4} = -\frac{7}{10} \)  
28. \( \frac{4}{15}x - \frac{5}{6} = \frac{2}{3} \)

29. \( \frac{5}{9}x - \frac{1}{2} = -\frac{7}{18} \)  
30. \( \frac{1}{2}x + \frac{3}{4} = \frac{1}{6} \)

31. \( \frac{3}{7}x + \frac{1}{2} = 3 \)  
32. \( -\frac{3}{4}x + \frac{5}{14} = -2 \)

33. \( \frac{2}{5}x = 3 \)  
34. \( \frac{5}{8}x = -\frac{2}{3} \)

35. \( x + \frac{2}{3} = \frac{3}{5} \)  
36. \( x - \frac{3}{4} = \frac{1}{2} \)

37. \( x + \frac{2}{5} = -\frac{3}{2} \)  
38. \( x - \frac{3}{4} = -\frac{9}{10} \)

39. \( 5x + 7 = 10 \)  
40. \( 6x - 8 = 20 \)

Simplify.

41. \( \frac{8y + 3}{6} + \frac{11y + 10}{6} \)  
42. \( \frac{17k + 14}{9} - \frac{12k + 1}{9} \)

43. \( \frac{8A - 3}{5} - \frac{10A - 12}{5} \)  
44. \( \frac{-6m + 4}{10} - \frac{-3m - 7}{10} \)
Answer Key for Additional Practice with Fractions.

1. \( \frac{4}{5} \)
2. \( -\frac{16}{25} \)
3. \( \frac{3}{4} \)
4. \( x^2 \)
5. \( \frac{3}{5m^3} \)
6. \( -\frac{5b}{11a^2} \)
7. \( -\frac{15}{56} \)
8. \( \frac{8}{9} \)
9. \( \frac{9k^4}{5} \)
10. \( -\frac{14}{15} \)
11. \( \frac{33}{40} \)
12. \( \frac{3m}{2} \)
13. \( -\frac{13}{24} \)
14. \( -\frac{29a}{35} \)
15. \( -\frac{7a}{18} \)
16. \( \frac{31a}{30} \)
17. \( -\frac{18}{5} \)
18. 8
19. \( -\frac{93}{40} \)
20. \( -\frac{97}{18} \)
21. \( -\frac{41m}{72} \)
22. \( -\frac{a}{6} \)
23. \( -\frac{14}{15} \)
24. \( \frac{2a^2}{3} \)
On the equations, this answer key shows the LCD used to clear the fractions. It then shows the resulting equation and the final solution.

25. LCD 24  
   \[9x + 20 = 30x + 16\]  
   \[x = \frac{4}{21}\]  
   LCD 18  
   \[4x - 6 = 15x - 9\]  
   \[x = \frac{3}{11}\]

27. LCD 20  
   \[8x + 5 = -14\]  
   \[x = -\frac{19}{8}\]  
   LCD 30  
   \[8x - 25 = 20\]  
   \[x = \frac{45}{8}\]

29. LCD 18  
   \[10x - 9 = -7\]  
   \[x = \frac{2}{10} = \frac{1}{5}\]  
   LCD 12  
   \[6x + 9 = 2\]  
   \[x = -\frac{7}{6}\]

31. LCD 14  
   \[6x + 7 = 42\]  
   \[x = \frac{35}{6}\]  
   LCD 28  
   \[-21x + 10 = -56\]  
   \[x = -\frac{66}{21} = -\frac{22}{7}\]

33. LCD 5  
   \[2x = 15\]  
   \[x = \frac{15}{2}\]  
   LCD 24  
   \[15x = -16\]  
   \[x = -\frac{16}{15}\]

35. LCD 15  
   \[15x + 10 = 9\]  
   \[x = -\frac{1}{15}\]  
   LCD 4  
   \[4x - 3 = 2\]  
   \[x = \frac{5}{4}\]

37. LCD 10  
   \[10x + 4 = -15\]  
   \[x = -\frac{19}{10}\]  
   LCD 20  
   \[20x - 15 = -18\]  
   \[x = -\frac{3}{20}\]

39. no LCD needed  
   \[x = \frac{3}{5}\]  
   40. no LCD needed  
   \[x = \frac{28}{6} = \frac{14}{3}\]
Chapter 4

Review

41. \( \frac{19y + 13}{6} \)

42. \( \frac{5k + 13}{9} \)

43. \( \frac{-2A + 9}{5} \)

44. \( \frac{-3m + 11}{10} \)
Chapter 4 Test

Reduce the following to lowest terms.

1. \( \frac{-24}{64} \)

2. \( \frac{-16}{-80} \)

3. \( \frac{x^2y^3}{xy} \)

4. \( \frac{15x^3y^4}{-60x^2y^2} \)

Simplify.

5. \( \left( \frac{-2}{3} \right)^2 \)

6. \( \left( \frac{2}{3} \right) \left( \frac{-3}{4} \right) \left( \frac{6}{7} \right) \)

7. \( \frac{3x^2 \cdot 5}{7 \cdot 6x} \)

8. \( \frac{-y^2}{2x^2} \div \frac{3y^2}{4x} \)

9. \( \frac{6y}{25} + \frac{4y}{25} \)

10. \( \frac{10x + 3}{8} - \frac{3x + 12}{8} \)

11. \( \frac{3x}{4} \cdot \frac{5x}{6} \)

12. \( \frac{5y + 3}{14} \cdot \frac{3}{7} \)

13. \( \frac{-5}{x} - \frac{3}{4} \)

14. \( \frac{5 \frac{2}{3} + \left( -2 \frac{1}{2} \right)}{x} \)

15. \( \frac{-1 \frac{1}{4}}{\left( -2 \frac{1}{3} \right)} \)

16. \( \frac{2 \frac{3}{4} + 1 \frac{1}{2}}{2} \)

17. \( \frac{3 \frac{2}{3} - 2 \frac{1}{2} - 1 \frac{1}{2}}{4} \)
Solve.

18. \( 8z + 9 = 3 \)
19. \( -4m - 12 = 5 \)
20. \( 8(k + 2) = 1 \)
21. \( 2y + 2 = 5y + 9 \)
22. \( 2a - (11a - 4) = 2 \)
23. \( x - \frac{2}{3} = -\frac{4}{5} \)
24. \( y + \frac{3}{4} = \frac{1}{3} \)
25. \( \frac{1}{2}z + \frac{1}{3} = -\frac{2}{5} \)
26. \( 4x - \frac{1}{6} = 3 \)
27. \( \frac{5}{9}x + 3 = \frac{2}{3} \)
28. \( \left(\frac{3}{7}\right)x + 5 = 9 \)
29. \( \left(\frac{11}{3}\right)x - \frac{1}{4} = 2 \)
30. \( \frac{5}{9}x - \frac{1}{6} = \frac{1}{2}x + \frac{2}{3} \)
Answer Key for Chapter 4 Test.

1. \(-\frac{3}{8}\)  
2. \(\frac{1}{5}\)  
3. \(x y^2\)  
4. \(-\frac{y^2}{4}\)  
5. \(\frac{4}{9}\)  
6. \(-\frac{3}{7}\)  
7. \(\frac{5x}{14}\)  
8. \(-\frac{2y^3}{3x}\)  
9. \(\frac{2y}{5}\)  
10. \(\frac{7x-9}{8}\)  
11. \(-\frac{x}{12}\)  
12. \(\frac{5y+6}{14}\)  
13. \(-\frac{20-3x}{4x}\)  
14. \(-\frac{85}{6} \text{ or } -14\frac{1}{6}\)  
15. \(\frac{15}{28}\)  
16. \(\frac{17}{4} \text{ or } 4\frac{1}{4}\)  
17. \(-\frac{1}{12}\)  
18. \(z = -\frac{3}{4}\)  
19. \(m = -\frac{17}{4}\)  
20. \(k = -\frac{15}{8}\)  
21. \(y = -\frac{7}{3}\)  
22. \(a = -\frac{2}{9} = \frac{2}{9}\)  
23. \(x = -\frac{2}{15}\)  
24. \(y = -\frac{5}{12}\)  
25. \(z = -\frac{22}{15} \text{ or } -1\frac{7}{15}\)  
26. \(x = \frac{19}{24}\)
27. \( x = -\frac{21}{5} \text{ or } -4 \frac{1}{5} \)

28. \( x = \frac{28}{3} \text{ or } 9 \frac{1}{3} \)

29. \( x = \frac{27}{44} \)

30. \( x = 15 \)
Cumulative Review #4 Through Chapter 4

1. Does $2^5 = 5^2$?

2. What is the sign (positive or negative) of the following (do not determine the numerical answer):
   
   $\frac{-3(-2)(-5)(4)}{(-6)(-8)(-9)(-1)}$

Reduce to lowest terms.

3. $\frac{14}{37}$

4. $\frac{-36x^2y^2}{48x^2y}$

Simplify.

5. $(3^2 + 2)^2 - (3 + 2)^2$

6. $(4^2 - 14) - (-2 + 3)$

7. $\left(-\frac{42}{7}\right)(-4) - (-3)(-2)$

8. $-6(-1 + 3x) - 6x$

9. $\frac{8k + 20}{-2}$

10. $7(2m - 8) - (3m - 5)$

11. $-5 - |-1-8| - (-10)$

12. $-\frac{9}{16} \left(\frac{6}{39}\right) \left(\frac{20}{27}\right)$

Simplify.

13. $\frac{a^3}{b^2} \div \frac{a^2}{b^3}$

14. $\frac{4 + 3k}{11} - \frac{7 - 2k}{11}$

15. $\frac{4}{9} + \frac{3}{4} - \frac{5}{6}$

16. $\frac{3}{4} - \frac{3x}{10}$

17. $-\frac{3\frac{1}{8}}{\div} 2\frac{1}{2}$

18. Evaluate $8 - x + (-3)$ when $x = -5$
Solve.

19. \[ k - 14 = -13 \]
20. \[ -2r - 6 + r = 3 \]
21. \[ A + A + 8 = 8 \]
22. \[ -3x + 5 - 2x - 2 = -3x - 5 \]

Solve.

23. \[ -4(x - 8) + (-x - 3) = 4 \]
24. \[ 3y - 8 = -6 \]
25. \[ -2(a + 5) = 11 \]
26. \[ x + \frac{2}{3} = \frac{7}{8} \]
27. \[ \frac{7}{15}x + \frac{5}{6} = \left( \frac{2}{3} \right)x + \frac{3}{5} \]

28. First, label the unknown. Second, translate the English words into math symbols.

The difference between 98 and three times a number

For each of the following, first label the unknowns. Second, set up an equation. Third, solve. Fourth, answer the question(s) asked.

29. Glenn took Michelle to a fancy dinner. If Michelle’s dinner cost $12 more than Glenn’s and the total cost was $86, how much did Michelle’s dinner cost?

30. Mary is 3 years older than twice Joe. The sum of their ages is 18. Find the age of each person.
Answer Key for Cumulative Review #4 through Chapter 4.

Reference Chapter

1. no 1
2. negative 2a
3. $\frac{14}{37}$ 4
4. $-\frac{3xy}{4}$ 4
5. 72 1
6. 1 2a
7. 18 2b
8. $-24x + 6$ 2b
9. $-4k - 10$ 2b
10. $11m - 51$ 2b
11. $-4$ 2b
12. $-\frac{5}{78}$ 4
13. $ab$ 4
14. $\frac{-3 + 5k}{11}$ or $\frac{5k - 3}{11}$ 4
15. $\frac{13}{36}$ 4
16. $\frac{15 - 6x}{20}$ 4
17. $-\frac{5}{4}$ or $-1\frac{1}{4}$ 4
18. 10  
19. \( k = 1 \)  
20. \( r = -9 \)  
21. \( A = 0 \)  
22. \( x = 4 \)  
23. \( x = 5 \)  
24. \( y = \frac{2}{3} \)  
25. \( a = -\frac{21}{2} \)  
26. \( x = \frac{5}{24} \)  
27. \( x = \frac{7}{6} \)  
28. the number = \( x \)  
\[98 - 3x\]  
29. cost of Glenn’s dinner = \( x \)  
cost of Michelle’s dinner = \( x + 12 \)  
\[ (x) + (x + 12) = 86 \]  
\[ x = 37 \]  
\[ x + 12 = 49 \]  
Michelle’s dinner cost $49.  
30. Joe’s age = \( x \)  
Mary’s age = \( 2x + 3 \)  
\[ (x) + (2x + 3) = 18 \]  
\[ x = 5 \]  
\[ 2x + 3 = 13 \]  
Joe is 5 years old; Mary is 15 years old.
Chapter 5: Decimals and Formulas

Chapter 5 presents ideas about decimals and formulas.

You have all the basic skills you need for algebra: how to simplify expressions, how to solve equations, and how to evaluate expressions. This chapter will give you the opportunity to further fine-tune those skills.

The first section will concentrate on solving equations that involve decimals. It will be assumed that you remember the arithmetic of decimals (how to add, subtract, multiply, and divide).

The other sections will present various formulas. Within those sections, you will be working with whole numbers, fractions, and decimals. You will also learn a little bit about percents. Further, you will use the algebra skills for evaluating expressions, simplifying expressions, and solving equations.

This chapter gives you another opportunity to be sure you understand what you have learned. It also gives you the opportunity to apply this learning.
Section 1: Solve Equations With Decimals

You are now ready to solve equations with decimals. You will use all the techniques for solving that you have previously learned. Observe.

\[ \text{a. } 0.2m + 5.6 = 8.4 \]
\[ -5.6 \]
\[ 0.2m = 2.8 \]
\[ \frac{0.2m}{0.2} = \frac{2.8}{0.2} \]
\[ m = 14 \]

\[ \text{b. } -3y - 6.12 = -8.34 \]
\[ +6.12 \]
\[ -3y = -2.22 \]
\[ \frac{-3y}{-3} = \frac{-2.22}{-3} \]
\[ y = 0.74 \]

\[ \text{c. } 1.5z + 9.531 = 2.79 \]
\[ -9.531 \]
\[ 1.5z = -6.741 \]
\[ \frac{1.5z}{1.5} = \frac{-6.741}{1.5} \]
\[ z = -4.494 \]

\[ \text{d. } -0.025k - 3.1 = -1.634 \]
\[ +3.1 \]
\[ -0.025k = 1.466 \]
\[ \frac{-0.025k}{-0.025} = \frac{1.466}{-0.025} \]
\[ k = -58.64 \]

As you can see, you already know the basic techniques for solving equations. These examples above, involving decimals, follow the same procedures.

\[ \text{e. } 0.4a - 9.32 = -2.76 \]
\[ 0.4a - 9.32 + 9.32 = -2.76 + 9.32 \]
\[ 0.4a = 6.56 \]
\[ \frac{0.4a}{0.4} = \frac{6.56}{0.4} \]
\[ a = 16.4 \]

Check:
\[ 0.4a - 9.32 = -2.76 \]
\[ 0.4(16.4) - 9.32 = -2.76 \]
\[ 6.56 - 9.32 = -2.76 \]
\[ -2.76 = -2.76 \]
\[ a = 16.4 \text{ is the solution.} \]
f. \[ 2x - 1 = 8 \]
\[
2x = 9 \\
x = \frac{9}{2} \text{ or } 4.5
\]

g. \[ 3x + 2 = 7 \]
\[
3x = 5 \\
x = \frac{5}{3} \text{ or } 1\frac{2}{3}
\]

Notice on example g, the result is left in fraction form because \( 5 \div 3 \) results in a nonterminating decimal (\( 5 \div 3 \) is approximately 1.667 after rounding off).

On example f, the three different results shown are all exact results. In most cases, it is better to leave a result in fraction form than to change to decimal form.
Homework for Chapter 5 Section 1

Solve.

1. \(2x + 8.5 = 20.4\)
2. \(3m + 5.7 = -18.3\)
3. \(5y - 6.23 = -2.4\)
4. \(4z - 9.81 = 0.215\)
5. \(0.2a + 9 = 11.4\)
6. \(0.5m + 17.6 = 5\)
7. \(-0.6y - 9.3 = 1.206\)
8. \(-0.08y - 12.42 = -4.7\)
9. \(4(x - 2) = 5\)
10. \(3k - 22 = 7k - 16\)
11. \(12z = 2\)
12. \(2x + 3 + x + 5 = 11\)
13. \(\frac{5}{12} + \frac{2}{3} = \frac{1}{8}\)
14. \(\frac{7}{15}x - 4 = 2\)
15. \(\left(\frac{3}{7}\right)x + \frac{1}{2} = \frac{3}{4}x - \frac{5}{14}\)

Simplify.

16. \(\frac{24}{3} + 3(5 - 7) - \left(\frac{20}{5}\right)^2\)
17. \(\frac{1}{2} \left(\frac{4}{9}\right) + \frac{5}{6}\)
18. \(\frac{17x + 11}{8} - \frac{2x - 6}{8}\)
19. \(5.7 + 3.2(4.1 + 7.3) + (2.5)^2\)

Set up and solve.

20. Nine cars more than three times the inventory of a dealership is 2754 cars. Find the number of cars.

21. The difference between the number of Mary's outfits and 27 is 19. How many outfits does Mary have?

22. There are 78 more trees than shrubs planted in the park. The sum of the trees and shrubs is 94. How many trees are planted in the park?

Evaluate.

23. \(3m + 9.2\) when \(m = 1.8\)
Chapter 5 Section 1

Solve equations with decimals

Answer Key for Chapter 5 Section 1.

1. \( x = 5.95 \)
2. \( m = -8 \)
3. \( y = 0.766 \)
4. \( z = 2.50625 \)
5. \( a = 12 \)
6. \( m = -25.2 \)
7. \( y = -17.51 \)
8. \( y = -96.5 \)
9. \( x = \frac{13}{4} \)
10. \( k = -\frac{6}{4} = -\frac{3}{2} \)
11. \( z = \frac{2}{12} = \frac{1}{6} \)
12. \( x = 1 \)
13. \( g = -\frac{13}{10} \)
14. \( x = \frac{90}{7} \)
15. \( x = \frac{-24}{-9} = \frac{8}{3} \)
16. \( 8 + 3(-2) - (4)^2 \)
17. \( \frac{2}{9} + \frac{5}{6} \)
18. \( \frac{(17x+11)-(2x-6)}{8} \)
19. \( 5.7 + 3.2(11.4) + 6.25 \)
20. There are 915 cars.
21. Mary has 46 outfits.
22. There are 86 trees planted in the park.
23. 14.6
Section 2: Percent Formula: \( \text{percent} \cdot (\text{whole}) = \text{part} \)

There are many ways to work problems containing percents. This section will present an algebraic way of approaching percent problems. First, though, a brief review of converting percents to decimals and vice versa.

Convert the following percents to decimals. Recall that the word percent means “per 100”; also recall that “per” means divide. To convert a percent to a decimal, divide by 100. The shortcut is to drop the percent symbol and to move the decimal point two places to the left. Observe.

- a. \( 18\% = 18 \div 100 = 0.18 \)
- b. \( 30\% = 30 \div 100 = 0.30 = 0.3 \)
- c. \( 9\% = 9 \div 100 = 0.09 \)
- d. \( 0.4\% = 0.4 \div 100 = 0.004 \)
- e. \( 112\% = 112 \div 100 = 1.12 \)

Convert the following decimals to percent. To do so, reverse the thinking from above. Multiply the decimal by 100 and write the percent symbol. The shortcut is to move the decimal point two places to the right and write a percent symbol. Observe.

- f. \( 0.29 = 0.29(100)\% = 29\% \)
- g. \( 0.476 = 0.476(100)\% = 47.6\% \)
- h. \( 0.02 = 0.02(100)\% = 2\% \)
- i. \( 0.008 = 0.008(100)\% = 0.8\% \)
- j. \( 1.043 = 1.043(100)\% = 104.3\% \)

In the following examples, you will be using the formula \( \text{percent} \cdot (\text{whole}) = \text{part} \).

As you are reading the problem, you will need to identify the percent (it will be identified by the percent symbol or by the word percent). Then identify the whole (all the items) and the part (only some of the items). As you are identifying, use a variable for anything unknown. Once you have identified these, substitute the numbers and the variable into the formula. Remember to convert any percent into a decimal as you are substituting into the formula. Then use your equation-solving skills to isolate the variable. Observe.
k. In a shipment, 5 airplane bolts were defective and thrown away. If that represented 4% of the shipment, how many airplane bolts were in the shipment to begin with?

percent = 4% = 0.04
whole = all the airplane bolts = x
part = 5 defective airplane bolts

Formula: \( \text{percent} \cdot (\text{whole}) = \text{part} \)
\[
0.04(x) = 5 \\
0.04x = 5 \\
\frac{0.04x}{0.04} = \frac{5}{0.04} \\
x = 125 \text{ airplane bolts}
\]

There were 125 airplane bolts in the shipment.

l. Seventy-five people attended a lecture. Fifty-five of them had seen the guest speaker before. What percent had seen the guest speaker before?

percent = x
whole = 75 total people
part = 55 had seen the guest speaker before

Formula: \( \text{percent} \cdot (\text{whole}) = \text{part} \)
\[
x(75) = 55 \\
\frac{75x}{75} = \frac{55}{75} \\
x = \frac{55}{75} \text{ or } 0.7333333... \\
\text{since this is a repeating decimal, round} \\
x = 0.73 \\
\text{use } = \text{ to indicate} \\
\text{“approximately equals”} \\
x = 73\% \\
\]

About 73% of the people had seen the guest speaker before.
An orchard contains 900 trees. During a windstorm, 13% of the trees were damaged. How many trees were damaged?

percent = 13% = 0.13
whole = 900 total trees
part = number of trees damaged = x

Formula: \[ \text{percent} \times \text{whole} = \text{part} \]
\[ 0.13(900) = x \]

117 trees = x

There were 117 trees damaged.

Notice that you incorporated your knowledge about solving equations as well as your knowledge about decimals (and now percents) in order to solve these problems. Knowing how to substitute into a formula and knowing how to isolate a variable is powerful!

NOTE: When you are using the formula \( \text{percent} \times \text{whole} = \text{part} \), you are using a decimal for the percent. You could change the decimal to a fraction and then proceed with your solving. Observe.

On a test, Joe received an 88%. If Joe had 44 problems correct, how many problems were on the test?

percent = 88% = 0.88
whole = total problems = x
part = 44 problems correct

Formula: \[ \text{percent} \times \text{whole} = \text{part} \]
\[ 0.88(x) = 44 \]

\[ x = \frac{44}{0.88} = 50 \]

The test had 50 problems.
Homework for Chapter 5 Section 2

Set up and solve.

1. A basketball player shoots 20 times and makes 18 baskets. What is this percent?

2. A statement in the newspaper says, "Nine out of 12 people surveyed are satisfied with the water district." What percent is this?

3. In a class of 40 people, 25% receive an A. How many people receive an A?

4. A store had to destroy 8 items. If this represented 10% of the shipment the store received, how many items were in the shipment?

5. At a football game, 12,000 people were rooting for the opposition. If this represents 30% of the number of people in attendance, how many people were in attendance?

6. In a theater of 250 people, 8% hated the movie. How many hated the movie?

7. A math instructor asked 6 people to redo a quiz and turn it back in. If this represents 12% of the students in class, how many students are in class?

8. A couple bought a stereo system for $1500. If the tax rate is 7%, how much tax did the couple pay?

Additional Practice with Percent \cdot (Whole) = Part

9. A math student took a 20 question test and had 17 correct. What percent is this?

10. A merchandiser has to return 10 items in a 200-item shipment. What percent is this?

11. A shipment of 3000 airplane parts has 5 defective parts. What percent is this?

12. A small country has 100,000 people; 20,000 left the country on vacation. What percent is this?

13. In a class of 32 people, 25% quit attending. How many people quit attending?

14. In an auditorium of 400 people, 12% had seen the speaker before. How many people had seen the speaker before? How many had NOT seen the speaker before?
15. A college has 30,000 people. If the college has a 15% drop-out rate, how many people drop out? How many people do not drop out?

16. A store had to destroy 12 items. If this represented 5% of the shipment the store received, how many items were in the shipment?

17. At a football game 15,000 people were rooting for the opposition. If this represents 20% of the number of people in attendance, how many people were in attendance?

18. At a restaurant, a group of people left $27 for the tip. If this represented 15% of the bill, what was the bill?

19. A newspaper reported, "After the recent flooding along the riverbanks, 2 out of 5 homeowners say they will move." What percent is this?

20. In a Nebraska town of 12,000 people, 80% once lived on a farm. How many once lived on a farm?

21. A math instructor asked 12 people to redo a quiz and turn it back in. If this represents 25% of the students in class, how many students are in class?

22. A couple bought a stereo system for $3,200. If the tax rate is 7%, how much tax did the couple pay? How much was the total bill?

23. A salesperson at a jewelry counter receives 4% commission on sales. If the salesperson sold $8,000 one week, what commission was earned?

24. The same salesperson who earns 4% commission received $800 as a commission payment. How much did the salesperson sell to earn this commission?

25. A mother had to buy school clothes. The clothes cost $356. If the tax rate is 6.5%, how much tax did she owe? How much was the total bill? If she paid for the clothes with 4 one-hundred-dollar bills, what was her change?

26. An appliance store advertises 20% off all refrigerators. What is the discount on a refrigerator that originally sold for $600? What is the sale price of this refrigerator?
Answer Key for Chapter 5 Section 2.

1. \( x = 90\% \)
   The basketball player makes 90% of the shots.

2. \( x = 75\% \)
   Seventy-five percent of the people are satisfied with the water district.

3. \( x = 10 \)
   Ten people receive an A.

4. \( x = 80 \)
   Eighty items are in the shipment.

5. \( x = 40,000 \)
   Forty thousand people are in attendance.

6. \( x = 20 \)
   Twenty people hated the movie.

7. \( x = 50 \)
   Fifty people are in class.

8. \( x = 105 \)
   The tax is $105.

9. \( x = 85\% \)
   The student had 85% correct.

10. \( x = 5\% \)
    The merchandiser returned 5% of the shipment.

11. \( x = 0.17\% \)
    About 0.17% of the parts were defective.

12. \( x = 20\% \)
    Twenty percent of the people left on vacation.

13. \( x = 8 \)
    Eight people quit attending.

14. \( x = 48 \)
    Forty-eight people had seen the speaker before; 352 people had not.

15. \( x = 4500 \)
    4500 people drop out; 25,500 people do not.
16. $x = 240$
   There were 240 items in the shipment.

17. $x = 75,000$
   There were 75,000 people in attendance.

18. $x = 180$
   The bill was $180.$

19. $x = 40\%$
   Forty percent say they will move.

20. $x = 9,600$
    9,600 people once lived on a farm.

21. $x = 48$
    There are 48 students in the class.

22. $x = 224$
    The tax was $224$; the total bill was $3,424.$

23. $x = 320$
    The commission was $320.$

24. $x = 20,000$
    The salesperson had $20,000$ in sales.

25. $x = 23.14$
    The tax was $23.14$; the total bill was $379.14$; the change was $20.86.$

26. $x = 120$
    The discount was $120$; the sale price was $480.$
Section 3: Perimeter Formulas

Recall: to find the perimeter of a shape, you add all the sides together. In this section, you will look at two particular perimeter formulas. One formula will be used to find the perimeter of a rectangle. The other formula will be used to find the perimeter (the circumference) of a circle.

The formula for the perimeter of a rectangle is

\[ P_{\text{rectangle}} = 2l + 2w, \]

where \( P \) stands for perimeter, \( l \) stands for length, and \( w \) stands for width. Here is an example of a rectangle:

\[ \text{width} \]

\[ \text{length} \]

How would you use this formula? Observe.

a. Find the perimeter of a rectangular parking lot that has a length of 80 feet and a width of 50 feet.

\[ P = 2l + 2w \]
\[ P = 2(80) + 2(50) \quad \text{substitute} \]
\[ P = 160 + 100 \quad \text{use order of operations} \]
\[ P = 260 \]

The perimeter of the rectangular parking lot is 260 feet.

This formula can also be used when the measurements are in decimals or in fractions. Observe.
b. Find the perimeter of a rectangular field that measures 120.6 yards by 95.8 yards.

\[ P = 2l + 2w \]

\[ P = 2(120.6) + 2(95.8) \]

\[ P = 241.2 + 191.6 \]

\[ P = 432.8 \]

The perimeter of the rectangular field is 432.8 yards.

c. A window measures \( \frac{2 \frac{1}{3}}{4} \) feet by \( \frac{3 \frac{3}{4}}{4} \) feet. Find the perimeter.

\[ P = 2l + 2w \]

\[ P = 2\left(\frac{7}{3}\right) + 2\left(\frac{19}{4}\right) \]

\[ P = \frac{14}{3} + \frac{19}{2} \]

\[ P = \frac{28}{6} + \frac{57}{6} \]

\[ P = \frac{85}{6} \]

The perimeter of the window is \( \frac{14 \frac{1}{6}}{6} \) feet.

d. Find the perimeter of a square with a side of 17 cm.

Recall that all four sides of a square are the same length. So length is 17 cm and width is also 17 cm.

\[ P = 2l + 2w \]

\[ P = 2(17) + 2(17) \]

\[ P = 34 + 34 \]

\[ P = 68 \]

The perimeter of the square is 68 cm.
You know what a circle looks like:

The perimeter of a circle is called the **circumference**. Observe the formula for the circumference of a circle: \( C_{\text{circle}} = 2 \pi r \), where \( C \) stands for circumference, the symbol \( \pi \) (pi) is **approximately** 3.14 (the \( \pi \) symbol on your calculator may show 3.141592654, and this number is still an approximation), and \( r \) stands for the radius of the circle. Recall the radius is the distance from the center of the circle to any point on the edge of the circle. Also, because \( \pi \) is approximately 3.14, all results obtained when using \( \pi \) as 3.14 are also approximations. This will be reflected in the final result.

e. Find the circumference of a circular swimming pool with a radius of 4 feet.

\[
C = 2 \pi r \\
C \approx 2 \times (3.14)(4) \\
C \approx 25.12 \\
\]

The circumference of the circular swimming pool is approximately 25.12 feet.

f. Find the circumference of a circular tank with a radius of 6.3 meters.

\[
C = 2 \pi r \\
C \approx 2 \times (3.14)(6.3) \\
C \approx 39.564 \\
\]

The circumference of the circular tank is approximately 39.564 meters.

g. Find the circumference of a circular window with a radius of \( \frac{11}{3} \) feet.

\[
C = 2 \pi r \\
C \approx 2 \left( \frac{314}{1} \right) \left( \frac{11}{3} \right) \\
C \approx \frac{2512}{3} \approx 837.3333... \\
C \approx 8.37 \\
\]

The circumference of the circular window is approximately 8.37 feet.
Homework for Chapter 5 Section 3

Choose the appropriate formula and solve.

1. Find the perimeter of a rectangular building that measure 77 feet by 56 feet.
2. Find the perimeter of a square with a side of 28 inches.
3. A fence is to be put up around a rectangular parking lot. How many feet of fence are needed if the lot measures 150.5 feet by 120.25 feet?
4. What is the perimeter of a square window that is 1.8 yards on a side?
5. Find the perimeter of a rectangular garden that measures $4 \frac{1}{3}$ feet by $3 \frac{5}{6}$ feet.
6. Find the perimeter of a square that measures $1 \frac{2}{3}$ feet on a side.
7. What is the circumference of a circle with a radius of 7 feet?
8. Find the circumference of a circular pan that has a radius of 4.5 inches.
9. Find the circumference of a circle with a radius of $2 \frac{1}{2}$ feet.
10. What is the circumference of a circle with a radius of $4 \frac{2}{3}$ inches?

Simplify.

11. $4 + 3(8 + 6) - 12 ÷ 4$
12. $8 + 7(3 + 2m) + 11 - 10m$
13. $4(3y - 12) - 2(5y - 6)$
14. $\frac{2x}{3} + \frac{5x}{7}$

Solve.

15. $2m - 17 = 21$
16. $16 - 5k = 24 + 3k$
17. $6 + 3(4d + 8) = 54$
18. $-8a + 20 = 14$
19. $\frac{1}{2}x + \frac{5}{6} = \frac{7}{12}$
20. $0.4y - 0.296 = 1.356$
Answer Key for Chapter 5 Section 3.

1. The perimeter of the rectangular building is 266 feet.

2. The perimeter of the square is 112 inches.

3. There are 541.5 feet of fence needed.

4. The perimeter of the square is 7.2 yards.

5. The perimeter of the rectangular garden is \(16 \frac{1}{3}\) feet.

6. The perimeter of the square is \(6 \frac{2}{3}\) feet.

7. The circumference of the circle is approximately 43.96 feet.

8. The circumference of the circular pan is approximately 28.26 inches.

9. The circumference of the circle is approximately 15.7 feet.

10. The circumference of the circle is approximately 29.31 inches.

11. \(43\)

12. \(4m + 40\)

13. \(2y - 36\)

14. \(\frac{29x}{21}\)

15. \(m = 19\)

16. \(k = -1\)

17. \(d = 2\)

18. \(a = \frac{-6}{-8} = \frac{3}{4}\)

19. clear with LCD 12

   \(6x + 10 = 7\)

   \(x = \frac{-3}{6} = -\frac{1}{2}\)

20. \(y = 4.13\)
Section 4: Area Formulas

In this section, you will find the areas of rectangles, parallelograms, triangles, and circles. Observe the formulas.

For a rectangle, the area formula is:

\[ A_{\text{rectangle}} = lw \]

where \( A \) represents area, \( l \) represents the length, and \( w \) represents the width.

For a parallelogram, the area formula is:

\[ A_{\text{parallelogram}} = bh \]

where \( A \) represents area, \( b \) represents the base, and \( h \) represents the height. Recall the height is the shortest (perpendicular) distance from the top to the base. The figure below is a parallelogram:

![Parallelogram](image)

For a triangle, the area formula is:

\[ A_{\text{triangle}} = \frac{1}{2}bh \]

where \( A \) represents area, \( b \) represents the base, and \( h \) represents the height. Recall in a triangle, the height is the shortest distance from the vertex to the base. The figure below is a triangle:

![Triangle](image)

For a circle, the area formula is:

\[ A_{\text{circle}} = \pi r^2 \]

where \( A \) represents area, \( \pi \) (pi) is approximately 3.14, and \( r \) represents the radius.

When you use these formulas, you will again be using whole numbers, decimals, and fractions. You will hone your skills with substituting and simplifying. As you read each problem, you will need to choose the appropriate formula. Observe.
Chapter 5 Section 4  

**Area formulas**

a. Find the area of a rectangular garden that measures 9.7 feet by 6.4 feet.

\[
A_{\text{rectangle}} = lw \\
A_{\text{rectangle}} = (9.7)(6.4) \\
A_{\text{rectangle}} = 62.08
\]

The area of the garden is 62.08 square feet.

Notice that perimeter is in units, but area is in square units.

b. The entry-way to a building is in the shape of a parallelogram. If the base is 18 feet and the height is 12 feet, find the area of the entry-way.

\[
A_{\text{parallelogram}} = bh \\
A_{\text{parallelogram}} = (18)(12) \\
A_{\text{parallelogram}} = 216
\]

The area of the entry-way is 216 square feet.

c. A rectangular wall measures \(15 \frac{2}{3}\) feet by \(8 \frac{1}{6}\) feet. What is the area?

\[
A_{\text{rectangle}} = lw \\
A_{\text{rectangle}} = \left(15 \frac{2}{3}\right) \left(8 \frac{1}{6}\right) \\
A_{\text{rectangle}} = \left(47 \frac{1}{3}\right) \left(49 \frac{1}{6}\right) \\
A_{\text{rectangle}} = \frac{2303}{18} \quad \frac{127}{18}
\]

The area of the wall is \(127 \frac{17}{18}\) square feet.
d. A triangular flag has a base of 3 feet and a height of 7 feet. What is the area?

\[
A_{\text{triangle}} = \frac{1}{2} \cdot b \cdot h
\]

\[
A_{\text{triangle}} = \left( \frac{1}{2} \right) \left( \frac{3}{1} \right) \left( \frac{7}{1} \right)
\]

\[
A_{\text{triangle}} = \frac{21}{2} \cdot 10 \cdot \frac{1}{2}
\]

The area of the flag is \(10 \frac{1}{2}\) square feet.

e. Find the area of a circular patio with a radius of 6.4 meters.

\[
A_{\text{circle}} = \pi r^2
\]

\[
A_{\text{circle}} = (3.14)(6.4)^2
\]

\[
A_{\text{circle}} = (3.14)(40.96)
\]

\[
A_{\text{circle}} = 128.6144
\]

The area of the patio is approximately 128.6144 square meters.

f. One side of a building is in the shape of a triangle. If the base of the triangle is 42.7 feet and the height is 64.9 feet, what is the area of that side of the building?

\[
A_{\text{triangle}} = \frac{1}{2} \cdot b \cdot h
\]

\[
A_{\text{triangle}} = \left( \frac{1}{2} \right) \left( \frac{42.7}{1} \right) \left( \frac{64.9}{1} \right)
\]

\[
A_{\text{triangle}} = \frac{2771.23}{2}
\]

\[
A_{\text{triangle}} = 1385.615
\]

The area of that side of the building is 1385.615 square feet.
g. Find the area of a circle that has a radius of $\frac{5}{12}$ inches.

$$A_{\text{circle}} = \pi r^2$$

$$A_{\text{circle}} = (3.14) \left(\frac{5}{12}\right)^2$$

$$A_{\text{circle}} = (3.14) \left(\frac{89}{12}\right)^2$$

$$A_{\text{circle}} = \left(\frac{3.14}{1}\right) \left(\frac{7921}{144}\right)$$

$$A_{\text{circle}} = \frac{2487194}{144}$$

$$A_{\text{circle}} = 172.7218056$$

$$A_{\text{circle}} = 172.72$$ if you round to two decimal places or

$$A_{\text{circle}} = 172.722$$ if you round to three decimal places

Some instructors follow the rules for significant digits when they talk about rounding; this is something you may learn from your instructor or from science classes. In these problems, you can choose how to round unless your instructor gives you different instructions.

The area of the circle is approximately 172.72 square inches.
Chapter 5 Section 4  

Area formulas

Homework for Chapter 5 Section 4

Choose the appropriate formula and solve.

1. Find the area of a rectangular door that measures 11 feet by 7.5 feet.

2. Find the area of a rectangular rug that measures 1.4 meters by 2.7 meters.

3. Find the area of a rectangular picture that measures $4 \frac{1}{2}$ inches by $3 \frac{1}{4}$ inches.

4. For a rectangle that measures 24 cm by 18 cm, find
   a. the perimeter.
   b. the area.

5. What is the area of a field in the shape of a parallelogram with a base of 32.8 yards and a height of 18 yards?

6. If a triangle has a base of $1 \frac{5}{6}$ inches and a height of $3 \frac{1}{3}$ inches, what is the area?

7. A triangular sail has a base of 7 feet and a height of 12 feet. What is the area?

8. What is the area of a circular pizza pan that has a radius of 5 inches?

9. What is the area of a circular window that has a radius of $1 \frac{1}{2}$ feet?

10. A circle has a radius of 5 yards. Find the
    a. circumference
    b. area

11. In an election, 590 people cast ballots. If this represents 25% of the student body, how many students are there?

12. On a test with 40 questions, a student answered 32 questions correctly. What percent is this?

13. In a town of 3800 people, 2% of the people moved away. How many people moved away?
14. Is $5 + 8 = 8 + 5$ an example of the commutative or associative property?

15. Describe in words the steps you would take to solve the equation $2x + 8 = 14$.

16. To simplify $3(5y + 7)$, what property would you use?

**Solve.**

17. $3 + 9z + 28 = 4(2z + 3)$

18. $-2(3a - 1) = a - 11$

19. $\frac{5}{6}x + \frac{2}{3} = \frac{7}{12}x + \frac{1}{2}$

20. $\frac{2}{3}x + \frac{1}{4} = \frac{2}{5}$

21. $\frac{5}{18}x + 2 = \frac{7}{9}$

22. $\frac{7}{8}x - 1 = 2x + \frac{1}{3}$

23. $\left(\frac{7}{11}\right)x + 2 = 5$

24. $0.3k - 4.41 = -1.65$

**Simplify.**

25. $-16x + 8 + 9x - 12$

26. $-4(3A - 7)$

27. $-3(4 - 6y) - (7 - 2y) - 5 - 9y$

28. $\frac{16a^3b^2c^5}{22a^4b^6c^3}$

29. $\frac{2x}{3} + \frac{9x}{4}$

30. $\frac{3m}{8} - \frac{5m}{6}$
Chapter 5 Section 4

Answer Key for Chapter 5 Section 4.

1. The area of the door is 82.5 square feet.

2. The area of the rug is 3.78 square meters.

3. The area of the picture is \( \frac{145}{8} \) square inches.

4. a. The perimeter of the rectangle is 84 cm.
   b. The area of the rectangle is 432 square cm.

5. The area of the field is 590.4 square yards.

6. The area of the triangle is \( \frac{31}{18} \) square inches.

7. The area of the sail is 42 square feet.

8. The area of the pizza pan is approximately 78.5 square inches.

9. The area of the window is approximately 7.065 square feet.

10. a. The circumference of the circle is approximately 31.4 yards.
    b. The area of the circle is approximately 78.5 square yards.

11. There are 2360 students.

12. The student answered 80% of the questions correctly.

13. Seventy-six people moved away.

14. commutative property

15. First, subtract the constant 8 from both sides of the equation. Second, divide both sides of the equation by the coefficient 2.

16. distributive property

17. \( z = -19 \)

18. \( a = \frac{13}{7} \)

19. \( x = -\frac{2}{3} \)

20. \( x = \frac{9}{40} \)
21. \( x = -\frac{22}{5} \)

22. \( x = -\frac{32}{27} \)

23. \( x = \frac{33}{7} \)

24. \( k = 9.2 \)

25. \( -7x - 4 \)

26. \( -12x + 28 \)

27. \( 11y - 24 \)

28. \( \frac{8e^2}{11ab^4} \)

29. \( \frac{35x}{12} \)

30. \( -\frac{11m}{24} \)
Section 5: Volume Formula

In this section, you will find the volume of a rectangular solid. A rectangular solid looks like a cardboard box. Here is an example of a rectangular solid:

![Rectangular Solid](image)

Observe the formula.

\[ V_{\text{rectangular solid}} = lwh, \] where \( V \) stands for volume, \( l \) for length, \( w \) for width, and \( h \) for height.

a. Find the volume of a rectangular solid with length 14 meters, width 8 meters, and height 5 meters.

\[
V_{\text{rectangular solid}} = lwh
\]
\[
V_{\text{rectangular solid}} = (14)(8)(5)
\]
\[
V_{\text{rectangular solid}} = 560
\]

The volume of the rectangular solid is 560 cubic meters.

Notice that volume is given in cubic units; recall that area is given in square units and perimeter is given in units.

b. A building is in the shape of a rectangular solid. Find the volume of the building if the length is 81.7 feet, the width is 55.9 feet, and the height is 41.2 feet.

\[
V_{\text{rectangular solid}} = lwh
\]
\[
V_{\text{rectangular solid}} = (81.7)(55.9)(41.2)
\]
\[
V_{\text{rectangular solid}} = 188161.636
\]

The volume of the building is 188,161.636 cubic feet.
c. A cardboard box has dimensions of $16\frac{1}{2}$ inches, $7\frac{1}{3}$ inches, and $5\frac{5}{6}$ inches. Find the volume.

$$V_{\text{rectangular solid}} = lwh$$

$$V_{\text{rectangular solid}} = \left(16\frac{1}{2}\right)\left(7\frac{1}{3}\right)\left(5\frac{5}{6}\right)$$

$$V_{\text{rectangular solid}} = \left(\frac{33}{2}\right)\left(\frac{22}{3}\right)\left(\frac{35}{6}\right)$$

$$V_{\text{rectangular solid}} = \frac{25,410}{36} = \frac{4235}{6} = 705\frac{5}{6}$$

or

The volume of the cardboard box is $705\frac{5}{6}$ cubic inches.
Homework for Chapter 5 Section 5

Choose the appropriate formula and solve.

1. A cabinet, in the shape of a rectangular solid, measures 5 feet by 2 feet by 4 feet. What is the volume?

2. A block, in the shape of a rectangular solid, has a length of 3 inches, a width of 2 inches, and a height of 4 inches. Find the volume.

3. A wooden crate has dimensions of 6.2 feet, 3.1 feet, and 2 feet. What is the volume?

4. What is the volume of a rectangular solid with length of \( \frac{1}{3} \) inches, width of \( \frac{1}{6} \) inches, and height \( \frac{7}{12} \) inches?

5. Find the circumference of a circular mirror with a radius of 2 feet.

6. Find the area of a rectangular lot that measures 45 feet by 39 feet.

7. Find the perimeter of a rectangular table that is 8 feet long and 6 feet wide.

8. Find the area of a circular clock with a radius of 6 inches.

9. In a college with 25,000 students, 35% are freshmen. How many are freshmen?

10. Twenty-eight people in a theater got bored and left. If this represents 4% of the audience, how many people were in the audience?

11. In a room with 150 people, 84 people are dressed casually.
   a. What percent are dressed casually?
   b. What percent are not dressed casually?

Simplify.

12. \( \frac{40}{5} - 9^2 \)

13. \(-28 + (-4) - 6(-5 - 2)\)

14. \(7y + 9 - 20y + 3\)

15. \(\frac{-60z + 45}{-3}\)
16. \(2(3x - 5) - (4x - 9)\)  

17. \(-\frac{9k - 2}{6} - \frac{2k - 17}{6}\)

18. \(-\frac{8k}{15} - \frac{k}{6}\)  

19. \(-\frac{5x}{6} + \frac{2x}{9}\)

Solve.

20. \(-5a + 19 = -11\)

21. \(-6k + 3 = -2k + 20\)

22. \(\frac{1}{2}m - 3 = 4\)

23. \(4y + 9.27 = 2.158\)

Set up and solve.

24. Twice the height of a tree, increased by 17 feet, is 90 feet. Find the height of the tree.

25. Joe walked 3 miles more than Mike. Together, they walked 15 miles. How far did each one walk?
Answer Key for Chapter 5 Section 5.

1. The volume of the cabinet is 40 cubic feet.

2. The volume of the block is 24 cubic inches.

3. The volume of the wooden crate is 38.44 cubic feet.

4. The volume of the rectangular solid is \(\frac{25}{54}\) cubic inches.

5. The circumference of the mirror is approximately 12.56 feet.

6. The area of the lot is 1755 square feet.

7. The perimeter of the table is 28 feet.

8. The area of the clock is approximately 113.04 square inches.

9. There are 8750 freshmen.

10. There were 700 people in the audience.

11. a. Fifty-six percent of people are dressed casually.
    b. Forty-four percent of people are not dressed casually.

12. \(-73\)  
13. 49

14. \(-13y + 12\)  
15. \(20z - 15\)

16. \(2x - 1\)  
17. \(-\frac{11k + 15}{6}\)

18. \(-\frac{21k}{30} \Rightarrow -\frac{7k}{10}\)  
19. \(-\frac{11x}{18}\)

20. \(a = 6\)  
21. \(k = \frac{17}{4}\)

22. \(m = 14\)  
23. \(y = -1.778\)

24. The height of the tree is 36.5 feet.

25. Mike walked 6 miles; Joe walked 9 miles.
Chapter 5 Summary

Solve Equations with Decimals

a. \[ 0.4x + 0.16 = -0.72 \]
   \[ -0.16 \quad -0.16 \]
   \[ 0.4x \quad = \quad -0.88 \]
   \[ 0.4 \quad \frac{0.4}{0.4} \]
   \[ x = -2.2 \]

b. Recall, you should also check your solutions.

Percent Formula

Percent \cdot (Whole) = Part

Identify percent (write as a decimal in the formula), whole, part; substitute into formula; isolate the variable

18 people left a theater; if this represented 2% of the audience, how many people were in the audience?

percent = 2\% (use 0.02 in the formula)
part = 18
whole = x

\[
0.02 \cdot (\text{whole}) = \text{part}
\]
\[0.02(x) = 18\]
\[x = 900\]

There were 900 people in the audience.

Perimeter Formulas

\[P_{\text{rectangle}} = 2l + 2w\]
\[C_{\text{circle}} = 2\pi r \quad (\pi \approx 3.14)\]
\[\quad = \text{means approximately}\]
Chapter 5

Area Formulas

\[ A_{\text{rectangle}} = l \, w \]

\[ A_{\text{parallelogram}} = b \, h \]

\[ A_{\text{triangle}} = \frac{1}{2} b \, h \]

\[ A_{\text{circle}} = \pi \, r^2 \]

Volume Formula

\[ V_{\text{rectangular solid}} = l \, w \, h \]
Chapter 5 Review

Solve.

1. \(0.4a + 3 = -3.4\)
2. \(-1.6y - 10.8 = 0.032\)
3. \(0.02R + 4.34 = -8.1\)

Simplify.

4. \(4.9 - 2.8(1.9 - 4.4) + (1.8)^2\)

Evaluate.

5. \(-4x - 9.3\) when \(x = -2.1\)

Solve the following problems.

6. A puppy ate 4 gingerbread cookies before being caught. This represented 20% of the total number baked. How many cookies had been baked?

7. A store employee received a 30% discount on a purchase of $420 worth of computer equipment. How much was the discount in dollars?

8. Six of the students in a class of 40 students failed to turn in their homework. What percent of the students failed to turn in their homework?

9. What is the circumference of a circle with a radius of 5.3 feet? Round to the nearest tenth inch.

10. Find the perimeter of a rectangular table with a length of 43 inches and a width of 27 inches.

11. A parallelogram has a base of 18 meters and a height of 8 meters. Find the area.

12. Find the area of a triangle with a base of 18 meters and a height of 8 meters.

13. A sprinkler system irrigates a circular area of ground with a radius of 50 meters. Find the area of the ground irrigated.

14. Find the volume of a box with a height of 8.3 inches, a width of 12.2 inches, and a length of 17 inches. Round to the nearest tenth inch.
**Answer Key for Chapter 5 Review.**

<table>
<thead>
<tr>
<th></th>
<th>Reference Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>a = (-16)</td>
</tr>
<tr>
<td>2.</td>
<td>y = (-6.77)</td>
</tr>
<tr>
<td>3.</td>
<td>R = (-622)</td>
</tr>
<tr>
<td>4.</td>
<td>15.14</td>
</tr>
<tr>
<td>5.</td>
<td>(-0.9)</td>
</tr>
</tbody>
</table>
| 6. | cookies baked = whole = x  
    | \(0.20(x) = 4\)  
    | x = 20 cookies baked |
| 7. | discount = part = x  
    | \(0.30(420) = x\)  
    | x = $126 discount |
| 8. | percent = x  
    | x(40) = 6  
    | x = 15\% |
| 9. | Circumference is approximately 33.3 feet. |
| 10. | Perimeter is 140 inches. |
| 11. | Area is 144 square meters. |
| 12. | Area is 72 square meters. |
| 13. | Area is approximately 7850 square meters. |
| 14. | Volume is approximately 1721.4 cubic inches. |
Chapter 5 Test

Solve.

1. \(3x + 4.8 = 8.1\)

2. \(-0.4x - 9.8 = 10.4\)

3. \(0.25y + 40 = 15\)

Simplify.

4. \(9.6 - 3.1(4.2 - 3) + (1.5)^2\)

Evaluate.

5. \(2x - 6.9\) when \(x = 3.1\)

Set up and solve.

6. In a recent local election, 38% of the voters voted for the independent candidate. 1550 people voted. How many people voted for the independent candidate?

7. On the last math quiz, John got 9 questions correct out of 12. What percent did he get correct?

8. There are 774 female employees at a company. Females make up 45% of the total number of employees. How many people does the company employ?

9. The tax on meals at a restaurant is 6%. My bill before tax was $21.50. What was my total bill, including tax?

Choose the appropriate formula and solve.

10. Find the perimeter of a rectangular room that measures 42 feet by 35 feet.

11. What is the perimeter of a square garden that measures 16.8 feet on a side?
Chapter 5 Test

Choose the appropriate formula and solve.

12. What is the circumference of a circular tablecloth with a radius of 3.2 feet?

13. Find the area of a rectangular pool cover that measures 16 feet by 32 feet.

14. What is the area of a piece of construction paper measuring \( \frac{8\frac{1}{2}}{2} \) inches by 11 inches?

15. A triangular flag has a base of 3 feet and a height of \( \frac{1}{2} \) feet. Find the area of the flag.

16. A circular lid has a radius of 1.75 inches. What is its area?

17. What is the volume of a room that measure 12 feet by 9 feet by 11 feet?

18. A storage unit in the shape of a rectangular solid measures 4.5 feet by 3 feet by 6.3 feet. What is its volume?

Set up and solve.

19. A piece of lumber is 12 feet long. It is cut into 2 pieces. The length of one piece is 3 feet more than twice the length of the other. Find the length of the pieces.

Solve.

20. \( 2(3x - 5) = 8x + 6 \)
Answer Key for Chapter 5 Test.

1. \( x = 1.1 \)
2. \( x = -50.5 \)
3. \( y = -100 \)
4. \( 8.13 \)
5. \(-0.7\)
6. \( x = 589 \)  589 people voted for the independent candidate.
7. \( x = 75\% \)  John got 75\% of the answers correct.
8. \( x = 1720 \)  There are 1720 employees.
9. \( x = $1.29 \)  My total bill was $21.50 + $1.29 = $22.79.
10. The perimeter is 154 feet.
11. The perimeter is 67.2 feet.
12. The circumference is approximately 20.096 feet.
13. The area is 512 square feet.
14. The area is \( 93\frac{1}{2} \) square inches.
15. The area is \( 2\frac{1}{4} \) square feet.
16. The area is approximately 9.61625 square inches.
17. The volume is 1188 cubic feet.
18. The volume is 85.05 cubic feet.
19. \( x = 3 \) feet
   \( 2x + 3 = 9 \) feet
   The pieces are 3 feet and 9 feet.
20. \( x = -8 \)
Cumulative Review #5 Through Chapter 5

1. To simplify an expression containing only the operations of multiplication and division, which would you do first?

2. Reduce to lowest terms: \(-\frac{x^2yz}{-xy^2z^2}\)

Simplify.

3. \(3(x + 2) + 6 \div 2 + \frac{8}{8}\)

4. \(-8T - 5T + (8 - T)\)

5. \(4(-5x + 3) - 3(2 - 4x)\)

6. \(3[-5 - 2(4)] + \left(\frac{20}{-5}\right)^2\)

7. \(\frac{24a - 16}{4}\)

8. \(\frac{8y - 11}{14} - \frac{-2y + 6}{14}\)

9. \(8\frac{1}{6} - 10\frac{1}{9}\)

10. \(-\frac{5a}{3} - \frac{3}{5}\)

11. \(\frac{3}{b} - \frac{a}{2}\)

12. \(\frac{R^2}{T} \div \frac{R}{T^3}\)

13. \((-5\frac{1}{3})(-2\frac{1}{2}) \div (3\frac{1}{3})\)
Evaluate.

14. \(-14 - x\) when \(x = -14\)

15. \(3a + 7.3\) when \(a = -2.3\)

Solve.

16. \(4x + 8 = 32\)

17. \(4x - 10 = -22\)

18. \(T + 28 + T = 4(T - 13)\)

19. \(4(x - 9) - 3(-x + 2) = 14\)

20. \(3x - 8 = 2\)

21. \(\left(\frac{1}{3}\right)x + 2 = \frac{5}{6}\)

22. \(-0.3x + 5.19 = -2.31\)

23. A restaurant adds 30% to the cost of a meal to have the meal delivered. If the basic cost of a meal is $56, what would be the charge to have the meal delivered?

24. Find the volume of a cube (a box with the same length, width, and height) with sides of length 3.2 inches. Round to the nearest tenth.

25. A mirror is in the shape of a circle. What is the area of the mirror if the radius is 5 feet?

26. 83 grams more than twice the weight of a bunch of bananas is 990.2 grams. Find the number of grams in this bunch of bananas.

Try this:

27. A table is in the shape of a semi-circle (one-half circle). If the radius of the circle is 4 feet, what is its perimeter? (Don’t forget perimeter will be the semi-circle plus the diameter.) Round to the nearest tenth.
Cumulative Review #5 through Chapter 5

Answer Key for Cumulative Review #5 through Chapter 5.

Reference Chapter

1. whichever comes first 1
2. \( \frac{x}{z^2} \) 4
3. 3x + 10 1
4. \(-14T + 8\) 2a
5. \(-8x + 6\) 2b
6. -23 2b
7. 6a - 4 2b
8. \( \frac{10y - 17}{14} \) 4
9. \(-\frac{35}{18} \text{ or } -1\frac{17}{18}\) 8 4
10. \(\frac{-25a - 9}{15}\) 4
11. \(\frac{6 - ab}{2b}\) 4
12. \(RT^2\) 4
13. 4 4
14. 0 2a
15. 0.4 5
16. \(x = 6\) 1
17. \(x = -3\) 2b
18. \(T = 40\) 3
19. \( x = 8 \)

20. \( x = \frac{10}{3} \)

21. \( x = -\frac{7}{2} \)

22. \( x = 25 \)

23. \( x = \) delivery charge = part
   \[
   0.30(56) = x \\
   x = 16.80 \\
   \]
   It cost $16.80 to have the meal delivered.

24. The volume of the cube is approximately 32.8 cubic inches.

25. The area of the mirror is approximately 78.5 square feet.

26. \( x = \) number of grams in this bunch of bananas
   \[
   2x + 83 = 990.2 \\
   x = 453.6 \\
   \]
   This bunch of bananas weighs 453.6 grams.

27. \( P = \) perimeter of table
   \[
   P = \text{semi-circle} + \text{diameter} \\
   P = \text{semi-circle} + 2 \text{ radii} \\
   P = \frac{1}{2} (2\pi r) + 2r \\
   P = \frac{1}{2} (2 \cdot 3.14 \cdot 4) + 2 \cdot 4 \\
   P \approx 20.6 \text{ feet} \\
   \]
   The perimeter of the semi-circular table is approximately 20.6 feet
Chapter 6: Polynomials

Chapter 6 presents ideas about polynomials. Before studying about polynomials, however, you will first learn a little more about exponents.

In the first section, you will once more concentrate on exponents. In particular, you will learn how to find the product when the bases are identical. You will also learn how to find the result when there is a term raised to a power.

The rest of this chapter will present ideas about polynomials. You will find out that polynomials are certain types of expressions. You will evaluate polynomials, add and/or subtract polynomials, and multiply polynomials. As you are doing these activities, you will incorporate what you have learned about signed numbers, order of operations, and combining like terms.

Your goal is to become very comfortable with exponents and with polynomials, along with practicing your algebra skills.
**Section 1: Exponents: Product Rule**

When you see $3^4$, you read that as “3 to the 4th power,” where 3 is the base and 4 is the exponent. Recall that $3^4$ means $(3)(3)(3)(3)$ or $3 \cdot 3 \cdot 3 \cdot 3$ or $3^4 \cdot 3^1 \cdot 3^1 \cdot 3^1$.

In this section, you will be multiplying. First, you will expand, count, and re-write with bases and exponents. Then you will look for a way to find the result without expanding. Observe.

a. $5^2 \cdot 5^4$

\[ (5 \cdot 5) \cdot (5 \cdot 5 \cdot 5 \cdot 5) \]

b. $7^3 (7^2)$

\[ (7 \cdot 7 \cdot 7) \cdot (7 \cdot 7 \cdot 7 \cdot 7 \cdot 7) \]

$5^6$

$7^8$

Did you notice that as you expanded, counted, and re-wrote with bases and exponents that you kept the original base? Did you also notice that you essentially added the exponents? This becomes the basis for the product rule of exponents.

**Product Rule:**

\[ x^a \cdot x^b = x^{a + b} \]

When you multiply with the same base, keep the base and add the exponents.

c. $9^1 \cdot 9 \cdot 9^4$

to $9^{1+1+4}$

\[ 9^8 \]

d. $x^{10}(x^4)$

\[ x^{10+4} \]

\[ x^{14} \]

Notice on example c, 9 means $9^1$.

If you forget the rule, fall back on expand, count, and re-write in exponent form:

\[ 9^1 \cdot 9 \cdot 9^4 \]

\[ (9 \cdot 9 \cdot 9)(9 \cdot 9 \cdot 9 \cdot 9) \]

\[ 9^8 \]

\[ x^{10}(x^4) \]

\[ (x \cdot x \cdot x \cdot x \cdot x)(x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x) \]

\[ x^{14} \]

e. $(x^9 y^2)(x^3 y^5)$

\[ x^9 \cdot x^3 \cdot y^2 \cdot y^5 \]

\[ x^{9+3} \cdot y^{2+5} \]

\[ x^{12} y^7 \]

f. $(a^2)(a^3 b^7)(b)$

\[ a^2 \cdot a^3 b^7 \cdot b \]

\[ a^{2+3} b^{7+1} \]

\[ a^5 b^8 \]

You used the commutative property to “re-arrange” the order of the factors.

g. $5a^6 \cdot 3a^2$

\[ 5 \cdot a^6 \cdot 3 \cdot a^2 \]

\[ 5 \cdot 3 \cdot a^6 \cdot a^2 \]

\[ 15 \cdot a^{6+2} \]

\[ 15a^8 \]

Notice on example g that you expand and use the commutative property to change the order; then coefficients are together and variables are together.

This allows you to multiply coefficients, keep the base, and add the exponents.
h. \( 4z^2(6z^4) \)  
\[ 24z^6 \]

i. \((7a^2b^7)(-4a^{10}b^9)\)  
\[ -28a^{12}b^{16} \]

What does the following expression mean? To find out, expand, count, and rewrite with base and exponents.

\[ (2^3)^4 \]
\[ (2^2 \cdot 2^1) \cdot (2^3) \cdot (2^4) \]
\[ (2 \cdot 2 \cdot 2) \cdot (2 \cdot 2 \cdot 2) \cdot (2 \cdot 2 \cdot 2) \cdot (2 \cdot 2 \cdot 2) \]
\[ 2^{12} \]

or \[ 2^{3+3+3+3} \]

Observe the following examples.

\[ (3^5)^2 \]  
\[ (3^5)(3^5) \]
\[ 3^{-2+5} \]
\[ 3^{10} \]

\[ (a^2b^1c^4)^3 \]  
\[ (a^2b^1c^4)(a^2b^1c^4)(a^2b^1c^4) \]
\[ a^{6+2+2} b^{1+1+1} c^{4+4+4} \]
\[ a^6b^3c^{12} \]

In examples j through n, there is a rule to help you remove parentheses when you have an exponent on the “outside” of the parentheses. You will learn that rule in another algebra class.

Observe the following examples.

\[ (x^4y^{10})^5 \]
\[ (x^4y^{10})(x^4y^{10})(x^4y^{10})(x^4y^{10})(x^4y^{10}) \]
\[ (x^{4+4+4+4+4}) \]
\[ x^{20} y^{50} \]

\[ (a^8)^3(5a^7)^2 \]
\[ (2a^8)(2a^8)(2a^8) \cdot (5a^7)(5a^7) \]
\[ (2 \cdot 2 \cdot a^{8+8+8}) \cdot (5 \cdot 5 \cdot a^{7+7}) \]
\[ 8 \cdot 25 \cdot a^{24+14} \]
\[ 200a^{38} \]

These exercises would say “Simplify”; you would be expected to know how to proceed. If you can’t remember exactly what to do, then you should expand, count, and rewrite with bases and exponents. Expanding will help you remember how to proceed.
Chapter 6 Section 1

Exponents: product rule

Homework for Chapter 6 Section 1

Simplify.

1. \((3^4)(3^6)\)  
2. \(5^7 \cdot 5^2 \cdot 5^4\)

3. \(7 \cdot 7^5\)  
4. \((x^3)(x^7)\)

5. \(a^3 \cdot a^4\)  
6. \(m^6(m)\)

7. \(y^2 \cdot y \cdot y^5\)  
8. \((x)(x^3)(x)\)

9. \((a^5b^8)(a^5b^{10})\)  
10. \(x^2y^3z^{11} \cdot x^4y^{18}z^6\)

11. \(9k^8 \cdot 6k\)  
12. \(-4z^3(7z^3)\)

13. \((-2m^3)(-3m^3)\)  
14. \(5x^2y^3(4x^4y^6)\)

15. \(-3a^7b^2(6a^4b^5)\)  
16. \(8j^3k^9(-2j^3k^2)\)

17. \(-4x^2y^7z^3(-5x^4y^6z^6)\)  
18. \((5^3)^6\)

19. \((d^{12})^3\)  
20. \((e^6f^2g)^5\)

21. \((2a^4)^5\)  
22. \((3x^3y^5)^4\)

23. \((4a^4b^6)^2\)  
24. \((p^7)^3(p^{11})^4\)

25. \((5e^4)^2(3e^7)^3\)  
26. \((3x^5y^7)^4(2x^3y^2)^2\)

Evaluate.

27. \(2x + 8\) when \(x = 0\)  
28. \(3y + 14\) when \(y = -1\)

29. \(4z - 6\) when \(z = 3\)  
30. \(-5m - 17\) when \(m = -2\)

31. \(x + \frac{2}{3}\) when \(x = \frac{5}{6}\)  
32. \(xy\) when \(x = 9\) and \(y = -2\)

Simplify.

33. \(-20 \div 4 - 6(-8 + 3)\)  
34. \(\left(\frac{10}{2}\right)\left(\frac{-16}{8}\right) + 4(2 - 5)^2\)

35. \(5a + 9b + 6b + 3a\)  
36. \(4(6m + 3) + (2m - 5)\)
Simplify.

37. \( \frac{x^5}{x^2} \)  
38. \( \frac{y^3}{y} \)

39. \( \frac{24a^2b^3}{30a^5b} \)  
40. \( \frac{-18A + 27B + 33}{-3} \)

41. \( \frac{-6k - 18}{4} - \frac{-9k - 3}{4} \)

Set up and solve.

42. Find the area of a circular painting if the radius is 4 feet.

43. Find the perimeter of a rectangular driveway which measures 70 feet by 15 feet.

44. Find the volume of a concrete sidewalk which is 80 feet long, 6 feet wide, and \( \frac{3}{4} \) feet deep.

45. Josh bought a stereo system; the marked price was $3284. If the tax rate is 7%, how much was the tax?

Solve.

46. \( 4A = 42 - 2A \)  
47. \( m + 19 = -73 \)

48. \( 2z - 3 - 7z - 17 = -55 \)  
49. \( 6(3a - 5) = 5(4a + 2) \)

50. \( 7x - 3 = 2 \)  
51. \( \frac{2}{3}y + \frac{1}{4} = \frac{5}{6} \)

52. \( 0.4R + 2.76 = 9.48 \)
Chapter 6 Section 1

Exponents: product rule

Answer Key for Chapter 6 Section 1.

1. \(3^{10}\)  
2. \(5^{13}\)

3. \(7^6\)  
4. \(x^{10}\)

5. \(a^7\)  
6. \(m^7\)

7. \(y^8\)  
8. \(x^5\)

9. \(a^8 \cdot b^{18}\)  
10. \(x^6 \cdot y^{21} \cdot z^{17}\)

11. \(54k^9\)  
12. \(-28z^8\)

13. \(6m^7\)  
14. \(20x^6 \cdot y^9\)

15. \(-18a^{11} \cdot b^7\)  
16. \(-16j^8 \cdot k^{11}\)

17. \(20x^6 \cdot y^{12} \cdot z^9\)  
18. \(5^{12}\)

19. \(d^{36}\)  
20. \(e^{30} \cdot f^{10} \cdot g^{5}\)

21. \(32a^{20}\)  
22. \(81x^{12} \cdot y^{20}\)

23. \(16a^8 \cdot b^{12}\)  
24. \(p^{65}\)

25. \(225e^{22}\)  
26. \(324x^{26} \cdot y^{32}\)

27. \(8\)  
28. \(11\)

29. \(6\)  
30. \(-7\)

31. \(\frac{9}{6} = \frac{3}{2}\) or \(1\frac{1}{2}\)  
32. \(-18\)

33. \(25\)  
34. \(26\)

35. \(8a + 15b\)  
36. \(26m + 7\)

37. \(x^3\)  
38. \(\frac{1}{y^4}\)

39. \(\frac{4b^5}{5a^6}\)  
40. \(6A - 9B - 11\)
Chapter 6 Section 1

41. \( \frac{3k - 15}{4} \)

42. The area of the painting is approximately 50.24 square feet.

43. The perimeter of the driveway is 170 feet.

44. The volume of the sidewalk is 360 cubic feet.

45. The tax was $229.88.

46. \( A = \frac{7}{2} \)  

47. \( m = \frac{-92}{2} \)  

48. \( z = \frac{7}{2} \)  

49. \( a = \frac{-20}{2} \)  

50. \( x = \frac{5}{7} \)  

51. \( y = \frac{7}{8} \)  

52. \( R = 16.8 \)
Section 2: Polynomials: Add, Subtract, Evaluate

At the beginning of the book, you learned about expressions. This chapter extends the concept of expressions. First, review the following.

a. \(5m + 9\)
   an expression with two terms

b. \(8z - 12\)
   an expression with two terms

c. \(9k\)
   an expression with one term

d. \(4x + 9y + 7z\)
   an expression with three terms

All of the above were expressions. The terms were separated by addition or subtraction symbols. All of the above are also polynomials.

e. A **monomial** is an expression with one term. Examples:
   \[7, 2m, -14y, 5x^2\]

f. A **binomial** is an expression with two terms. Examples:
   \[4a + 3, 5z - 17, -2x + 6y, x^2 + 3\]

g. A **trinomial** is an expression with three terms. Examples:
   \[2x + 3y + 4z, 4x^2 + 3x + 8\]

h. Polynomials with more than three terms aren’t given a special name.

When a polynomial is in the same variable (such as \(4x^2 + 3x + 8\)), it is usually written in **descending order**, meaning the term with the largest exponent is written first, then the term with the next largest exponent is written second, and so forth until the constant is written last.

i. \(9k - 4k^3 + 5k^4 - 7 + 6k^2\)
   polynomial with 5 terms

\(5k^4 - 4k^3 + 6k^2 + 9k - 7\)
   same polynomial in descending order
Polynomials can be added and subtracted. When you do that, you need to combine like terms. In order for terms to be alike, the variables must be identical AND the exponents must be the same. For example, $4x$ and $9x^2$ are NOT alike because the exponents are different; $3x^2$ and $6y^2$ are NOT alike because the variables are different; $7x^2$ and $5x^2$ ARE alike because the variables are identical and the exponents are the same.

When you **combine like terms**, you combine the coefficients and leave the variable part the same. Observe.

j. $5k^2 + 6k^2$
   
   $11k^2$

k. $-7z^3 + 2z^3$
   
   $-5z^3$

l. $9a^2 + 3 + 7a - 2 + 4a^2 - 19a$
   
   $9a^2 + 4a^2 + 7a - 19a + 3 - 2$
   
   $13a^2 - 12a + 1$

Sometimes before you can combine like terms, you need to remove parentheses. Observe.

m. $2(3y^2 - 4y - 6) + 4(-2y^2 - 5y + 7)$
   
   $6y^2 - 8y - 12 - 8y^2 - 20y + 28$
   
   $6y^2 - 8y^2 - 8y - 20y - 12 + 28$
   
   $-2y^2 - 28y + 16$

n. $(-9z^3 + 17z^2 + 2z - 24) + (10z^3 + 5z + 30)$
   
   $-9z^3 + 17z^2 + 2z - 24 + 10z^3 + 5z + 30$
   
   $-9z^3 + 10z^3 + 17z^2 + 2z + 5z - 24 + 30$
   
   $z^3 + 17z^2 + 7z + 6$

On example n, you could say that there was an “understood 1” in front of each set of parentheses; multiplying by 1 would not change any numbers.

**o.** $-2(7y^2 + 6y - 5) + 5(2y - 10)$
   
   $-14y^2 - 12y + 10 + 10y - 50$
   
   $-14y^2 - 12y + 10y + 10 - 50$
   
   $-14y^2 - 2y - 40$
Chapter 6 Section 2  Polynomials: Add, subtract, evaluate

p. \(3(8m^2 - 30m + 3) - 4(-2m^2 - 3m + 7)\)
\[
= 24m^2 - 90m + 9 + 8m^2 + 12m - 28 \\
= 24m^2 + 8m^2 - 90m + 12m + 9 - 28 \\
= 32m^2 - 78m - 19
\]

On the second set of parentheses, you were multiplying with negative 4. Therefore, all the signs in the second set of parentheses changed.

q. \((2z^2 + 11z - 6) - (5z^2 - 4z - 7)\)
\[
= 2z^2 + 11z - 6 - 5z^2 + 4z + 7 \\
= 2z^2 - 5z^2 + 11z + 4z - 6 + 7 \\
= -3z^2 + 15z + 1
\]

On the second set of parentheses, you were multiplying by an “understood negative 1”; that is why all the signs changed.

When you **evaluate a polynomial**, you remove the variable and replace it with a number. Remember to place the number inside parentheses; this will help with the order of operations. Once you have substituted the number for the variable, simplify using order of operations. Observe.

r. **Evaluate** \(6x^2 + 7x - 11\) when \(x = 4\).
\[
16(4)^2 + 7(4) - 11 \\
= 96 + 28 - 11 \\
= 113
\]

s. **Evaluate** \(2y^2 + 8y + 1\) when \(y = -5\).
\[
2(-5)^2 + 8(-5) + 1 \\
= 50 - 40 + 1 \\
= 11
\]

t. **Evaluate** \(-16t^2 + 3t - 5\) for \(t = 2\).
\[
-16(2)^2 + 3(2) - 5 \\
-16(4) + 6 - 5 \\
= -64 + 6 - 5 \\
= -63
\]
Homework for Chapter 6 Section 2

State whether each polynomial is a monomial, binomial, trinomial, or a polynomial with no special name.

1. \(4x + 9\)  
2. \(8\)  
3. \(5y^2 + 7y - 8\)  
4. \(-5m - 6\)  
5. \(8z\)  
6. \(8w^3 + 2w^2 - w + 5\)

Simplify. Write the result in descending order.

7. \(9h - 12 + 7h^2 + 5 + 2h - 3h^2\)  
8. \(10a^3 + 17 + 4a - 10a^2 - 7a^3 + 5 + 2a\)  
9. \(2(3f^2 + 2f - 6) + 3(4f^2 - 3f - 5)\)  
10. \((14a^2 - 11a + 3) + (-a^2 + 12a - 7)\)  
11. \((12x^2 + 5x + 14) + (2x^2 - 6x - 10)\)  
12. \(4(8m^2 - 2m - 10) - 5(m^2 - 6m - 12)\)  
13. \(-7(8y - 1) + 3(4y^2 + 3y - 6)\)  
14. \((6z^2 + 17z + 1) - (-3z^2 - 18z + 2)\)  
15. \((3k^2 - 9k - 16) - (18k^2 + 10k - 8)\)  
16. \((y^2 - 2y - 1) - (6y^2 + 2y + 3) + (5y^2 + 9y + 5)\)

Evaluate.

17. \(3x^2 + 7x + 2\)  
   a. when \(x = 6\)  
   b. when \(x = -4\)  
   c. when \(x = 0\)  
18. \(-5x^2 + 6x - 11\)  
   a. when \(x = 9\)  
   b. when \(x = -2\)  
   c. when \(x = 0\)
Chapter 6 Section 2
Polynomials: Add, subtract, evaluate

Simplify.

19. a. \(2x + 3x\)

b. \(2x(3x)\)

c. What makes a and b different from each other?

20. a. \(5x^2 - 9x\)

b. \(5x^2(-9x)\)

c. What makes a and b different from each other?

Simplify appropriately.

21. \(a^4 \cdot a^3\)

22. \(m^3(m^2)\)

23. \(y^7 \cdot y \cdot y^4\)

24. \(x^2y^3(xy^5)\)

25. \(4x^5(6x^3)\)

26. \(-5x \cdot 4x^2\)

27. \((9c^6d^4)(7c^2d^9)\)

28. \((-3x^4y)(5x^3y^5)\)

29. \((x^4)^5\)

30. \((m^2)^4\)

31. \((2f^3g^5)^4\)

32. \((x^2)^3(x^3)^2\)

33. \(4(3m + 2) - 5(6m - 3)\)

34. \(9 + 3(6 - 10) + \frac{28}{-4}\)

35. \(\frac{a^3b^6c^9}{a^5b^2c^3}\)

36. \(\frac{7y}{10} + \frac{y}{10}\)

37. \(\frac{5x + 8}{5} - \frac{3x + 7}{5}\)

38. \(\frac{-42y + 6}{-6}\)

Set up and solve.

39. Mary has a rectangular table that measures 5.3 feet by 4.7 feet.
   a. Find the perimeter.
   b. Find the area.
Solve.

40. \(9a + 15 - 12a + 3 = -24\)

41. \(8 + 3(6 + 2k) = 18 + 4k\)

42. \(2y = 9 - 3y\)

43. \(\frac{3}{8}x + 1 = \frac{2}{3}\)

44. \(\left(\frac{5}{11}\right)x + \frac{3}{4} = 1\)

45. \(\frac{7}{9}x - \frac{2}{3} = \frac{5}{6}x - \frac{3}{4}\)
Answer Key for Chapter 6 Section 2.

1. binomial
2. monomial
3. trinomial
4. binomial
5. monomial
6. polynomial with no special name

7. $4h^2 + 11h - 7$
8. $3a^3 - 10a^2 + 6a + 22$
9. $18f^2 - 5f - 27$
10. $13a^2 + a - 4$
11. $14x^2 - x + 4$
12. $27m^2 + 22m + 20$
13. $12y^2 - 47y - 11$
14. $9z^2 + 35z - 1$
15. $-15k^2 - 19k - 8$
16. $5y + 1$

17. a. 152  
b. 22  
c. 2

18. a. $-362$  
b. $-43$  
c. $-11$

19. a. $5x$  
b. $6x^2$  
c. In a, you combine like terms; in b, you multiply.

20. a. $5x^2 - 9x$ is already simplified.  
b. $-45x^3$  
c. In a, already simplified (not like terms so you can’t combine); in b, you multiply.

21. $a^7$  
22. $m^4$

23. $y^7$  
24. $x^3 y^8$

25. $24x^8$  
26. $-20x^3$

27. $63c^6 d^{14}$  
28. $-15x^7 y^6$

29. $x^{20}$  
30. $m^8$

31. $16f^{12} g^{20}$  
32. $x^{16}$

33. $-18m + 23$  
34. $-10$
35. \( \frac{b^4c^6}{a^2} \)  

36. \( \frac{8y}{10} \Rightarrow \frac{4y}{5} \)

37. \( \frac{2x + 1}{5} \)  

38. \( 7y - 1 \)

39. a. The perimeter is 20 feet  
   b. The area is 24.91 sq ft

40. \[ 9a + 15 - 12a + 3 = -24 \]  
   \[ -3a + 18 = -24 \]  
   \[ -3a = -42 \]  
   \[ a = 14 \]

41. \[ 8 + 3(6 + 2k) = 18 + 4k \]  
   \[ 8 + 18 + 6k = 18 + 4k \]  
   \[ 6k + 26 = 4k + 18 \]  
   \[ 2k + 26 = 18 \]  
   \[ 2k = -8 \]  
   \[ k = -4 \]

42. \[ 2y = 9 - 3y \]  
   \[ 5y = 9 \]  
   \[ y = \frac{9}{5} \]

43. clear; multiply by LCD 24
   \[ 9x + 24 = 16 \]  
   \[ 9x = -8 \]  
   \[ x = -\frac{8}{9} \]

44. clear; multiply by LCD 44
   \[ 20x + 33 = 44 \]  
   \[ 20x = 11 \]  
   \[ x = \frac{11}{20} \]

45. clear; multiply by LCD 36
   \[ 28x - 24 = 30x - 27 \]  
   \[ -2x - 24 = -27 \]  
   \[ -2x = -3 \]  
   \[ x = \frac{-3}{-2} = \frac{3}{2} \]
Section 3: Multiply Binomials

In this section, you will learn to multiply binomials. First, review the following examples.

a. \(x(x)\)  
   \(x^2\)

b. \(2m(3m)\)  
   \(6m^2\)

c. \(3(2y - 7)\)  
   \(6y - 21\)

d. \(-4(z + 2)\)  
   \(-4z - 8\)

e. \(5a(3a + 6)\)  
   \(15a^2 + 30a\)

f. \(7y(y - 6)\)  
   \(7y^2 - 42y\)

Generally, on examples e and f, you don’t show the middle step. Instead, you think about what distribution means: Multiply the outside by each inside term.

To multiply binomials, you will use the skills from the examples above. Observe.

\(g.\) \((3k + 8)(4k + 5)\) means \(3k(4k + 5) + 8(4k + 5)\)  
\(12k^2 + 15k + 32k + 40\)  
\(12k^2 + 47k + 40\)

Notice you are applying the distributive property twice.

\(h.\) \((10z + 1)(z - 4)\) means \(10z(z - 4) + 1(z - 4)\)  
\(10z^2 - 40z + 1z - 4\)  
\(10z^2 - 39z - 4\)

When you multiply binomials, you generally don’t show the “means” step. Instead you usually think this way: Multiply the first term in the first binomial by EACH term in the second binomial; then multiply the second term in the first binomial by EACH term in the second binomial. However, you can always show more steps if you want to so you will understand.

\(i.\) \((m - 9)(m - 6)\)  
\(m^2 - 6m - 9m + 54\)  
\(m^2 - 15m + 54\)

\(j.\) \((8y - 5)(3y - 2)\)  
\(24y^2 - 16y - 15y + 10\)  
\(24y^2 - 31y + 10\)

OR \((m - 9)(m - 6)\)  
\(m(m - 6) - 9(m - 6)\)  
\(m^2 - 6m - 9m + 54\)  
\(m^2 - 15m + 54\)

OR \((8y - 5)(3y - 2)\)  
\(8y(3y - 2) - 5(3y - 2)\)  
\(24y^2 - 16y - 15y + 10\)  
\(24y^2 - 31y + 10\)

Notice you can show additional steps and get the same result!
Chapter 6 Section 3

Multiply binomials

k. \((9z - 2)(3z + 5)\)
\[27z^2 + 45z - 6z - 10\]
\[27z^2 + 39z - 10\]

l. \((a + 6)(a - 6)\)
\[a^2 - 6a + 6a - 36\]
\[a^2 - 36\]

Can you see why this result is a binomial?

How would you approach the following? Observe.

m. \((5d + 2)^2\)
\[(5d + 2)(5d + 2)\]
\[25d^2 + 10d + 10d + 4\]
\[25d^2 + 20d + 4\]

This result was a trinomial! Why? Whenever you see a binomial raised to the second power, the result will ALWAYS be a trinomial!

The instructions for multiplication of binomials will say “Simplify”; you will be expected to know how to proceed.
Chapter 6 Section 3

Multiply binomials

Homework for Chapter 6 Section 3

Simplify.

1. \( z(z) \)  
2. \( 5a(a) \)
3. \( 4k(-3k) \)  
4. \( -6m(-2m) \)
5. \( 3(4y + 2) \)  
6. \( 7(2x - 5) \)
7. \( -5(d + 3) \)  
8. \( -2(x - 8) \)
9. \( -(9f + 1) \)  
10. \( -(3w - 5) \)
11. \( x(x + 3) \)  
12. \( 5m(2m + 4) \)
13. \( 3y(2y - 7) \)  
14. \( 8r(3r - 8) \)
15. \( (a + 2)(a + 9) \)  
16. \( (y + 11)(y + 3) \)
17. \( (m + 3)(m - 6) \)  
18. \( (z + 7)(z - 1) \)
19. \( (k - 4)(k + 10) \)  
20. \( (w - 9)(w + 2) \)
21. \( (d - 3)(d - 6) \)  
22. \( (e - 12)(e - 4) \)
23. \( (5x + 3)(4x + 7) \)  
24. \( (2y + 8)(y - 3) \)
25. \( (4z - 7)(3z + 2) \)  
26. \( (9a - 5)(3a - 10) \)
27. \( (b + 4)(b - 4) \)  
28. \( (z - 7)(z + 7) \)
29. \( (5g + 8)(5g - 8) \)  
30. \( (6k - 5)(6k + 5) \)
31. \( (x + 9)^2 \)  
32. \( (y - 7)^2 \)
33. \( (8z + 3)^2 \)  
34. \( (2a + 5)^2 \)
35. \( (9m - 4)^2 \)  
36. \( (4q - 1)^2 \)
37. \( (2x^2 + 3x + 4)(5x^2 + 7x + 6) \)
38. Evaluate: \( 3x^3 + 2x^2 - 4x + 9 \) for \( x = 5 \)
39. \( m^3(m^2) \)  
40. \( (m^3)^2 \)
Chapter 6 Section 3

Multiply binomials

Answer Key for Chapter 6 Section 3.

1. $z^2$
2. $5a^2$
3. $-12k^2$
4. $12m^2$
5. $12y + 6$
6. $14x - 35$
7. $-5d - 15$
8. $-2x + 16$
9. $-9f - 1$
10. $-3w + 5$
11. $x^2 + 3x$
12. $10m^2 + 20m$
13. $6y^2 - 21y$
14. $24r^2 - 64r$
15. $a^2 + 11a + 18$
16. $y^2 + 14y + 33$
17. $m^2 - 3m - 18$
18. $z^2 + 6z - 7$
19. $k^2 + 6k - 40$
20. $w^2 - 7w - 18$
21. $d^2 - 9d + 18$
22. $e^2 - 16e + 48$
23. $20x^2 + 47x + 21$
24. $2y^2 + 2y - 24$
25. $12z^2 - 13z - 14$
26. $27a^2 - 105a + 50$
27. $b^2 - 16$
28. $z^2 - 49$
29. $25g^2 - 64$
30. $36k^2 - 25$
31. $x^2 + 18x + 81$
32. $y^2 - 14y + 49$
33. $64z^2 + 48z + 9$
34. $4a^2 + 20a + 25$
35. $81m^2 - 72m + 16$
36. $16q^2 - 8q + 1$
37. $10x^4 + 14x^3 + 12x^2 + 15x^3 + 21x^2 + 18x + 20x^2 + 28x + 24$
$10x^4 + 29x^3 + 53x^2 + 46x + 24$
38. $414$
39. $m^7$
40. $m^{10}$
Chapter 6 Summary

Product Rule of Exponents

\[ x^a \cdot x^b = x^{a+b} \]

- \((x^3)(x^4) = x^7\)
- \(9a^2(-2a^3) = -18a^5\)
- \(-3xy^4(-5x^3y^3) = 15x^4y^6\)

Power Rule of Exponents

\[(x^a)^b = x^{ab}\]

- \((x^2)^3 = x^{12}\)
- \((3^5)^2 = 3^{10}\)
- \((x^2y^5)^6 = x^{12}y^{30}\)

Add and Subtract Polynomials

- Remove any parentheses
- Combine like terms

\[
(5x^2 + 3x - 7) - (4x^2 - 6x - 10)
5x^2 + 3x - 7 - 4x^2 + 6x + 10
x^2 + 9x + 3
\]

Evaluate Polynomials

- \(2x^2 + 7x + 1\) for \(x = -5\)

\[
2(-5)^2 + 7(-5) + 1
2(25) + 7(-5) + 1
50 - 35 + 1
16
\]

Multiply Polynomials

- Apply the distributive property; combine like terms as necessary

\[
(2x + 9)(7x - 3)
2x(7x - 3) + 9(7x - 3)
14x^2 - 6x + 63x - 27
14x^2 + 57x - 27
\]
Chapter 6 Review

Simplify. Write polynomials in descending order.

1. \(a^4(a^4)\)  
2. \(x^2y^7(x^3y^5)\)
3. \(2m^3(5m^6)\)  
4. \(-4k^2(5k)\)
5. \((-4A^2)(A^4B^3)(2A^2B^3)\)  
6. \((y^2)^6\)
7. \((x^3y^4)^5\)  
8. \((2a)^5\)
9. \((3m^6)^3\)  
10. \((x^{14})^2(x^5)^3\)
11. \((3x^5y^4z^3)^2(-x^3y^4z^5)^4\)  
12. \(\frac{9b^2c^3}{27b^5c^7}\)
13. \(5K - 3K^2 + 7 - 3K^2 + 7K - 7 - 2K^2\)
14. \(4(3y^2 + 4y - 3) - 2(-y + 5 - 9y^2)\)
15. \(-4y(4y^2 - 2y + 10)\)
16. \((3x + 2)(4x - 3)\)
17. \((5X - 7)(5X + 7)\)
18. \((3a - 2)^2\)
19. \((x^2 - 2x + 3)(x - 1)\)

Evaluate.

20. \(4x^3 - 3x + 4\) when \(x = -2\)
21. \(x^{20} - x^{10}\) when \(x = -1\)
**Answer Key for Chapter 6 Review.**

<table>
<thead>
<tr>
<th>Question</th>
<th>Answer</th>
</tr>
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<tbody>
<tr>
<td>1.</td>
<td>$a^7$</td>
</tr>
<tr>
<td>2.</td>
<td>$x^3y^{12}$</td>
</tr>
<tr>
<td>3.</td>
<td>$10m^9$</td>
</tr>
<tr>
<td>4.</td>
<td>$-20k^3$</td>
</tr>
<tr>
<td>5.</td>
<td>$-8A^8B^8$</td>
</tr>
<tr>
<td>6.</td>
<td>$y^{12}$</td>
</tr>
<tr>
<td>7.</td>
<td>$x^{15}y^{20}$</td>
</tr>
<tr>
<td>8.</td>
<td>$2^5a^3$ or $32a^5$</td>
</tr>
<tr>
<td>9.</td>
<td>$3^3m^{18}$ or $27m^{18}$</td>
</tr>
<tr>
<td>10.</td>
<td>$x^{43}$</td>
</tr>
<tr>
<td>11.</td>
<td>$9x^{24}y^{24}z^{26}$</td>
</tr>
<tr>
<td>12.</td>
<td>$\frac{1}{3b^2c^2}$</td>
</tr>
<tr>
<td>13.</td>
<td>$-8K^2 + 12K$</td>
</tr>
<tr>
<td>14.</td>
<td>$30y^2 + 18y - 22$</td>
</tr>
<tr>
<td>15.</td>
<td>$-16y^3 + 8y^2 - 40y$</td>
</tr>
<tr>
<td>16.</td>
<td>$12x^3 - x - 6$</td>
</tr>
<tr>
<td>17.</td>
<td>$25X^2 - 49$</td>
</tr>
<tr>
<td>18.</td>
<td>$9a^2 - 12a + 4$</td>
</tr>
<tr>
<td>19.</td>
<td>$x^3 - 3x^2 + 5x - 3$</td>
</tr>
<tr>
<td>20.</td>
<td>$-22$</td>
</tr>
<tr>
<td>21.</td>
<td>$0$</td>
</tr>
</tbody>
</table>

**Reference Section**

- Question 1: p1
- Question 2: p1
- Question 3: p1
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- Question 20: p2
- Question 21: p2

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Chapter 6 Test

Simplify. As much as possible, write results with bases and exponents.

1. \((2^5)(2^6)\)  \hspace{2cm} 2. \(3^4 \cdot 3^5 \cdot 3\)
3. \(x^3 \cdot x^4\)  \hspace{2cm} 4. \(-3y^2z^2(2y^3z^5)\)
5. \((x^3)^2\)  \hspace{2cm} 6. \((x^2y^3)^4\)
7. \((2x^3)^2\)  \hspace{2cm} 8. \((4x^3y^4z)^3\)
9. \((5x^2y^3)^3(2x^3y)^4\)  \hspace{2cm} 10. \(\frac{18x^2y^5}{6x^2y^2}\)

State whether the polynomial is a monomial, binomial, or trinomial.

11. \(-4x^2 + 3x - 1\)  \hspace{2cm} 12. \(4y + 5\)
13. \(3x^2\)

Simplify. Write in descending order.

14. \(3a + 2a^2 - 7 - 5a^2 + 7 + 2a\)
15. \((5x^2 + 3x + 7) + (-3x^2 - 2x + 1)\)
16. \((7y^2 - 4y - 3) - (y^2 - 5y + 4)\)
17. \(2(3x - 1) + 3(2x^2 + x - 1)\)

Evaluate.

18. \(4x^2 - 3x + 1\) when \(x = -2\)
Simplify.

19. $3x(-2x)$
20. $-(4y - 7)$
21. $4a(3a + 5)$
22. $6b(b - 4)$
23. $(x + 2)(x + 3)$
24. $(y - 3)(y + 2)$
25. $(3z + 2)(4z - 1)$
26. $(7z - 2)(3z - 4)$
27. $(2x - 3)^2$
28. $(3x + 5)^2$
29. $(x + 4)^2$
30. $(x - 7)^2$
Answer Key for Chapter 6 Test.

1. \(2^{11}\)  
2. \(3^{10}\)
3. \(x^9\)  
4. \(-6y^5z^{13}\)
5. \(x^6\)  
6. \(x^8y^{20}\)
7. \(4x^6\)  
8. \(64x^9y^{12}z^3\)
9. \((16)(125)x^{18}y^{19} = 2000x^{18}y^{19}\)  
10. \(\frac{3y^3}{x}\)
11. trinomial  
12. binomial
13. monomial  
14. \(-3a^2 + 5a\)
15. \(2x^2 + x + 8\)  
16. \(6y^2 + y - 7\)
17. \(6x^2 + 9x - 5\)  
18. 23
19. \(-6x^2\)  
20. \(-4y + 7\)
21. \(12a^2 + 20a\)  
22. \(6b^2 - 24b\)
23. \(x^2 + 5x + 6\)  
24. \(y^2 - y - 6\)
25. \(12z^2 + 5z - 2\)  
26. \(21z^2 - 34z + 8\)
27. \(4x^2 - 12x + 9\)  
28. \(9x^2 + 30x + 25\)
29. \(x^2 + 8x + 16\)  
30. \(x^2 - 14x + 49\)
Cumulative Review #6 Through Chapter 6

1. Reduce: \[ \frac{35x^3y^5}{-28xy^3} \]

Simplify.

2. \[ (8 - 4)^3 \cdot 2 \]

3. \[ -4x - 3x + (-2x) - (-7x) + x \]

4. \[ -2 - 8 - \left( \frac{-12}{4} \right) + (3^2 - 4)^2 \]

5. \[ -16 \div 4 - (3 - 4^2) + (5 - 2)^2 \]

6. \[ \frac{-35a + 56}{-7} \]

7. \[ \frac{18a + 13b + 6}{11} - \frac{3a - 17b + 9}{11} \]

8. \[ \frac{8}{9} - \frac{1}{6} + \frac{x}{3} \]

9. \[ -\frac{3}{5} \left( -1\frac{1}{6} \right) \]

10. \[ \left( -\frac{2}{5} \right)^4 \]

11. \[ 3.8 - (3.2)^2 + 1.8(-0.2) \]

12. \[ (-3x^3)(x^3y^2)(-2xy^3) \]

13. \[ (a^2)^2 (a^3)^3 \]

14. \[ \frac{14x^2y^5}{-42xy^3} \]
Simplify.

15. \(4(a^2 + 3a - 2) - 5(-a - 8 + a^2)\)

16. \((x^2 - x + 1)(x + 1)\)

17. \(-3x(2x^2 - 3x + 8)\)

Evaluate.

18. \(-7x - 8\) for \(x = -1\)

19. \(3x^2 - 2x + 1\) when \(x = -2\)

Solve.

20. \(-4r - 3 + 2r = 7\)

21. \(4x - 3(x + 2) = 7(-x + 3) - 3\)

22. \(4(-3x + 3) - 2(-4x - 2) = -12\)

23. \(7(2a + 1) = 11\)

24. \(\frac{1}{4}x - \frac{1}{8} = \frac{2}{3}\)

25. \(\left(\frac{2}{3}\right)x + 1 = \frac{5}{6}\)

26. \(\frac{3}{7}x - \frac{5}{2} = \frac{5}{21}x + \frac{1}{6}\)

27. \(0.7T - 0.5 = 73\)

28. Four times the quotient of a number and 8 equals 36. Find the number.

29. Jennifer’s phone and internet bills added to $92.38. If the phone bill was $18.42 more than the internet bill, find the amount of each bill.

30. Fourteen of the 56 fish in a tank were guppies. What percent were guppies?
Answer Key for Cumulative Review #6 through Chapter 6.

Reference Chapter

1. \(-\frac{5x^3y^2}{4}\)
2. 32
3. \(-x\)
4. 18
5. 18
6. 5a \(- 8\)
7. \(\frac{15a + 30b - 3}{11}\)
8. \(\frac{6x + 13}{18}\)
9. \(\frac{7}{10}\)
10. \(\frac{16}{625}\)
11. \(-6.8\)
12. \(6x^7y^4\)
13. \(a^{13}\)
14. \(-\frac{xy^3}{3}\)
15. \(-a^2 + 17a + 32\)
16. \( x^3 + 1 \)  
17. \(-6x^3 + 9x^2 - 24x\)  
18. \(-1\)  
19. \(17\)  
20. \(r = -5\)  
21. \(x = 3\)  
22. \(x = 7\)  
23. \(a = \frac{4}{14} = \frac{2}{7}\)  
24. \(x = \frac{19}{6} \text{ or } 3\frac{1}{6}\)  
25. \(x = -\frac{1}{4}\)  
26. \(x = 14\)  
27. \(T = 105\)  
28. the number = \(x\)  
\[4 \left( \frac{x}{8} \right) = 36\]  
\(x = 72\)  
The number is 72.  
29. The internet bill was $36.98; the phone bill was $55.40.  
30. 25% of the fish were guppies.
Chapter 7: Graphing

Chapter 7 presents ideas about graphing, using two number lines. These number lines are the x-axis and y-axis of the rectangular coordinate system.

You will begin the chapter with vocabulary and plotting points. Then you will be introduced to graphing equations of lines.

Your goal is to become familiar with the rectangular coordinate system, plotting points, and graphing lines. As you are learning the new ideas, you will also use some skills from previous chapters, including signed numbers, evaluation, and order of operations.
Section 1: Plotting Points

The **rectangular coordinate system** consists of two number lines. The horizontal number line is called the **x-axis**. The vertical number line is called the **y-axis**. The x-axis and y-axis are perpendicular (they intersect in a 90° angle). The point where they intersect is called the **origin**. Observe.

A **point** \((x, y)\) consists of two numbers in \((x, y)\) order, where \(x\) and \(y\) are the **coordinates** of the point, and where the parentheses must be written. Since order matters, points are also called **ordered pairs**. The \(x\) coordinate describes movement right (for positive numbers) or left (for negative numbers) from the origin. From there, the \(y\) coordinate describes movement up (for positive numbers) or down (for negative numbers). Therefore, a point \((x, y)\) is essentially a set of instructions for movement from the origin to get to an ending location.

To **plot a point** means to find the ending location after moving as indicated by the \(x\) and \(y\) coordinates. Observe. Notice on all examples that points are written with parentheses.

a. Plot \((0, 0)\)

From the origin, don’t move left or right and don’t move up or down. Stay at the origin! Therefore, the **origin** is also called the \((0, 0)\) point.
b. Plot (2, 4)

The x coordinate indicates moving 2 lines to the RIGHT of the origin; then the y coordinate indicates moving UP 4 lines.

c. Plot (3, −6)

The x coordinate indicates moving 3 lines to the RIGHT of the origin; then the y coordinate indicates moving DOWN 6 lines.

d. Plot (−1, 5)

From the origin, move 1 line LEFT and then 5 lines UP.
**e. Plot (-7, -4)**

From the origin, move 7 lines LEFT and then 4 lines DOWN.

![Graph showing point (-7, -4)](image)

**f. Plot (2, 0)**

From the origin, move 2 lines RIGHT and don’t move up or down.

![Graph showing point (2, 0)](image)

**g. Plot (-5, 0)**

From the origin, move 5 lines LEFT and don’t move up or down.

![Graph showing point (-5, 0)](image)
h. Plot (0, 4)

From the origin, don’t move left or right and do move 4 UP.

![diagram showing point (0, 4)]

i. Plot (0, -7)

From the origin, don’t move left or right and do move 7 DOWN.

![diagram showing point (0, -7)]

j. Plot $\left(2\frac{1}{2}, 3\frac{1}{2}\right)$

From the origin, move $2\frac{1}{2}$ RIGHT and then $3\frac{1}{2}$ UP.

![diagram showing point $\left(2\frac{1}{2}, 3\frac{1}{2}\right)$]
k. State the coordinates of the following points.

A = (4, 1)
B = (−2, −3)
C = (0, 1)
D = (3, −2)
E = (−5, 4)
Chapter 7 Section 1  

Homework for Chapter 7 Section 1

1. The horizontal number line is called the ____ axis.
2. The vertical number line is called the ____ axis.
3. The (0, 0) point is also called the ____.
4. In the point (4, 9), what is the y coordinate?
5. In the point (−3, 6), what is the x coordinate?
6. The x coordinate indicates movement in what direction for a positive number? for a negative number?
7. The y coordinate indicates movement in what direction for a positive number? for a negative number?

Plot the points. You can reproduce the graph paper located at the beginning of this chapter.

8. (6, 2)  
9. (2, −5)  
10. (0, 4)  
11. (−3, 5)  
12. (1, 0)  
13. (−4, −6)  
14. (0, −7)  
15. (−2, 0)  
16. \( \left( \frac{5}{2}, \frac{-3}{2} \right) \)  
17. (0, 0)  
18. (−5, 6)  
19. (−8, −6)
Chapter 7 Section 1

Plotting points

State the coordinates of the following points.

20. A =
21. B =
22. C =
23. D =
24. E =
25. F =
26. G =
27. H =
28. I =
29. J =
Chapter 7 Section 1  

Plotting points

Simplify.

30. \( \frac{-24}{-4} \div 2 + 3(-5 + 1)^2 \)  
31. \( 4 + 3(2y + 9) - 30y \)

32. \( 7x^3y^8(-4x^2y^4) \)  
33. \( (3a^4b^2)^3 \)

34. \( (4x + 7)(3x + 2) \)  
35. \( (6m + 1)(2m - 5) \)

36. \( \frac{a^3b^4c^6}{a^5b^2c^7} \)  
37. \( \frac{-15k + 30}{3} \)

38. \( \frac{20a - 35}{11} - \frac{16a - 4}{11} \)  
39. \( \frac{5m}{6} + \frac{3m}{4} \)

Solve.

40. \( 4k + 7 - 6k + 11 = 30 \)  
41. \( 3(5y + 4) = 2(8y + 11) \)

42. \( 6z + 4 = 8 \)  
43. \( \frac{3}{4}x - \frac{2}{3} = \frac{1}{6}x - \frac{7}{9} \)

44. \( \left( \frac{2}{7} \right)x - 1 = 4 \)  
45. \( x + \frac{7}{18} = \frac{4}{9} \)
Answer Key for Chapter 7 Section 1.

1. x  
2. y  
3. origin  
4. 9  
5. −3  
6. right (positive number)  
left (negative number)  
7. up (positive number)  
don (negative number)
<table>
<thead>
<tr>
<th>Number</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>20.</td>
<td>A = (-7, 3)</td>
</tr>
<tr>
<td>21.</td>
<td>B = (6, 0)</td>
</tr>
<tr>
<td>22.</td>
<td>C = (-1, -6)</td>
</tr>
<tr>
<td>23.</td>
<td>D = (4, 8)</td>
</tr>
<tr>
<td>24.</td>
<td>E = (2, -2)</td>
</tr>
<tr>
<td>25.</td>
<td>F = (0, 3)</td>
</tr>
<tr>
<td>26.</td>
<td>G = (0, 0)</td>
</tr>
<tr>
<td>27.</td>
<td>H = (-5, -9)</td>
</tr>
<tr>
<td>28.</td>
<td>I = (-7, 0)</td>
</tr>
<tr>
<td>29.</td>
<td>J = (0, -7)</td>
</tr>
<tr>
<td>30.</td>
<td>51</td>
</tr>
<tr>
<td>31.</td>
<td>-24y + 31</td>
</tr>
<tr>
<td>32.</td>
<td>-28x^5 y^{12}</td>
</tr>
<tr>
<td>33.</td>
<td>27a^{12} b^{6}</td>
</tr>
<tr>
<td>34.</td>
<td>12x^2 + 29x + 14</td>
</tr>
<tr>
<td>35.</td>
<td>12m^2 - 28m - 5</td>
</tr>
<tr>
<td>36.</td>
<td>(\frac{b^2}{a^2 c})</td>
</tr>
<tr>
<td>37.</td>
<td>-5k + 10</td>
</tr>
<tr>
<td>38.</td>
<td>(\frac{4a - 31}{11})</td>
</tr>
<tr>
<td>39.</td>
<td>(\frac{19m}{12})</td>
</tr>
<tr>
<td>40.</td>
<td>k = -6</td>
</tr>
<tr>
<td>41.</td>
<td>y = -10</td>
</tr>
<tr>
<td>42.</td>
<td>z = (\frac{4}{6} = \frac{2}{3})</td>
</tr>
<tr>
<td>43.</td>
<td>x = -(\frac{4}{21})</td>
</tr>
<tr>
<td>44.</td>
<td>x = (\frac{35}{2})</td>
</tr>
<tr>
<td>45.</td>
<td>x = (\frac{1}{18})</td>
</tr>
</tbody>
</table>
Section 2: Graphing Lines

In the prior chapters, equations had one variable only (such as, $x + 3 = 5$ where $x = 2$). In this section you will be working with equations that contain two variables. For example, $y = 2x + 3$. These equations are called **linear equations in two variables**.

For the equation $y = 2x + 3$, two numbers are required for a solution. Is the ordered pair $(5, 13)$ a solution for that equation? Check by substitution:

\[
y = 2x + 3 \\
13 = 2(5) + 3 \\
13 = 10 + 3 \\
13 = 13
\]

This checks; therefore, $(5, 13)$ is a solution for the equation $y = 2x + 3$. Is the point $(-1, 1)$ a solution? Is the point $(0, 3)$ a solution? Yes, they are. There are infinitely many solutions for a linear equation in two variables.

If you plot all the points that are solutions to a linear equation in two variables, the plotted points form a straight line. This is why these are called linear equations.

In this section, you will learn to graph the lines of a linear equation when $y$ is isolated.

When the equation has $y$ isolated on the left of the equal sign, you should choose several numbers for $x$. Substitute those numbers in for $x$ and figure out the $y$ values. Plot the points and connect them with a straight line. To organize your information, make an $x$ and $y$ table of values. Place the $x$ values under the $x$ column; after you figure out the $y$ values, place those numbers under the $y$ column. Observe.
Chapter 7 Section 2

Graphing lines

a. \( y = 2x - 3 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>( y = 2x - 3 )</th>
<th>( y = 2x - 3 )</th>
<th>Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>-5</td>
<td>( y = 2(-1) - 3 = -2 - 3 = -5 )</td>
<td>( -1, -5 )</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>-3</td>
<td>( y = 2(0) - 3 = 0 - 3 = -3 )</td>
<td>( 0, -3 )</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>-1</td>
<td>( y = 2(1) - 3 = 2 - 3 = -1 )</td>
<td>( 1, -1 )</td>
<td></td>
</tr>
</tbody>
</table>

Now plot the points and connect with a straight line to show a “picture” of the line, which extends forever in both directions.

b. \( y = 2x - 3 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>( y = 2x - 3 )</th>
<th>( y = 2x - 3 )</th>
<th>Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-3</td>
<td>( y = 2(0) - 3 = 0 - 3 = -3 )</td>
<td>( 0, -3 )</td>
<td></td>
</tr>
<tr>
<td>-3</td>
<td>-9</td>
<td>( y = 2(-3) - 3 = -6 - 3 = -9 )</td>
<td>( -3, -9 )</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>( y = 2(2) - 3 = 4 - 3 = 1 )</td>
<td>( 2, 1 )</td>
<td></td>
</tr>
</tbody>
</table>

Notice you can choose any numbers for \( x \); then figure out your corresponding \( y \) values. The graph will be the same.
When you choose your x values, usually choose “easy” numbers to calculate with. Recall, two points are required to draw a line. However, a third point helps you check your work.

c. \( y = x + 4 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>( y = x + 4 )</th>
<th>Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>2</td>
<td>( y = (-2) + 4 = -2 + 4 = 2 )</td>
<td>(-2, 2)</td>
</tr>
<tr>
<td>0</td>
<td>4</td>
<td>( y = (0) + 4 = 0 + 4 = 4 )</td>
<td>(0, 4)</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>( y = (1) + 4 = 1 + 4 = 5 )</td>
<td>(1, 5)</td>
</tr>
</tbody>
</table>


d. \( y = -3x + 5 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>( y = -3x + 5 )</th>
<th>Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>8</td>
<td>( y = -3(-1) + 5 = 3 + 5 = 8 )</td>
<td>(-1, 8)</td>
</tr>
<tr>
<td>0</td>
<td>5</td>
<td>( y = -3(0) + 5 = 0 + 5 = 5 )</td>
<td>(0, 5)</td>
</tr>
<tr>
<td>2</td>
<td>-1</td>
<td>( y = -3(2) + 5 = -6 + 5 = -1 )</td>
<td>(2, -1)</td>
</tr>
</tbody>
</table>
Chapter 7 Section 2

Graphing lines

e. \( y = -x - 1 \)

Remember: this means \( y = -1x - 1 \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>( y = -1x - 1 )</th>
<th>Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-1</td>
<td>( y = -1(0) - 1 = 0 - 1 = -1 )</td>
<td>(0, -1)</td>
</tr>
<tr>
<td>1</td>
<td>-2</td>
<td>( y = -1(1) - 1 = -1 - 1 = -2 )</td>
<td>(1, -2)</td>
</tr>
<tr>
<td>2</td>
<td>-3</td>
<td>( y = -1(2) - 1 = -2 - 1 = -3 )</td>
<td>(2, -3)</td>
</tr>
</tbody>
</table>

f. \( y = 2x \)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
<th>( y = 2x )</th>
<th>Points</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>-2</td>
<td>( y = 2(-1) = -2 )</td>
<td>(-1, -2)</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>( y = 2(0) = 0 )</td>
<td>(0, 0)</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>( y = 2(1) = 2 )</td>
<td>(1, 2)</td>
</tr>
</tbody>
</table>

To summarize: When \( y \) is isolated, choose ANY \( x \) values you want. Substitute into the equation and find the corresponding \( y \) value. This forms your points \((x, y)\) that you place on the graph. Connect the points to form a straight line, which extends forever in both directions.
Homework for Chapter 7 Section 2

Show your x and y table of values. Graph the line.

1. \( y = x + 3 \)
2. \( y = x - 1 \)

3. \( y = 3x \)
4. \( y = -2x \)

5. \( y = 2x + 2 \)
6. \( y = 3x - 5 \)

7. \( y = -3x + 4 \)
8. \( y = -2x - 1 \)

9. \( y = -x + 3 \)
10. \( y = -x - 2 \)

Simplify appropriately.

11. \( a^3 \cdot a^4 \)
12. \( y^3(y^3) \)

13. \( x^3 \cdot x \cdot x^2 \)
14. \( 4m^2(5m^3) \)

15. \(-8x^3y^4(2x^2y)\)
16. \((x^3)^3 \)

17. \((a^3)^4 \)
18. \((2x)^4 \)

19. \((4a^3)^3 \)
20. \((2a^4)^3(5a^3)^2 \)

21. \(4m^2 + 3m - 8 + 2m^2 - m - 3 \)
22. \((4m^2 + 3m - 8) - (2m^2 - m - 3) \)

23. \((x + 3)(x + 4)\)
24. \((2x - 5)(3x + 4)\)

25. \((x - 6)(2x - 7)\)
26. \((x + 3)^2 \)

27. \((x - 6)^2 \)
28. \((2x + 7)^2 \)

29. \((3x - 4)^2 \)

Solve.

30. \(2(9k + 1) = 4(4k + 5)\)
31. \(\frac{5}{9} x - \frac{7}{8} = \left(\frac{5}{36}\right)x + \frac{3}{4}\)

32. \(0.2m - 2.104 = -7.3\)
Answer Key for Chapter 7 Section 2.

The tables of values will vary.

1. \( y = x + 3 \)

2. \( y = x - 1 \)

3. \( y = 3x \)

4. \( y = -2x \)

5. \( y = 2x + 2 \)

6. \( y = 3x - 5 \)
Chapter 7 Section 2  
Graphing lines

7. \( y = -3x + 4 \)  
8. \( y = -2x - 1 \)

9. \( y = -x + 3 \) \( y = -1x + 3 \)  
10. \( y = -x - 2 \) \( y = -1x - 2 \)

11. \( a^3 \)  
12. \( y^8 \)
13. \( x^6 \)  
14. \( 20m^5 \)
15. \(-16x^4y^5 \)  
16. \( x^6 \)
17. \( a^{12} \)  
18. \( 16x^4 \)
19. \( 64a^6 \)  
20. \( 8a^{12} \cdot 25a^6 = 200a^{18} \)
21. \( 6m^2 + 2m - 11 \)  
22. \( 2m^3 + 4m - 5 \)
23. \( x^2 + 7x + 12 \)  
24. \( 6x^2 - 7x - 20 \)
25. \( 2x^2 - 19x + 42 \)  
26. \( x^2 + 6x + 9 \)
27. \( x^2 - 12x + 36 \)  
28. \( 4x^2 + 28x + 49 \)

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Chapter 7 Section 2

Graphing lines

29. \(9x^2 - 24x + 16\)

30. \(k = 9\)

31. clear; multiply by LCD 72
   \[40x - 63 = 10x + 54\]
   \[30x - 63 = 54\]
   \[30x = 117\]
   \[x = \frac{117}{30} = \frac{39}{10}\]

32. \(0.2m = -5.196\)
   \(m = -25.98\)
Chapter 7 Summary

x-axis  
Horizontal number line

y-axis  
Vertical number line

(0, 0)  
The origin; the point where the x-axis and y-axis intersect in a 90° angle

(x, y)  
A point (ordered pair) whose numbers x and y indicate movement from the origin

Plot a Point  
To find the ending location of a point (x, y) where x indicates movement right or left from the origin and, from there, where y indicates movement up or down

(2, 3) would indicate moving 2 units right of the origin and from there moving 3 units up

(-1, -8) would indicate moving 1 unit left of the origin and from there moving 8 units down

Graph a Line  
Two points are needed to graph a line; choose at least two separate x values to substitute into the equation and solve for their corresponding y values; plot and connect the points
Chapter 7 Review

1. The vertical number line is called the ______axis.

Plot the points on graph paper.

2. (4, -3) 3. (0, 5)
4. (-4, -2) 5. (-2.4, 3.1)
6. (0, 0) 7. (3, 0)
8. (2 ½, 4 ¼) 9. (-1, 4)

State the coordinates of the following points:

10. A = 11. B =
14. E = 15. F =
Show your x and y table of values. Graph the line.

16. \( y = -2x \)  
17. \( y = 4x - 5 \)  
18. \( y = -x + 3 \)  
19. \( y = 2x + 3 \)  
20. \( y = 3x - 3 \)
Answer Key for Chapter 7 Review.

Reference Section

1. y-axis

2-9. See the graph.

10. A = (-3, 2)

11. B = (2, -6)

12. C = (0, 5)

13. D = (4, 3)

14. E = (-9, -1)

15. F = (2, 0)
16-19. The tables of value will vary.

16.\[ y = -2x \]

17.\[ y = 4x - 5 \]

18.\[ y = -x + 3 \]
19. \( y = 2x + 3 \)

20. \( y = 3x - 3 \)
Chapter 7 Test

1. Circle the correct choice: The x-axis is the (horizontal, vertical) number line in a rectangular coordinate system.

2. Fill in the blanks: The center (origin) is labeled (____, ____).

3. In the point (−2, −3), what is the y-coordinate?

4. Circle the correct choice: A negative x coordinate indicates movement to the (left, right).

Plot the points.

5. (2, 3)  6. (4, −3)

7. (−5, 1)  8. (−2, −3)

9. (0, 0)  10. \( \left(2\frac{1}{2}, 0\right) \)

Show your x and y table of values. Graph.

11. \( y = x − 2 \)  12. \( y = x + 4 \)

13. \( y = 2x \)  14. \( y = −3x \)

15. \( y = −2x − 3 \)
**Answer Key for Chapter 7 Test.**

1. horizontal .......................................................... 2. (0, 0)

3. $-3$ is the y-coordinate ........................................ 4. left

Answers for problems 5 through 10 on the graph:

![Graph with points labeled 5, 6, 7, 8, 9, 10]
For 11 through 15, the tables of value will vary.

11. \( y = x - 2 \)

12. \( y = x + 4 \)

13. \( y = 2x \)

14. \( y = -3x \)

15. \( y = -2x - 3 \)
Cumulative Review #7 Through Chapter 7

1. a. Is \(-3\) an integer?
   b. Is \(-3\) a whole number?

Simplify.

2. \[
\frac{2a^3 \cdot 14b^4}{7b^5} \cdot \frac{20a^5}{14}
\]

3. \[
-28 \div 4 - 5(-4 + 3) - 6^2
\]

4. \[
\left(\frac{14}{7}\right)(-4) + 8(6 - 9)^2
\]

5. \[
\frac{18m - 12}{7} + \frac{-2m - 9}{7}
\]

6. \[
\frac{-3x}{14} + \frac{4x}{21}
\]

7. \[
\frac{xy^3}{x^2} \div \frac{-3x^3y}{xy^2}
\]

8. \[
-4 \frac{1}{4} + 2 \frac{1}{2} - 3 \frac{1}{3}
\]

9. \[
(-3x^3y^2z^3)^2 (2x^2y^3z)^3
\]

Simplify.

10. \[
3y - 2y^2 + y - 7y + 9y^2 - 2
\]

11. \[
(3x + 2)(2x - 3)
\]

Evaluate.

12. \[
-5x - 7.3 \text{ for } x = -1.9
\]

13. \[
a^5 - a^4 \text{ when } a = -1
\]
Solve.

14. \(-4x - 7 = 9\)

15. \(6 - (3 - 2x) - 2x = -(3x + 3)\)

16. \(9(2x - 8) - 7(3x + 2) = -(x - 6)\)

17. \(5y + 6 = 12y + 10\)

18. \(\frac{1}{2}m + \frac{2}{3} = \frac{5}{6}\)

19. \(2.8a + 3.5 = -4.2\)

20. A coat costs 6 times as much as a jacket, and a sweater costs twice as much as the jacket. It the total cost of the three items is $171, find the cost of the jacket.

21. The volume of a certain cube is \(\frac{3}{8}\) the volume of a particular sphere. If the volume of the cube is 24 cubic inches, find the volume of the sphere.

22. Three students in a class received an A on an English essay. If this represents 15% of the class, how many students are in the class?

23. Plot the points on graph paper:
   a. \((-3, 2)\)
   b. \((0, -2)\)

Show your x and y table of values. Graph the line.

24. \(y = x + 2\)

25. \(y = -2x - 2\)
Answer Key for Cumulative Review #7 through Chapter 7.

Reference Chapter

1. a. yes
   b. no

2. \( \frac{1}{5a^3b} \)

3. -38

4. 64

5. \( \frac{20m - 3}{7} \)

6. \( -\frac{x}{42} \)

7. \( -\frac{y^6}{3x^3} \)

8. \( -\frac{61}{12} \) or \( -5\frac{1}{12} \)

9. \( 72x^{12}y^{19}z^{13} \)

10. \( 7y^2 - 3y - 2 \)

11. \( 6x^2 - 5x - 6 \)

12. 2.2

13. -2
14. \( x = -4 \) 
15. \( x = 2 \) 
16. \( x = -23 \) 
17. \( y = -\frac{4}{7} \) 
18. \( m = \frac{1}{3} \) 
19. \( a = -2.75 \) 
20. The cost of the jacket is $19. 
21. The volume of the sphere is 64 cubic inches. 
22. There are 20 students. 

See the graph for 23.
The tables of value for 24 and 25 will vary.

24. \[ y = x + 2 \]

25. \[ y = -2x - 2 \]
Feedback Form

If you have any feedback about this book that you would like to share, please send that feedback to:

Michelle Wyatt
College of Southern Nevada– Sort Code S1A
3200 E Cheyenne
N Las Vegas, NV  89030

You may also e-mail any feedback:  michelle.wyatt@csn.edu

Feedback could include your thoughts about why the book helped you; how the book could be improved; any typographical errors; any errors in the answer keys; or any other idea you wish to share.

If you would like, feel free to copy this page and write your comments on this page. Then give the page to your instructor, who can send it by campus mail to the Mathematics Department, Sort Code S1A, Attn: Michelle Wyatt.
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